Density of dark matter in R^2 gravity

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based on joint works with A.D. Dolgov and R.S. Singh
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Hot Topics in Modern Cosmology

Spontaneous Workshop XIV

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Outline

- Dark Matter Mystery
- Cosmological evolution in R^2-gravity
- Kinetics and freezing of massive stable relics in cosmic plasma and bounds on the masses of DM particles
  - Minimally coupled scalars mode
  - Massive fermions mode
  - Gauge bosons mode
- Possible observational manifestations
- Conclusions

Dark Matter Mystery

- invisible form of matter disclosing itself through its gravitational action
- electrically neutral, since doesn't scatter light
- properties are practically unknown

Particles of many different types can be DM candidates

The fractional mass density of dark matter:

$$\Omega_{DM} = \frac{\varrho_{DM}}{\varrho_{crit}} \approx 0.265$$

The critical energy density of the universe:

$$\varrho_{crit} = \frac{3H_0^2 M_{Pl}^2}{8\pi} \approx 5\ \mathrm{keV/cm^3}, \quad M_{Pl} = 1.22 \cdot 10^{19} GeV = 2.18 \cdot 10^{-5} g$$

H_0 is the present day value of the Hubble parameter:

$$H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \approx 70\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$$

The observed mass density of DM in contemporary universe:

$$\varrho_{DM} \approx 1\,\mathrm{keV/cm^3}$$

Observational Evidence and Possible Carriers of DM

Independent pieces of data:

- flat rotational curves around galaxies;
- equilibrium of hot gas in rich galactic clusters;
- the spectrum of the angular fluctuations of Cosmic Microwave Background (CMB) Radiation;
- onset of Large Scale Structure (LSS) formation at the redshift $z_{LSS} = 10^4$ prior to hydrogen recombination at $z_{rec} = 1100$.

Possible carriers of dark matter:

- WIMP (Weakly Interacting Massive Particle): axions ($\sim 10^{-5}$ eV), heavy neutral leptons ($\sim$ GeV), particles of mirror matter, the Lightest Supersymmetric Particle (LSP), ...
- MACHO (Massive Astrophysical Compact Halo Object): primordial black holes (PBH) (from 10^{20} g up to tenth $M_\odot$), topological or non-topological solitons, possible macroscopic objects consisting e.g. from the mirror matter, ...

SUSY Dark Matter

Low energy minimal SUSY model:

- Predicts the existence of stable LSPs with mass $M_{LSP} \sim 100$–1000 GeV
- No manifestation at LHC $\Longrightarrow$ restricted parameter space open for SUSY

The LSP's energy density

$$\varrho_{LSP} \sim \varrho_{DM}^{(obs)} (M_{LSP}/1\ TeV)^2, \quad \varrho_{DM}^{(obs)} \approx 1\ keV/cm^3$$

- For $M_{LSP} \sim 1$ TeV, ϱ_{LSP} is of the order of the observed DM energy density
- For larger masses LSPs would overclose the universe.

LSPs are practically excluded as DM particles in the conventional cosmology.

In $(R + R^2)$-gravity the energy density of LSPs may be much lower $\Longrightarrow$ it reopens for them the chance to be the dark matter, if $M_{LSP} \geq 10^6$GeV.

- EA, A. D. Dolgov and R. S. Singh, "Dark matter in $R + R^2$ cosmology," JCAP **04** (2019) 014, arXiv:1811.05399 [astro-ph.CO]

General Relativity (GR):

$$S_{EH} = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\,R$$

Beyond the frameworks of GR:

$$S_{tot} = -\frac{M_{Pl}^2}{16\pi}\int d^4x\sqrt{-g}\left[R-\frac{R^2}{6M_R^2}\right]+S_m$$

Magnitude of temperature fluctuations of CMB demands $M_R \approx 3\cdot 10^{13}$ GeV.

- R^2-term leads to exponential cosmological expansion (Starobinsky inflation).
- It creates considerable deviation from the Friedmann cosmology in the post-inflationary epoch. (EA, A.D. Dolgov and R. Singh, "Distortion of the standard cosmology in $R+R^2$ theory," JCAP **1807** (2018) no.07, 019)
- Kinetics of massive species and the density of DM particles differ significantly from those in the conventional cosmology.

