Signatures of Primordial Gravitational Waves on the Large-Scale Structure of the Universe

Pritha Bari
University of Padova
Department of Physics and Astronomy

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Based on

- Signatures of Primordial Gravitational Waves on the Large-Scale Structure of the Universe (P. Bari, A. Ricciardone, N. Bartolo, D. Bertacca & S. Matarrese, arxiv:2111.06884)
- Signatures of Primordial Gravitational Waves on the Large-Scale Structure of the Universe: A Complete Study (P.Bari *et. al*, in prep.)

Introduction

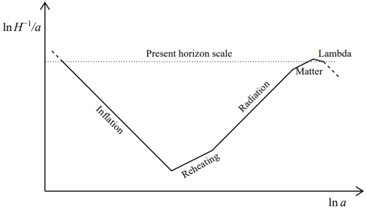

Figure 1: Inflation generates inhomogeneities which can explain the observed large-scale structures (LSS). (Liddle and Leach 2003)

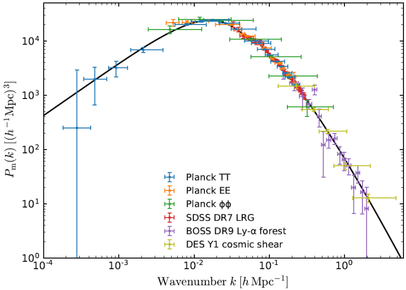

Figure 2: The linear matter power spectrum (at $z = 0$) inferred from different cosmological probes.(Planck 2018 results, Aghanim *et. al.*)

Effect of GW on LSS:

- Long-wavelength tensor fossils generate a local anisotropy in LSS (Jeong & Kamionkowski 2012).
- GWs induce volume distortion to affect the observed angular positions and redshifts of LSS (Jeong & Schmidt 2012).

Can we find another imprint?

Motivation

- On small and intermediate scales, second-order metric perturbations come into play
  $\longrightarrow$ mode-mixing happens
  $\longrightarrow$ perturbations can be composed of other kinds of perturbations of lower orders.
- For example, scalar induced gravitational waves are thoroughly investigated over the years (Tomita 1967, Matarrese *et. al* 1998...).
- We study the opposite effect here, generating matter-density perturbations from the primordial tensor modes
  $\longrightarrow$ can provide a new way to constrain GW parameters.

Set-up

We consider-

- Collisionless cold dark matter plus a cosmological constant for the matter content
- co-moving, synchronous, and time-orthogonal gauge

$$ds^2 = a^2(\eta)[-d\eta^2 + \gamma_{ij}(\boldsymbol{x}, \eta)dx^i dx^j]$$

where $\gamma_{ij} = \delta_{ij} + \chi_{ij}^{(1)T} + \frac{1}{2}\left(-2\phi^{(2)}\delta_{ij} + D_{ij}\chi^{(2)\|}\right)$,
$\eta \to$ conformal time, $a(\eta) \to$ scale factor.

Second order density contrast from first order tensor modes

- Evolution equation of the second order density contrast (Matarrese *et. al* 1998, Bruni *et. al* 2014)-

$$\delta^{(2)''} + \mathcal{H}\delta^{(2)'} - 4\pi G a^2 \overline{\rho}_m \delta^{(2)} = \frac{1}{2}\chi'^{ij}\chi'_{ij} \, .$$

$\Rightarrow$ no super-horizon contribution!
$\Rightarrow$ essentially a linear effect, sourced by the gravitational radiation only on subhorizon scales (See Wu *et al* 2007 for an analogous effect sourced by EM radiation):

$$\rho_{GW} = \frac{1}{32\pi G a^2}\langle \chi'^{ij}\chi'_{ij} \rangle$$

Focusing on matter domination only, we have, for the Fourier space density contrast,

