Hot Topics in Modern Cosmology
Spontaneous Workshop XIV
May 8–14, 2022 • Institut d'Études Scientifiques de Cargèse, France

A boosted gravitational wave background for primordial black holes with broad mass distributions and thermal features

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(arXiv : 2110.07487)

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Introduction

- Intriguing properties of BHs : low **effective spins**, binaries in the low or pair instability **mass gaps**, low **mass ratios** (e.g. GW190521, GW190814)
- Alternative explanation: **primordial origin** of BHs (Primordial Black Holes : PBHs)
- PBHs could have any mass (above 10^{11} kg), but the **thermal history** of the Universe naturally produces a strong **peak at the solar-mass scale** (QCD transition)
- PBHs with **wide mass distributions** could explain the rates, masses and spins of the LIGO/Virgo detections and the totality of the DM
- Searching for signatures of wide-mass models is important to distinguish the **primordial and stellar origin** of BHs
- Goal : Computation of the GWB from PBH binaries with a wide mass function

I. The GWB from black hole mergers

A GWB is produced by a population of binaries (of BHs) experiencing merging

$$\Omega_{\rm gw} \equiv \frac{1}{\rho_{\rm c}} \frac{{\rm d}\rho_{\rm gw}}{{\rm d}\log f}$$

$$\Omega_{\rm gw}h^2 = \frac{\pi h^2}{4G\rho_{\rm c}} f^2\, h_{\rm c}^2(f)$$

$\rho_{\rm GW}$: GW energy density

$\rho_{\rm c}$: critical density today

The **GW energy spectrum** is a superposition of redshifted GW radiation coming from merging BH binaries over the whole cosmic history

The **characteristic strain today** (in the Newtonian limit, with approximation of circular orbits)

$$h_{\rm c}^2(f) = \frac{4\,G^{5/3}}{3\pi^{1/3}\,c^2} f^{-4/3} \int_0^\infty \frac{{\rm d}z}{(1+z)^{4/3} H(z)} \times \int\!\!\int {\rm d}\ln m_1\,{\rm d}\ln m_2\, \tau_{\rm merg}(m_1, m_2, z)\, M_{\rm c}^{5/3}$$

$\tau_{\rm merg}$: merging rate (per unit of logarithmic mass)

$H(z)$: Hubble parameter

$M_{\rm c}$: chirp mass of the binary system

II. Broad PBH mass functions with thermal features

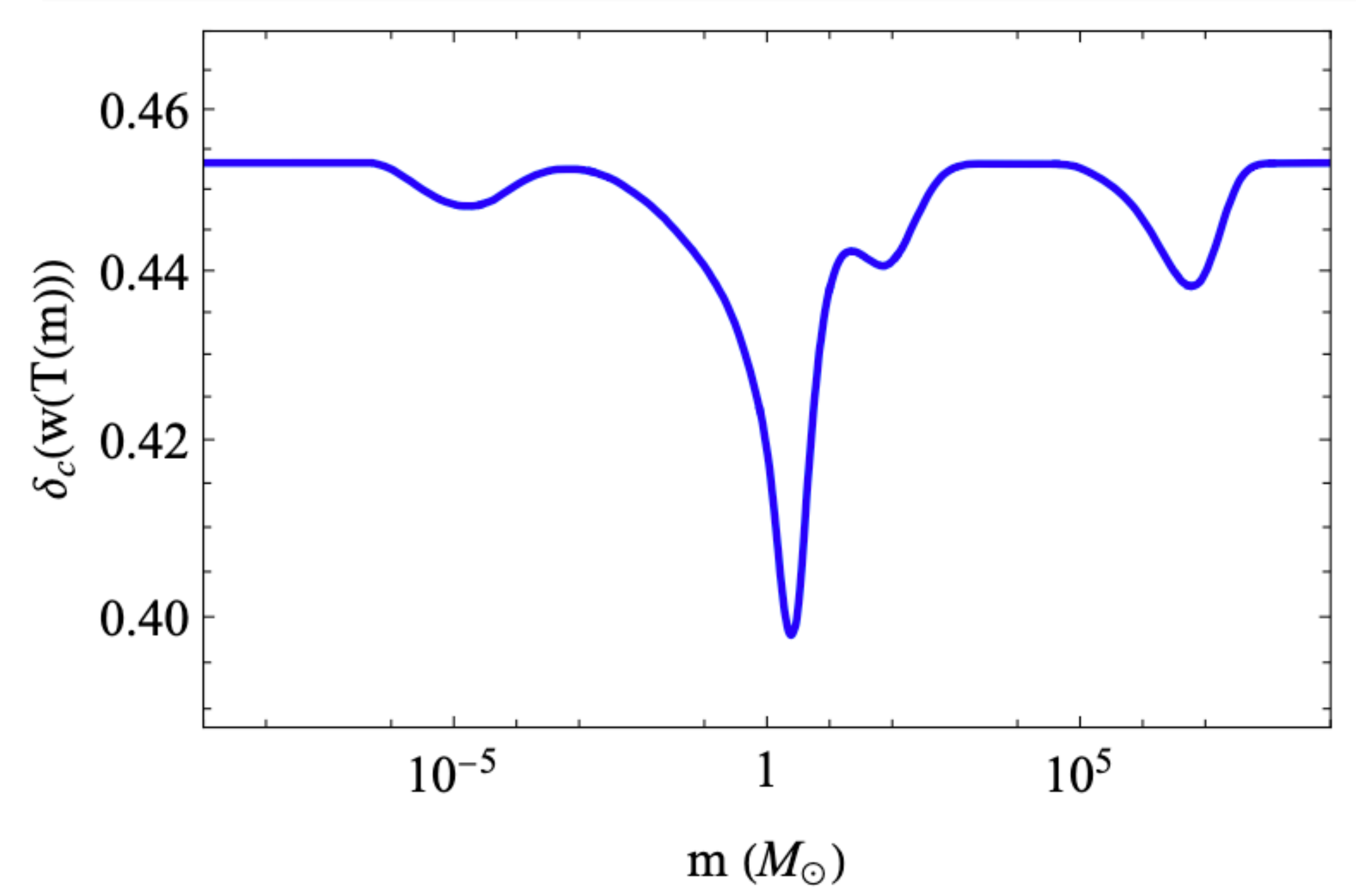
A primordial origin would imply an extended mass function