The modified Einstein equations:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R-\frac{1}{3M_R^2}\left(R_{\mu\nu}-\frac{1}{4}Rg_{\mu\nu}+g_{\mu\nu}D^2-D_\mu D_\nu\right)R=\frac{8\pi}{M_{Pl}^2}T_{\mu\nu}$$

FLRW: $ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2\right], \;\; H = \dot{a}/a$

Trace equation:

$$D^2 R + M_R^2 R = -\frac{8\pi M_R^2}{M_{Pl}^2}\, T^\mu_\mu$$

For homogeneous field, $R = R(t)$, and with equation of state $P = w\varrho$:

$$\ddot{R} + 3H\dot{R} + M_R^2 R = -\frac{8\pi M_R^2}{M_{Pl}^2}(1 - 3w)\varrho$$

w is usually a constant parameter:

- non-relativistic: $w = 0$, relativistic: $w = 1/3$, vacuum-like: $w = -1$

The curvature scalar:

$$R = -6\dot{H} - 12H^2$$

The covariant conservation condition $D_\mu T^\mu_\nu = 0$ in FLRW-metric:

$$\dot{\varrho} = -3H(\varrho + P) = -3H(1 + w)\varrho$$

Equation for the Curvature Scalar Evolution

$$\ddot{R} + 3H\dot{R} + M_R^2 R = -\frac{8\pi M_R^2}{M_{Pl}^2}(1-3w)\varrho$$

- does not include the effects of particle production by the curvature scalar;
- is a good approximation at inflationary epoch, when particle production by $R(t)$ is absent, because R is large and friction is large, so $R \to 0$ slowly.

At some stage, when $\boldsymbol{H}$ becomes smaller than $\boldsymbol{M_R}$, $\boldsymbol{R}$ starts to oscillate efficiently producing particles.

- It commemorates the end of inflation, the heating of the universe, and the transition from the accelerated expansion (inflation) to a de-accelerated one.
- The latter resembles the usual Friedmann matter dominated expansion regime but differs in many essential features.

Curvature $\boldsymbol{R(t)}$ can be considered as an effective scalar field (scalaron) with the mass $\boldsymbol{M_R}$ and with the decay width $\boldsymbol{\Gamma}$.

Dimensionless Equations

Dimensionless time variable and dimensionless functions

$$\tau = tM_R, \quad H = M_R h, \quad R = M_R^2 r, \quad \varrho = M_R^4 y$$

The system of dimensionless equations

$$\boldsymbol{r'' + 3hr' + r = -8\pi\mu^2(1-3w)y}$$
$$\boldsymbol{h' + 2h^2 = -r/6}$$
$$\boldsymbol{y' + 3(1+w)hy = 0}$$

- prime denotes derivative over $\boldsymbol{\tau}$, $\boldsymbol{\mu} = M_R/M_{Pl}$

Inflationary stage: $y = 0$ $(\varrho = 0)$ – "empty" Universe.

Inflation: $a(t) \sim \exp(\int_0^t h\, dt')$, $a_{inf} = \exp(N_e)$

With sufficiently large initial value of R, the devoid of matter universe would expand quasi exponentially long enough to provide solution of flatness, horizon and homogeneity problems existing in Friedmann cosmology.

The initial conditions should be chosen in such a way that at least 70 e-foldings during inflation are ensured:

$$N_e = \int_0^{\tau_{inf}} h\, d\tau \geq 70$$

- τ_{inf} is the moment when inflation terminated.
- $N_e \geq 70$ can be achieved if the initial value of $\boldsymbol{r}$ is sufficiently large, practically independently on the initial value of $\boldsymbol{h}$.