$$\delta^{(2)}(\boldsymbol{k},\eta) = \sum_{\sigma,\sigma'} \int \frac{d^3\boldsymbol{k}_2}{(2\pi)^3} A_{\sigma'}(\boldsymbol{k}_2) A_\sigma(\boldsymbol{k}-\boldsymbol{k_2})$$
$$\times \epsilon^{\sigma'}_{ij}(\hat{\boldsymbol{k}}_2)\epsilon^{\sigma ij}(\widehat{\boldsymbol{k}-\boldsymbol{k_2}})$$
$$\times \left[\frac{\eta^2}{10}\int_0^\eta \frac{d\tilde{\eta}}{\tilde{\eta}}\left(\frac{3j_1(k_2\tilde{\eta})}{k_2\tilde{\eta}}\right)'\left(\frac{3j_1(|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta})}{|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta}}\right)'\right.$$
$$\left. - \frac{1}{10\eta^3}\int_0^\eta d\tilde{\eta}\,\tilde{\eta}^4\left(\frac{3j_1(k_2\tilde{\eta})}{k_2\tilde{\eta}}\right)'\left(\frac{3j_1(|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta})}{|\boldsymbol{k}-\boldsymbol{k}_2|\tilde{\eta}}\right)'\right].$$

$\epsilon^\sigma_{ij}(\hat{\boldsymbol{k}}) \to$ polarisation tensor
$A_\sigma(\boldsymbol{k}) \to$ GW stochastic variable

Result for different GW sources

- On CMB scales, r is tightly constrained ($r < 0.032$, BK18 + PR4). CMB-S4, LiteBIRD $\rightarrow$ more accuracy ($r < 0.001$).
- Moving away from CMB scales, we have less stringent bounds on the amplitude of the GW spectrum, which allows us to choose larger values for their amplitude.
- Many mechanisms can produce a large GW background at small scales (gauge field coupling, PBH...)

Models of GW sources

- **Blue-tilted spectrum ($n_T = 0.2$)**

$$\Delta_T^2(k) = A_T\left(\frac{k}{k_*}\right)^{n_T}$$

- **Monochromatic source**

$$\Delta_T^2\,(k) = A_T\delta_D\left(\ln\frac{k}{k_*}\right)$$

$$\Delta_\delta^2(k) = 4\times 10^{-5}\left(k\,\eta_0\right)^4 A_T^2 \times\left(\frac{8k_*^2}{k^2}+\frac{k^6}{16k_*^6}-\frac{k^4}{2k_*^4}+3\frac{k^2}{k_*^2}-8\right)\Theta(2k_*-k)$$

- **Gaussian-bump spectrum**

$$\Delta_T^2(k) = A_T\left(\frac{k}{k_*}\right)^{n_T} e^{-\frac{1}{2\sigma^2}\ln^2\left(\frac{k}{k_p}\right)}$$

Comparison with linear power spectrum

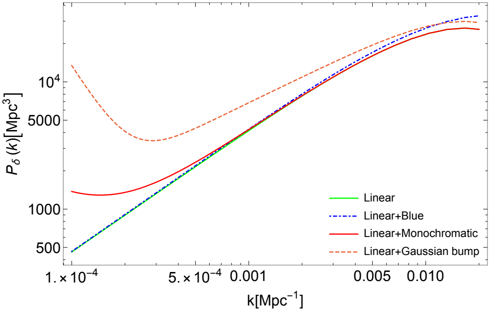

Figure 3: *Impact of different GW power spectra on the matter power spectrum* $P_\delta(k) = (2\pi^2/k^3)\Delta^2_\delta(k)$*: i) blue-tilted* $\left(A_T = 5\times10^{-7}, n_T = 0.2\right)$*, ii) monochromatic* $\left(A_T = 10^{-5}, k_* = 0.005\mathrm{Mpc}^{-1}\right)$*, iii) Gaussian bump* $\left(A_T = 10^{-5}, \sigma = 2, k_* = 0.005\mathrm{Mpc}^{-1}, k_p = 0.05\mathrm{Mpc}^{-1}\right)$*.*

$P_\delta\left(z=0,\Omega_m=0.32\right) \simeq 0.59 P_\delta\left(z=0,\Omega_m=1\right)$ considering growth suppression