The **thermal history** of the early Universe induces a reduction in the EoS parameter w

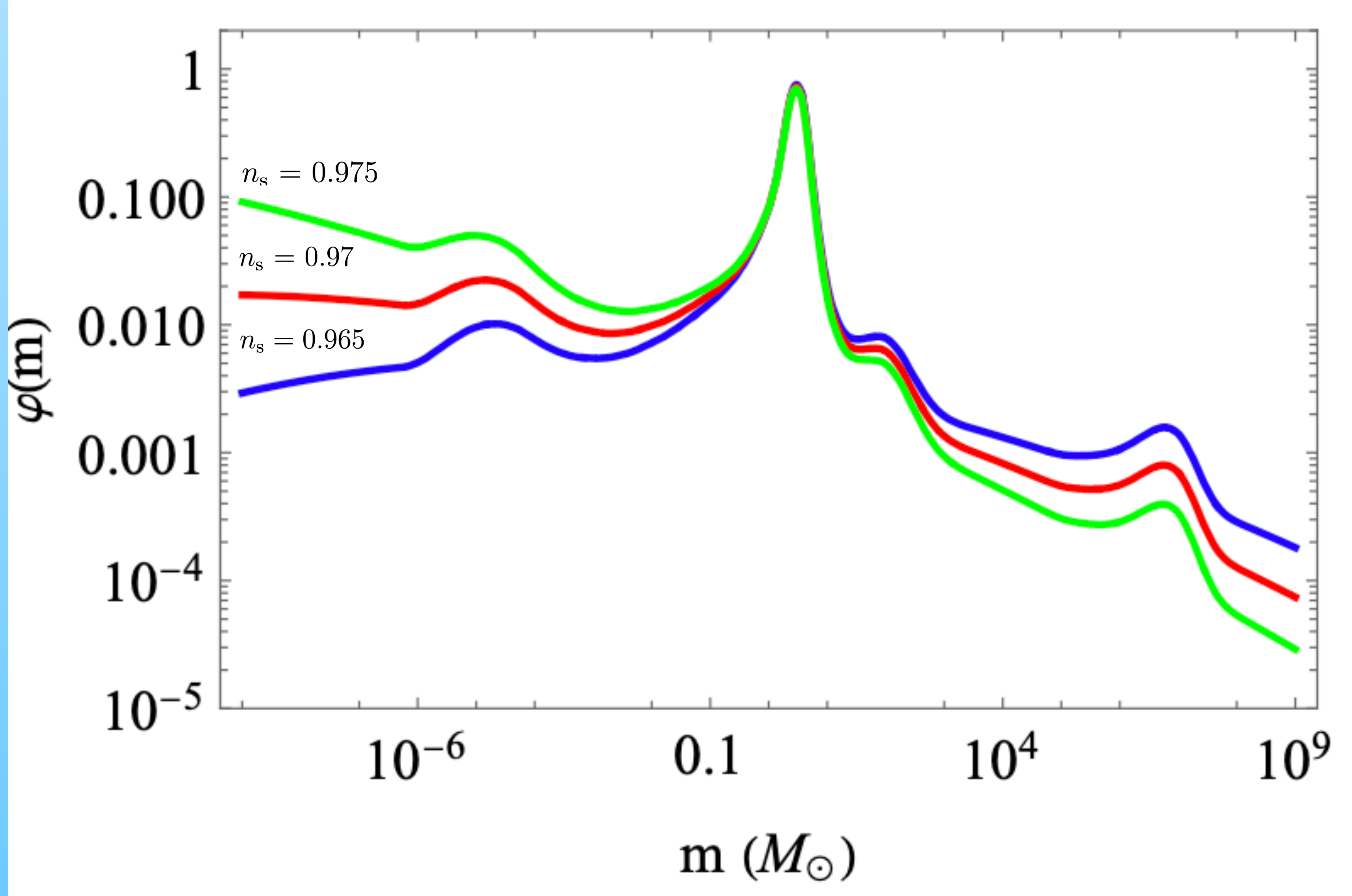
This reduction decreases the value of the critical overdensity threshold δ_c for PBH formation