Simplified system to estimate the duration of inflation ($y = 0$):

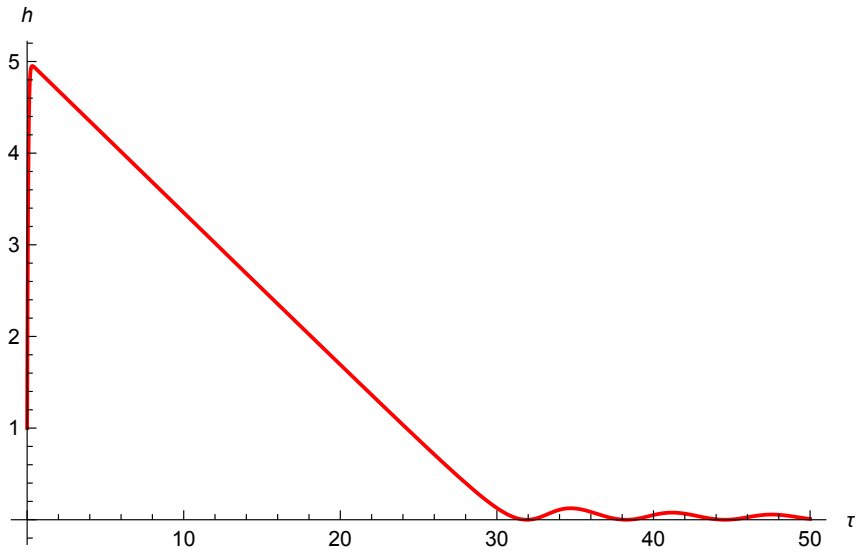
$$h^2 = -r/12, \qquad 3hr' = -r$$

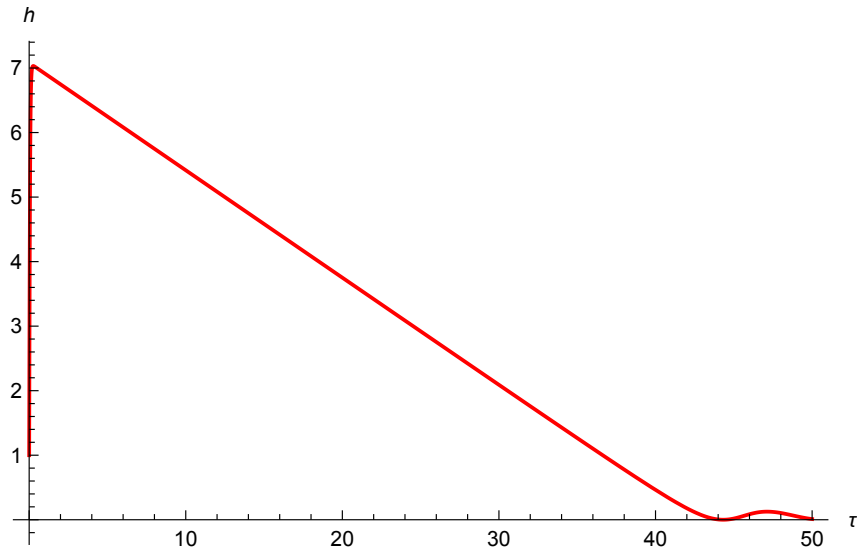
The duration of inflation is roughly determined by the condition $\boldsymbol{h = 0}$, i.e.

$$\boldsymbol{\tau_{inf} = \sqrt{-3r_0} \;\Rightarrow\; N_e \approx |r_0|/4, \quad r_0 = r(\tau = 0)}$$

Inflationary stage: numerical solutions

Evolution of $\boldsymbol{h(\tau)}$ at inflation





- Initial values of dimens-less curvature $|r_0| = 300$ (left) and $|r_0| = 600$ (right).
- The numbers of e-foldings: $\boldsymbol{N_e \approx |r_0|/4 = 75}$ (left) and **150** (right).