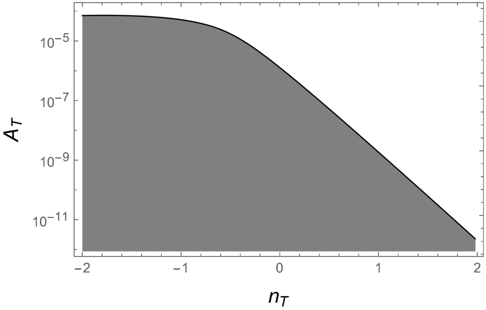

Figure 4: *Parameter space at $k = 0.006\mathrm{Mpc}^{-1}$ for n_T and A_T for a power-law GW spectrum, assuming a 4% error uncertainty on the linear matter power spectrum.*

Extension to radiation domination

We consider

- co-moving (with CDM) and time-orthogonal gauge

$$ds^2 = a^2(\eta)\left[-\left(1+2\psi\right)d\eta^2 + \gamma_{ij}\left(\boldsymbol{x},\eta\right)dx^i dx^j\right],$$

  where

$$\psi = \frac{\psi^{(2)}}{2},$$
$$\gamma_{ij} = \delta_{ij} + \chi^{(1)}_{ij} - \phi^{(2)}\delta_{ij} + \frac{1}{2}D_{ij}\chi^{(2)||}$$

  .

- Conservation equation $\rightarrow$ synchronous gauge regained!

Extension to radiation domination

Radiation domination		
Dominant component	Deep RD	Intermediate phase
In background	radiation	radiation $+$CDM
Perturbation	$\delta\rho_{\mathrm{r}}^{(2)}$	$\delta\rho_{\mathrm{m}}^{(2)}$

Two phases

- deep radiation domination
- towards matter-radiation equality

Deep RD

$$F \equiv \frac{\delta^{(2)}\rho_m}{\delta^{(2)}\rho_r} = \frac{\bar{\rho}_m}{\bar{\rho}_r}\frac{\delta_m^{(2)}}{\delta_r^{(2)}}. \tag{1}$$

$\delta^{(2)}\rho_m$ becomes of the same order as $\delta^{(2)}\rho_r$ at $F \sim 1$, i.e. at a particular time $y_M(y = a/a_{\rm eq} = \bar{\rho}_m/\bar{\rho}_r)$,

$$y_M \delta_m^{(2)}(y_M) = \delta_r^{(2)}(y_M).$$

Towards matter-radiation equality

- Meszaros' equation with a source term quadratic in GWs.

$$\frac{d^2\delta_{\rm m}^{(2)}}{dy^2} + \frac{2+3y}{2y(y+1)}\frac{d\delta_{\rm m}^{(2)}}{dy} - \frac{3}{2y(y+1)}\delta_{\rm m}^{(2)} = \frac{1}{2}\frac{d\chi^{ij}}{dy}\frac{d\chi_{ij}}{dy}.$$

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$$\delta_{\rm m}^{(2)}(\boldsymbol{x}, y) = P_1(\boldsymbol{x})D_1(y) + P_2(\boldsymbol{x})D_2(y) + \frac{1}{2}\int d\tilde{y}\, G(y,\tilde{y})\frac{d\chi^{ij}}{d\tilde{y}}\frac{d\chi_{ij}}{d\tilde{y}},$$

- $P_1(\boldsymbol{x})$ and $P_2(\boldsymbol{x})$ can be found by matching $\delta_{\rm m}^{(2)}$ and its derivative at y_M.

Summary and discussion

- We analyzed a new effect of 'tensor-induced scalar modes' on the present day matter power spectrum, and found that a large GW power spectrum can leave a significant imprint.
- Our effect mimics the linear perturbation in the subhorizon limit, whereas completely vanishes in the superhorizon, leaving no CMB temperature anisotropy on large scales.
- We extend the study to radiation domination for a complete picture. We also intend to explore its high intrinsic non-Gaussianity in the future.

Thank You