PBHs would have formed at different times in the early Universe, with different masses

Variation of the threshold δ_c with the Horizon mass



Broad PBH mass function



III. PBH merging rates

2 binary formation channels : capture in dense clusters or in the early Universe

Early PBH binaries

$$\frac{\mathrm{d}\tau_{\mathrm{prim}}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} = \frac{1.6\times10^6}{\mathrm{Gpc^3 yr}} \times \underbrace{f_{\mathrm{sup}}}_{2.5\times10^{-3}}(m_1,m_2,z) f_{\mathrm{PBH}}^{53/37} \times \phi(m_1)\phi(m_2)\left[\frac{t(z)}{t_0}\right]^{-34/37} \times \left(\frac{m_1+m_2}{M_\odot}\right)^{-32/37} \times \left[\frac{m_1 m_2}{(m_1+m_2)^2}\right]^{-34/37}$$

Effects that lead to binary **disruption** and **rate suppression**:

1. Close PBH falling into binary
2. Binary absorption in early cluster
3. Surrounding matter fluctuations

Late PBH binaries in clusters

$$\frac{\mathrm{d}\tau_{\mathrm{clust}}}{\mathrm{d}\ln m_1 \mathrm{d}\ln m_2} = \underbrace{R_{\mathrm{clust}}}_{460} \times \phi(m_1)\phi(m_2) \times \frac{(m_1+m_2)^{10/7}}{(m_1 m_2)^{5/7}} \mathrm{yr}^{-1}\mathrm{Gpc}^{-3}$$