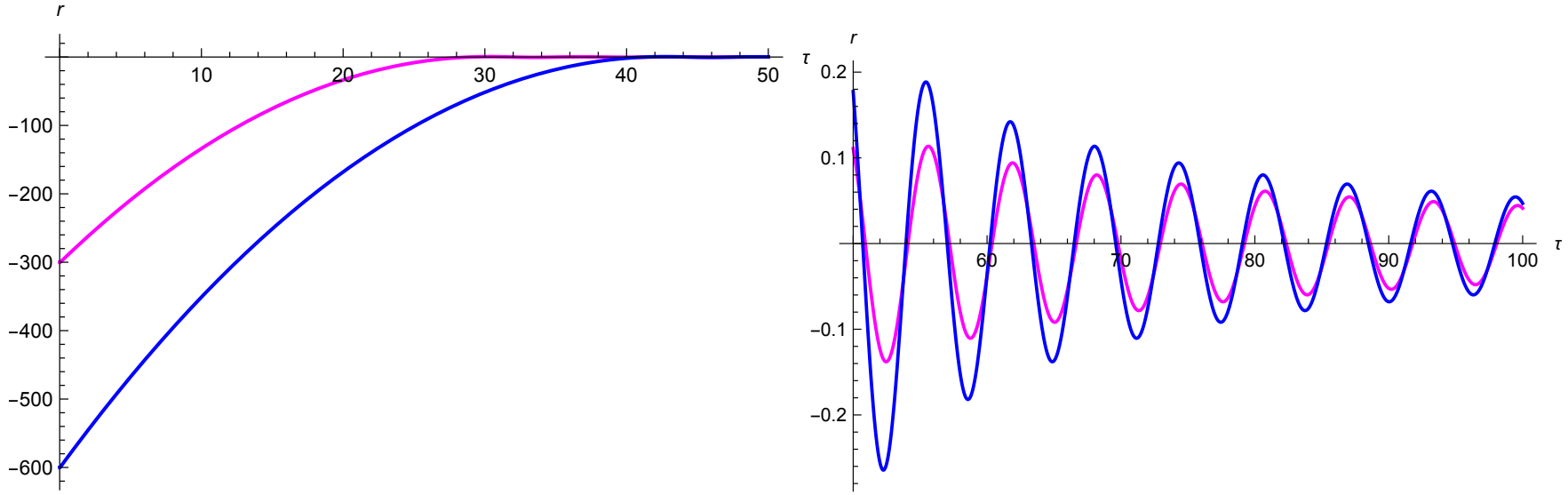
The number of e-folding is equal to the area of the triangle below the line $h(\tau)$.

It is an excellent agreement with the numerical solutions.

After $h(t)$ reaches 0 it started to oscillate with the amplitude decreasing as $2/(3t)$; the exponential rise of $a(t)$ turns into a power law one.

Inflationary stage: numerical solutions

Evolution of the dimensionless curvature scalar $r(\tau)$ for $r_{in} = -300$ (magenta) and $r_{in} = -600$ (blue)



- *Left panel*: evolution during inflation.
- *Right panel*: evolution after the end of inflation, the curvature scalar starts to oscillate (scale differs from the left graph).

Curvature oscillations lead to the creation of particles and to the heating of the Universe.

Universe Heating in R^2-gravity

Particle production: friction term approximation for harmonic oscillations of $R(t) \Longrightarrow \Gamma\dot{R}$-term

$$\ddot{R} + (3H + \Gamma)\dot{R} + M_R^2 R = -\frac{8\pi M_R^2}{M_{Pl}^2}(1 - 3w)\varrho$$

Particle production leads to an emergence of the source term in Eq. for ϱ:

$$\dot{\varrho} + 3H(1 + w)\varrho = \bar{S}[R] \neq 0.$$

The system of dimensionless equations:

$$h' + 2h^2 = -r/6$$
$$r'' + (3h + \gamma)r' + r = -8\pi\mu^2(1 - 3w)y$$
$$y' + 3(1 + w)hy = S[r]$$

- $\mu = M_R/M_{Pl}$, $\Gamma = M_R\gamma$.

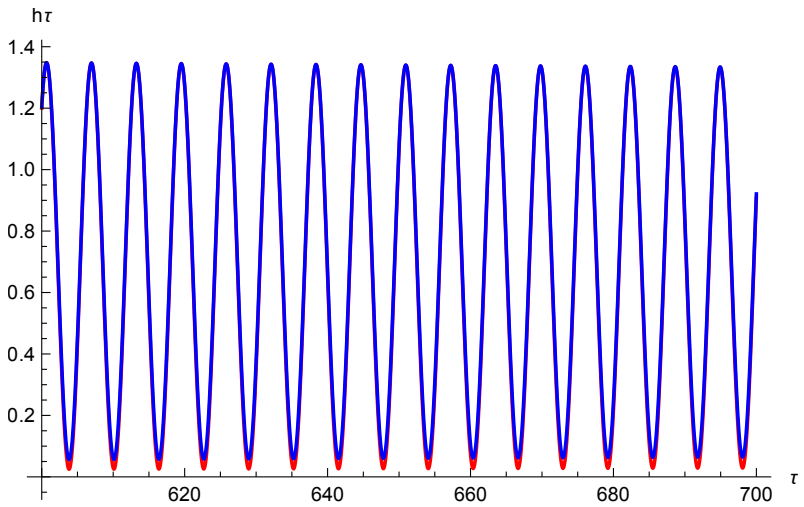
The value of Γ depends on the decay channel of the scalaron