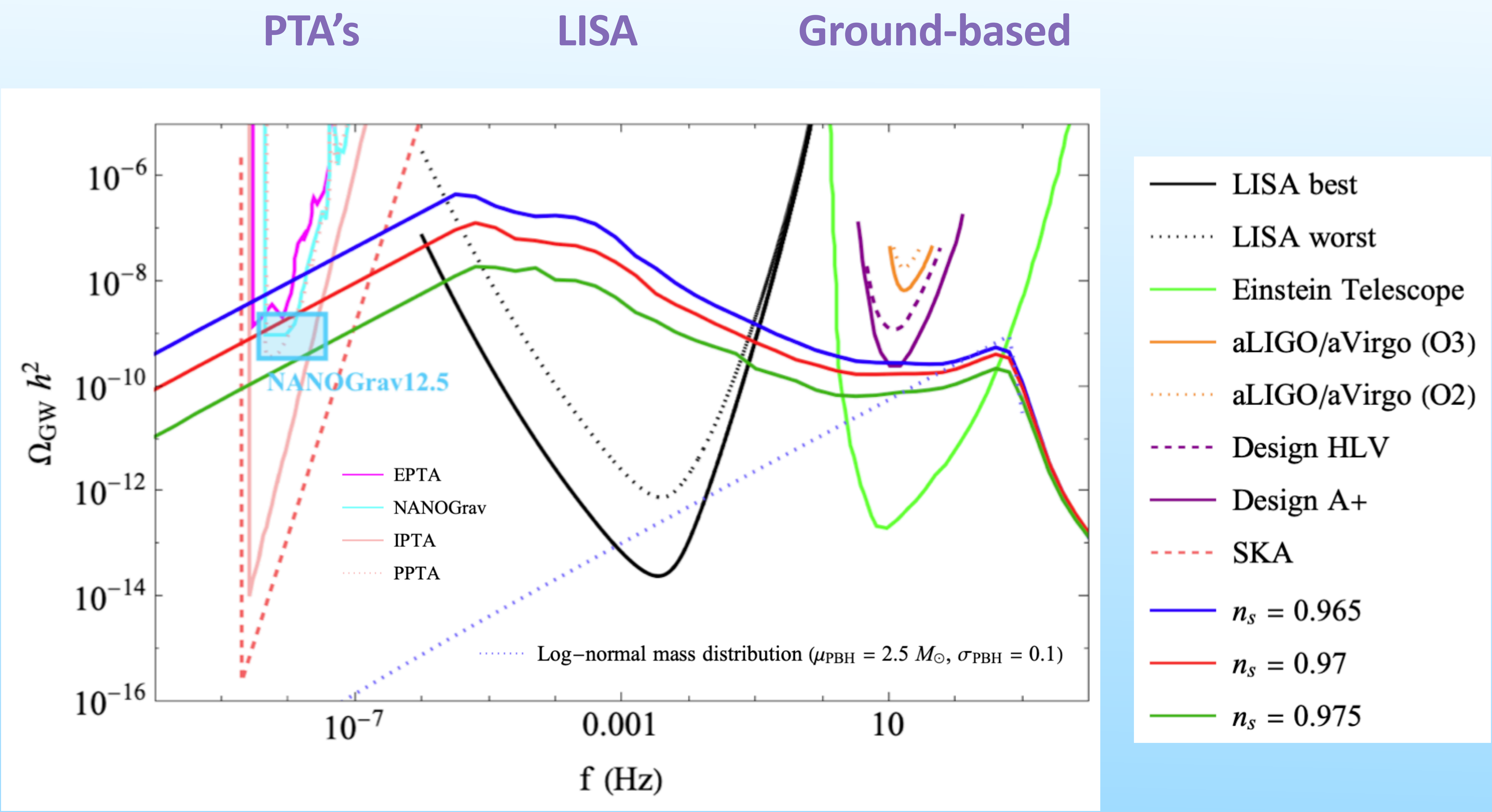
- The formula comes from the calculation of the 2-body binary capture
- R_{clust} is a scaling factor that depends on the PBH clustering properties and velocity distribution (~ 460, based on GW190425)
- If most PBH are regrouped in clusters, microlensing limits on their abundance can be evaded, allowing $f_{\mathrm{PBH}} = 1$

In both cases, one obtains the LIGO/Virgo merging rates at the QCD peak

IV. Results and detectability of the GWB

a) GWB from late PBH binaries in clusters

$$h_c(f) \simeq 1.15 \times 10^{-25} \left(\frac{\tau_{\rm clust}}{\rm yr^{-1} Gpc^{-3}}\right)^{1/2} \times \left(\frac{f}{\rm Hz}\right)^{-2/3} \left(\frac{M_c}{M_\odot}\right)^{5/6}$$



Ground-based :

- O2 and O3 LV runs impose no constraints on the background yet
- The GWB reaches the sensitivity of ET and A+ ($n_s = 0.965$)

LISA :

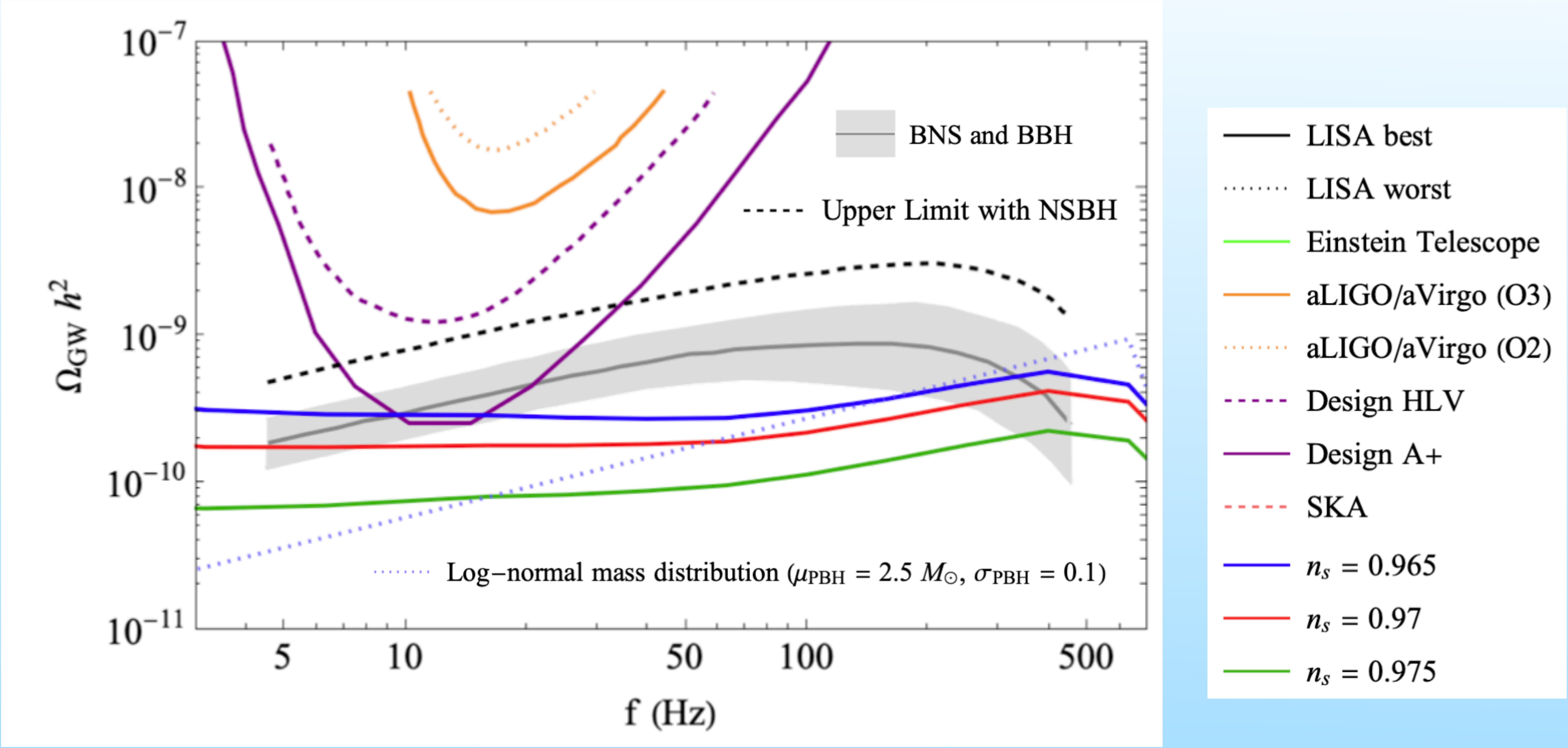
- Negative background spectral index, leading to a strong enhancement and comes from merging binaries with low mass ratios (intermediate + solar)

PTA's :

- The signal is now detectable, and the NANOGrav12.5 signal could be explained by this model

$$h_c(f) \simeq 1.15 \times 10^{-25} \left(\frac{\tau_{\rm clust}}{\rm yr^{-1} Gpc^{-3}} \right)^{1/2} \times \left(\frac{f}{\rm Hz} \right)^{-2/3} \left(\frac{M_c}{M_\odot} \right)^{5/6}$$

- Combined (astrophysical) GWB from the merging of BBH, BNS, NSBH in range 5-500 Hz
- The GWB is lower at ~100 Hz due to merging binaries with low mass ratios (subsolar + solar)