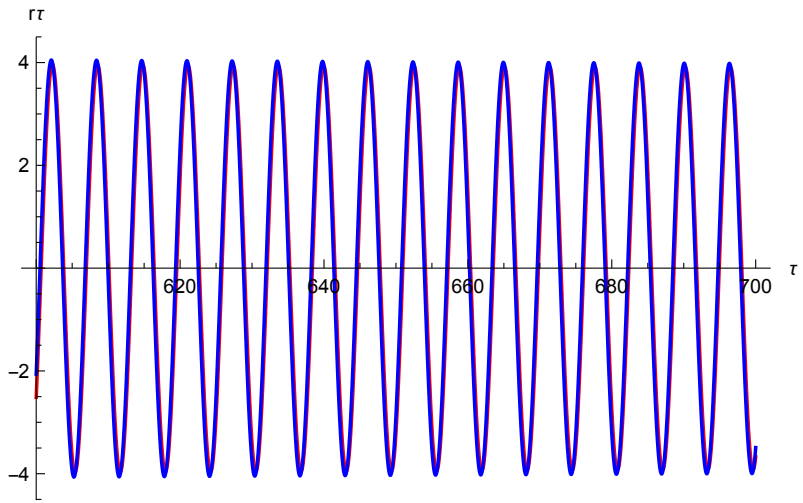
Evolution of $H(t)$ and $R(t)$ at post-inflationary stage

Asymptotic solutions:

$$h = \frac{2}{3\tau}\left[1 + \sin\left(\tau + \theta\right)\right], \quad r = -\frac{4\cos(\tau + \theta)}{\tau} - \frac{4}{\tau^2}$$

Comparison of numerical calculations with analytical estimates for the adjusted "by hand" phase $\theta = -2.9\pi/4$





- *Left panel*: comparison of numerical solution for $h\tau$ (red) with analytic estimate (blue). *Right panel*: the same for numerically calculated $r\tau$.

The difference between the red and blue curves is not observable.

Cosmological evolution in R^2-modified gravity: 4 distinct epochs

EA, A. Dolgov, R. Singh, *R^2-Cosmology and New Windows for Superheavy Dark Matter*, Symmetry 13 (2021) 5, 877

1. Inflation: R slowly decreases from large value $R/M_R^2 \gtrsim 10^2$ down to zero
2. Curvature oscillations:

$$R(t) = 4M_R \frac{\cos(M_R t + \theta)}{t}$$

   leading to efficient particle production through the scalaron decay and consequently to the universe heating (scalaron dominated regime)
3. Transition of the scalaron domination regime to the dominance of the produced matter of mostly relativistic particles.
4. Transition to the conventional cosmology governed by the General Relativity.

We consider the epoch of the universe heating and calculate the freezing of the massive species, X, in plasma, created by scalaron decays into heavier particles:

- massless minimally coupled scalars
- massive fermions
- massless gauge bosons

Cosmological energy density for different decay channels

Scalaron decay into 2 massless scalars minimally coupled to gravity

$$\Gamma_s = \frac{M_R^3}{24 M_{Pl}^2}, \qquad \varrho_s = \frac{M_R^3}{240\pi t}$$

Scalaron decay into a pair of fermions with mass m_f:

$$\Gamma_f = \frac{M_R m_f^2}{6 M_{Pl}^2}, \qquad \varrho_f = \frac{M_R m_f^2}{240\pi t}$$

Scalaron decay into gauge bosons induced by the conformal anomaly:

$$\Gamma_{an} = \frac{\beta_1^2 \alpha^2 N}{96\pi^2} \frac{M_R^3}{M_{Pl}^2}, \qquad \varrho_{an} = \frac{\beta_1^2 \alpha^2 N}{4\pi^2} \frac{M_R^3}{120\pi t}$$

β_1 is the first coefficient of the beta-function, N is the rank of the gauge group
α is the gauge coupling constant (at high energies it depends upon the theory).