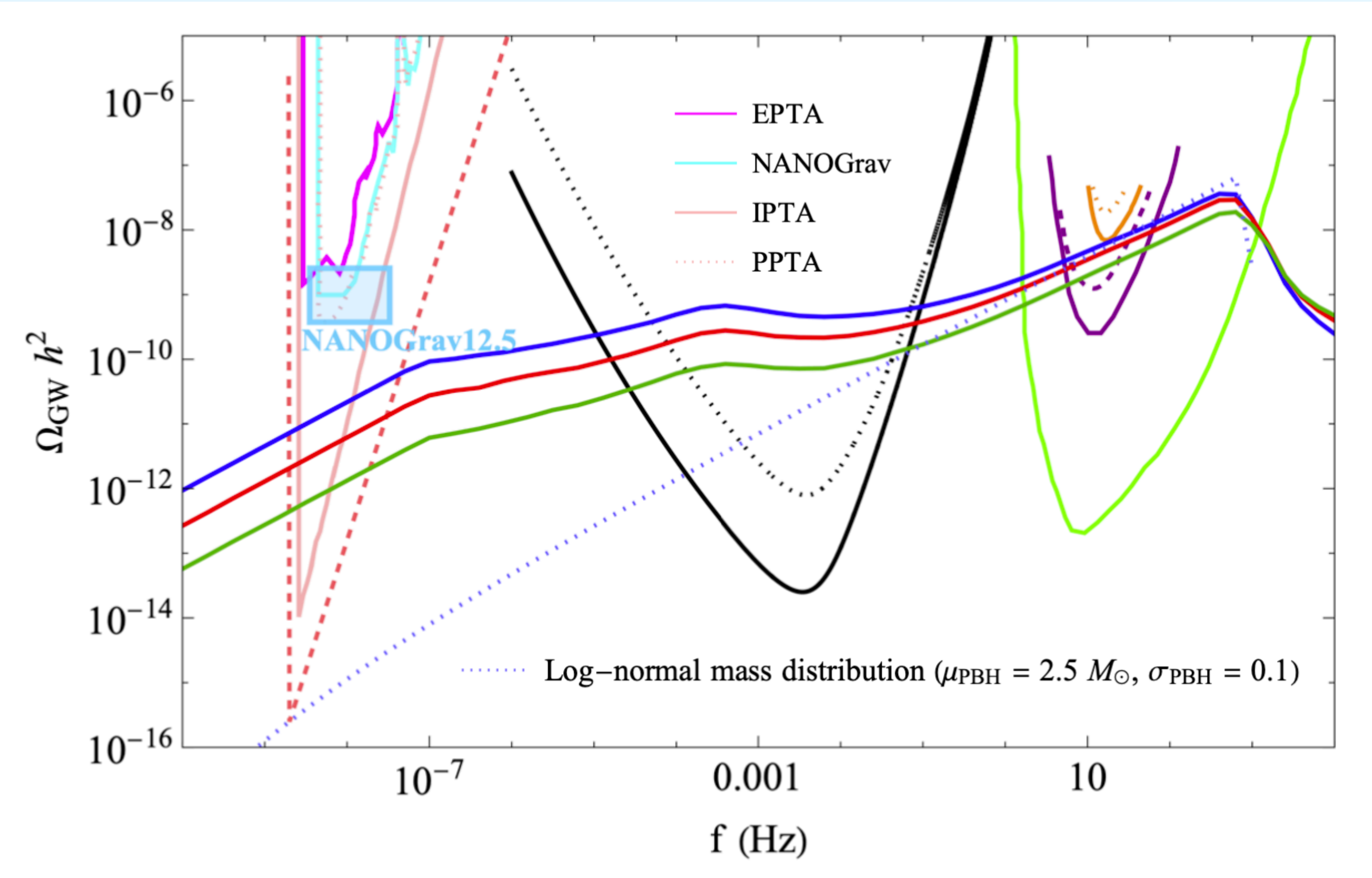
- The background spectral index is negative, whereas for astrophysical sources it is positive