Much slower decrease of the energy density of matter than normally for relativistic matter is ensured by the influx of energy from the scalaron decay.

- Normally for relativistic matter: $\varrho \sim 1/a^4(t) \sim 1/t^{8/3}$, since $a(t) \sim t^{(2/3)}$ at SD.

Connection of the temperature with time: GR $\Longleftrightarrow$ R^2

In thermalized plasma with $\varrho_{therm} = \pi^2 g_* T^4/30$

$$\varrho_{GR} = \frac{3M_{Pl}^2}{32\pi t^2} = \frac{\pi^2 g_* T^4}{30} \quad \Longrightarrow \quad (tT^2)_{GR} = \left(\frac{90}{32\pi^3 g_*}\right)^{1/2} M_{Pl} = const$$

- g_* is the number of relativistic species in the plasma, $g_* \sim 100$.

R^2-theory:

$$\varrho_s = \frac{M_R^3}{240\pi t} = \frac{\pi^2 g_* T^4}{30} \quad \Longrightarrow \quad (tT^4)_s = \frac{M_R^3}{8\pi^3 g_*} = const$$

$$\varrho_{an} = 2.6 \cdot 10^{-2} \alpha_R^2 \frac{M_R^3}{t} \quad \Longrightarrow \quad (tT^4)_{an} = \frac{0.78}{\pi^2 g_*} \alpha_R^2 M_R^3 = const$$

- The coupling α_R is taken at the energies equal to the scalaron mass.

Correspondingly

$$\left(\frac{\dot{T}}{T}\right)_{GR} = -\frac{1}{2t} \qquad \left(\frac{\dot{T}}{T}\right)_{s;an} = -\frac{1}{4t}$$

Evolution of X-particles in thermal plasma

Freezing of massive species $X \Longrightarrow$ Zeldovich Eq., 1965 (Lee-Weinberg, 1977):

$$\dot{n}_X + 3Hn_X = -\langle\sigma_{ann}v\rangle\left(n_X^2 - n_{eq}^2\right), \;\; n_{eq} = g_s\left(\frac{M_X T}{2\pi}\right)^{3/2} e^{-M_X/T}$$

- $\langle\sigma_{ann}v\rangle$ is the thermally averaged annihilation cross-section of X-particles
- n_{eq} is their equilibrium number density, g_s is the number of spin states.

For annihilation of non-relativistic particles:

$$\langle\sigma_{ann}v\rangle = \sigma_{ann}v = \frac{\pi\alpha^2\beta_{ann}}{2M_X^2} \;\; \text{(S-wave)},$$

$$\langle\sigma_{ann}v\rangle = \frac{3\pi\alpha^2\beta_{ann}}{2M_X^2}\frac{T}{M_X} \;\; \text{(P-wave, Majorana fermions)}$$

- M_X is a mass of X-particle, $\boldsymbol{\alpha}$ is a coupling constant, in SUSY theories $\boldsymbol{\alpha \sim 0.01}$
- $\boldsymbol{\beta_{ann}}$ is a numerical parameter $\sim$ the number of annihilation channels, $\boldsymbol{\beta \sim 10}$.

We assume that direct X-particle production by $\boldsymbol{R(t)}$ is suppressed in comparison with inverse annihilation of light particles into $X\bar{X}$-pair.

Some comments

Two possible channels to produce massive stable X-particles:

- Directly through the scalaron decay into a pair $X\bar{X}$,
- By inverse annihilation of relativistic particles in thermal plasma.

Direct production of $X\bar{X}$-pair by scalaron gives

$$\varrho_X^{(0)} \approx \varrho_{DM} \approx 1\mathrm{keV}/\mathrm{cm}^3, \text{ if } M_X \approx 10^7 \text{ GeV}$$

"Catch-22":

- For such small mass thermal production results in too large $\boldsymbol{\varrho_X}$.
- For larger masses $\varrho_X^{(0)}$ would be unacceptably larger than ϱ_{DM}.

A possible way out:

- Since oscillating curvature scalar creates particles only in symmetric state, the direct production of $\boldsymbol{X}$-particles is forbidden, if they are Majorana fermions, which must be in antisymmetric state.

Scalaron decay into massless non-conformal scalars

Dimensionless Zeldovich equation

$$\frac{df}{dx} = -\frac{0.03 g_s \alpha^2 \beta_{ann}}{\pi^3 g_*} \left(\frac{M_R}{M_X}\right)^3 \frac{f^2 - f_{eq}^2}{x^5}, \quad n_X = n_{in} \left(\frac{a_{in}}{a}\right)^3 \boldsymbol{f}, \quad x = \frac{M_x}{T}$$

For $g_* = 100$, $\alpha = 0.01$, $\beta_{ann} = 10$, $M_R = 3 \times 10^{13}$ GeV, and $n_\gamma = 412/\text{cm}^3$

The present day energy density of the X-particles:

$$\boldsymbol{\varrho_X = M_X n_\gamma f_{fin} \approx 10^8 \left(\frac{10^{10}\text{GeV}}{M_X}\right)} \text{GeV/cm}^3$$

To be compared with the observed energy density of DM: $\boldsymbol{\varrho_{DM} \approx 1}$ keV/cm^3.

- X-particles must have mass $M_X \gg M_R$ to make reasonable DM density.
- If $\boldsymbol{M_X > M_R}$, then classical scalaron field can still create X-particles, but the probability of their production would be strongly suppressed $\Longrightarrow$ such LSP with the mass somewhat larger than $\boldsymbol{M_R}$ could successfully make the cosmological DM.

Scalaron decay into a pair of fermions

The decay width and the energy density:

$$\Gamma_f = \frac{M_R m_f^2}{6M_{Pl}^2}, \qquad \varrho_f = \frac{M_R m_f^2}{120\pi t}$$

The largest contribution into the cosmological energy density at scalaron dominated regime is presented by the decay into the heaviest fermion species.

We assume:

- The mass of the LSP is considerably smaller than the masses of the other decay products, $\boldsymbol{m_X < m_f}$, at least as $\boldsymbol{m_X \lesssim 0.1 m_f}$.
- The direct production of X-particles by $\boldsymbol{R(t)}$ can be neglected.

In such a case LSPs are dominantly produced by the secondary reactions in plasma, which was created by the scalaron production of heavier particles.

Kinetic equation for freezing of fermionic species

$$\frac{df}{dx} = -\frac{\alpha^2 \beta_{ann}}{2\pi^3 g_*} \frac{n_{in} M_R m_f^2}{m_X^6} \frac{f^2 - f_{eq}^2}{x^5}$$

$n_{in} = 0.09 g_s m_X^3$ is the initial number density of X-particles at $T \sim m_X$.

$$\varrho_X = m_X n_\gamma \left(\frac{n_X}{n_{rel}}\right)_{now} = 7 \cdot 10^{-9} \frac{m_f^3}{m_X M_R} \mathrm{cm}^{-3}$$

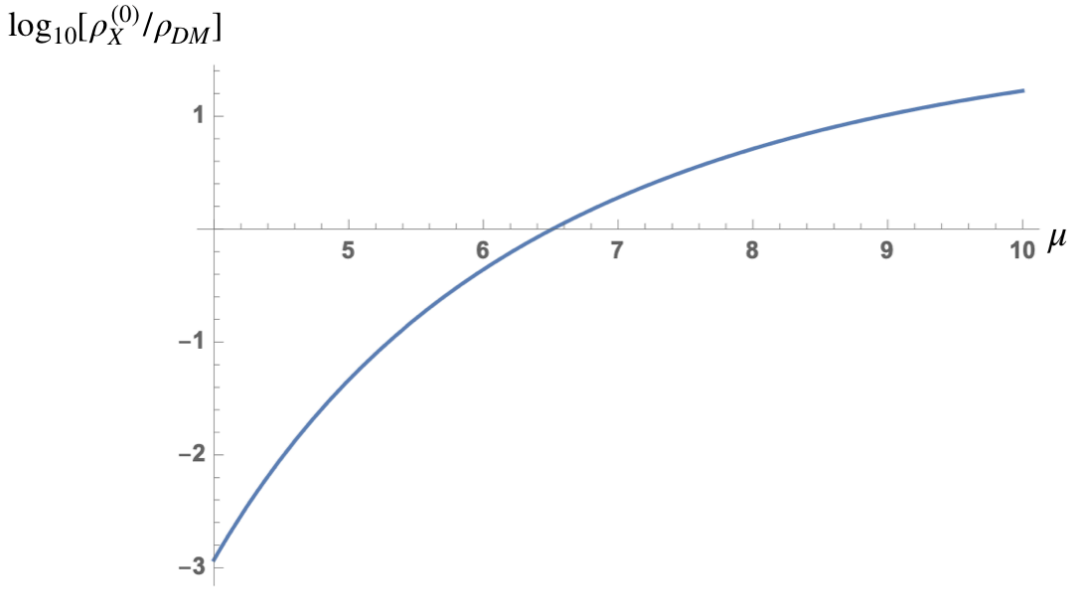
- $\alpha = 0.01$, $\beta_{ann} = 10$, $g_* = 100$, $n_\gamma \approx 412/\mathrm{cm}^3$, $n_{rel} \approx \varrho^{rel}/3T$
- If we take $m_f = 10^5$ GeV and $m_X = 10^4$ GeV, then $\varrho_X \ll \varrho_{DM}$.