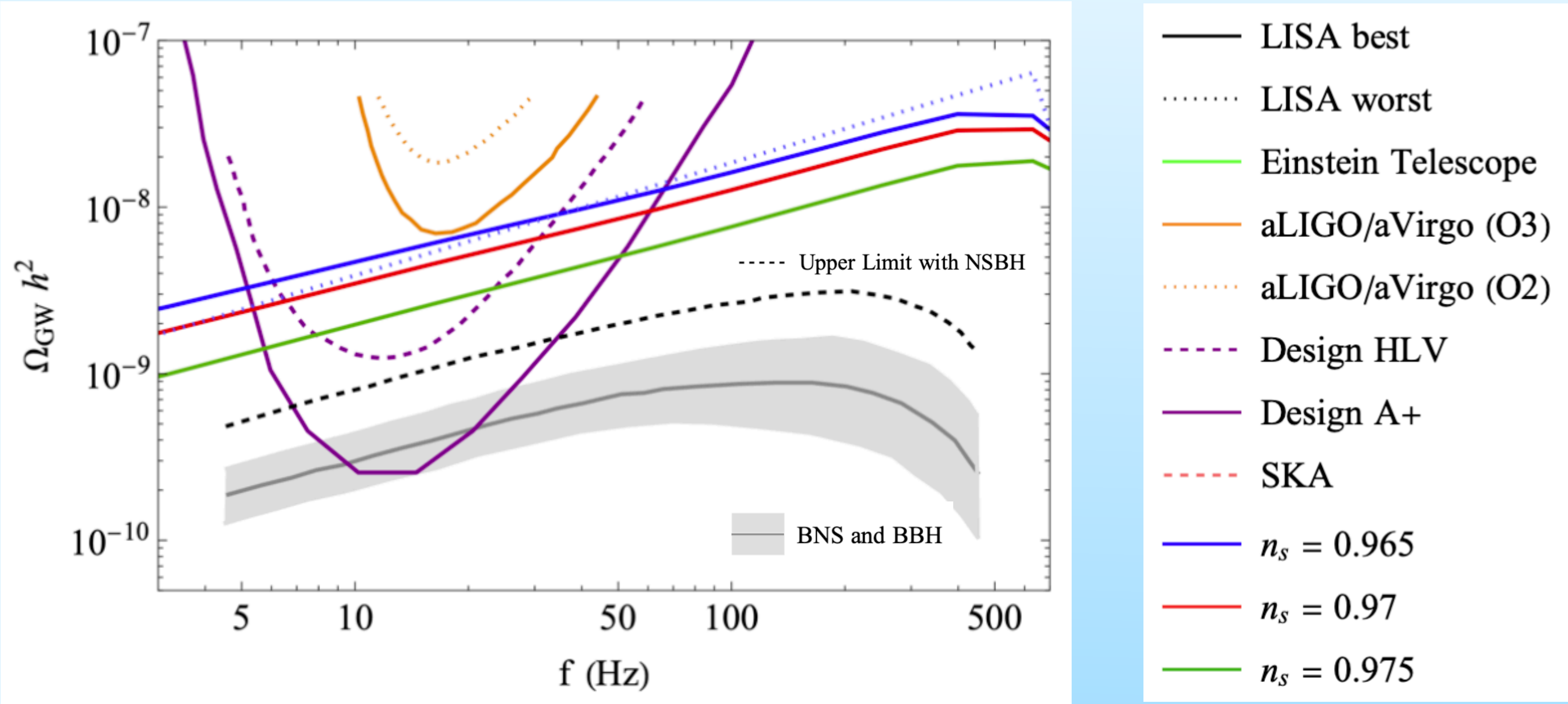
Probing the background spectral index with ground-based detectors could help distinguish between an astrophysical and primordial origin

b) GWB from early PBH binaries

$$h_c(f) \simeq 3.21 \times 10^{-25} \left(\frac{\tau_{\mathrm{prim}}}{\mathrm{yr}^{-1}\mathrm{Gpc}^{-3}} \right)^{1/2} \times \left(\frac{f}{\mathrm{Hz}} \right)^{-2/3} \left(\frac{M_c}{M_\odot} \right)^{5/6}$$

PTA's LISA Ground-based





Ground-based :

- The spectrum is just below the current O3 LV limits and above expectations for astrophysical sources

LISA :

- The GWB amplitude is lower (by ~ 3 orders of magnitude) and the spectral index remains positive
- The bump around 10^{-3} Hz comes from the bump in the mass function around 10^6 $M_\odot$

Summary

Late PBH binaries in clusters

☑ The GWB amplitude is greatly enhanced for frequencies below 10 Hz due to low mass ratio binaries

☑ The background spectral index will allow us to distinguish between an astrophysical and primordial origin

☑ The GWB coincides with a signal from NANOGrav12.5

Early PBH binaries

☑ The GWB typically dominates the one from late and astrophysical binaries

☑ It could potentially be detected in the next observing runs of LV

Thank you!

Backup Slides

A primordial origin would imply an extended mass function

The **thermal history** of the early Universe induces a reduction in the EoS parameter w

↓

This reduction decreases the value of the critical overdensity threshold δ_c for PBH formation

↓

PBHs would have formed at different times in the early Universe, with different masses

Spectrum of the root-mean-square amplitude of the Gaussian inhomogeneities from which the PBHs have formed:

$$\boxed{\delta_{\rm rms}(m) = A_{\rm s}\left(\frac{m}{M_\odot}\right)^{(1-n_{\rm s})/4}}$$

$n_{\rm s}$: small-scale scalar spectral index

$A_{\rm s}$: spectrum amplitude

Fraction of the Universe that collapses to form PBHs of mass m at formation :

$$\boxed{\beta(m) = \mathrm{erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2}\delta_{\rm rms}(m)}\right)}$$

$\delta_{\rm c}$: critical overdensity threshold leading to PBH formation

$$\boxed{\phi(m) \equiv \frac{1}{\rho_{\rm DM}}\frac{\mathrm{d}\rho_{\rm PBH}}{\mathrm{d}\ln m} \approx \frac{3.8}{f_{\rm PBH}}\left(\frac{M_{\rm eq}}{m}\right)^{1/2}\beta(m)}$$