ϱ_X becomes comparable with the energy density of the cosmological DM, $\varrho_{DM} \approx 1$ keV/cm^3, if $m_X \sim 10^6$ GeV, $m_f \sim 10^7$ GeV:

$$\varrho_X = 0.23 \left(\frac{m_f}{10^7\,\mathrm{GeV}}\right)^3 \left(\frac{10^6\,\mathrm{GeV}}{m_X}\right) \frac{\mathrm{keV}}{\mathrm{cm}^3}$$

Scalaron decay into gauge bosons due to conformal anomaly

- $X, \bar{X}$ are Majorana fermions $\Longrightarrow$ direct production by scalaron is forbidden.
- $X\bar{X}$-pairs are produced through the inverse annihilation of relativistic particles in the thermal plasma.



Log of the ratio of the energy density of X-particles to the observed energy density of DM as a function of $\mu = M_R/M_X$ calculated through the Zeldovich equation.

X-particles may be viable candidates for the carriers of the cosmological dark matter, if their mass $\boldsymbol{M_X \approx 5 \cdot 10^{12}}$ GeV.

Possible observations

According to our results, the mass of DM particles, with the interaction strength typical for supersymmetric ones, can be in the range from 10^6 to 10^{13} GeV.

Possibilities to make X-particles visible:

1. Annihilation effects in clusters of dark matter in galaxies and galactic halos, in which, according to

   - V. S. Berezinsky, V. I. Dokuchaev and Y. N. Eroshenko, *Small-scale clumps of dark matter*, *Phys. Usp.* **57** (2014) 1 [arXiv:1405.2204]

   the density of DM is many times higher than DM cosmological density.

2. The decay of superheavy DM particles, which could have a lifetime long enough to manifest themselves as stable DM, but at the same time lead to the possibly observable contribution to the UHECR spectrum.

3. Furthermore, instability of superheavy DM particles can arise due to Zeldovich mechanism through virtual black holes formation.

The existence of stable particles with interaction strength typical for SUSY and heavier than several TeV is in tension with conventional Friedmann cosmology.

R^2-gravity opens a way to save life of such X-particles, because in this theory the density of heavy relics with respect to the plasma entropy could be noticeably diluted by radiation from the scalaron decay.

The range of allowed masses of X-particles to form cosmological DM depends upon the dominant decay mode of scalaron.

Dominant decay channel of the scalaron	Allowed M_X to form DM
Minimally coupled scalars mode: $$\Gamma_s = \frac{M_R^3}{24 M_{Pl}^2}$$	$M_X \gtrsim M_R \approx 3 \cdot 10^{13}$ GeV
Massive fermions mode: $$\Gamma_f = \frac{m_f^2 M_R}{6 M_{Pl}^2}$$	$M_X \sim 10^6$ GeV
Gauge bosons mode: $$\Gamma_g = \frac{\alpha^2 \kappa^2 M_R^3}{24 M_{Pl}^2}$$	$M_X \sim 5 \cdot 10^{12}$ GeV

THE END

THANK YOU FOR ATTENTION