$M_{\rm eq} \simeq 2.8\cdot 10^{17} M_\odot$: Hubble mass at matter-radiation equality

GW amplitude as a function of the masses m_1 and m_2

At LIGO frequencies

Contribution of subsolar and solar-mass binaries

At LISA and PTA frequencies

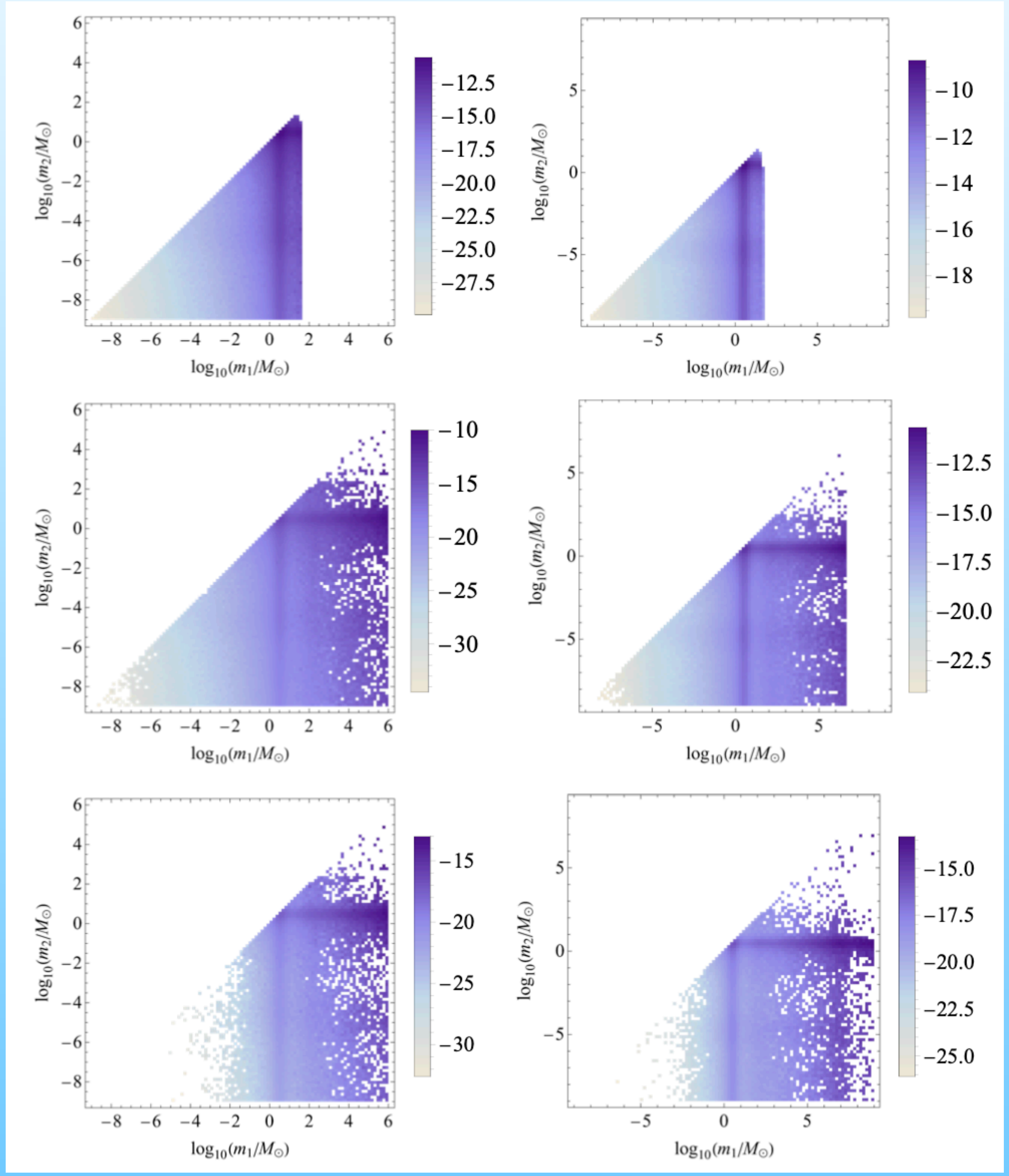
Contribution of solar and intermediate-mass binaries

In general

Binaries with very low mass ratios contribute importantly to the GWB

Late PBHs binaries in clusters

Early PBHs binaries



$f = 100$ Hz

$f = 10^{-3}$ Hz

$f = 10^{-8}$ Hz