de Sitter Space as a Coherent State of Gravitons

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de Sitter space as a coherent state of gravitons

Coherent state description of de Sitter space: Dvali, Gomez '13, Dvali, Gomez, Zell '17

Q: What are the properties of constituent quanta? Vacuum?

A: Need for off-shell longitudinal gravitons, over the Minkowski spacetime.

Repercussions: quantum non-eternity/inconsistency.

Coherent states in QFT

$$|C\rangle = e^{-i\int d^3x\left(\phi_{cl}(x)\hat{\pi}(x)-\pi_{cl}(x)\hat{\phi}(x)\right)}|\Omega\rangle.$$

$$\langle C|\hat{\phi}|C\rangle(t=0) = \phi_{cl}(x)\,,$$

$$\langle C|\hat{\pi}|C\rangle(t=0) = \pi_{cl}(x)\,.$$

Classical source in scalar theory

$$\hat{\mathcal{H}} = \frac{1}{2}\hat{\pi}^2 + \frac{1}{2}(\partial_j\hat{\phi})^2 + \Lambda\hat{\phi}$$

Coherent state description of $\phi_{cl} = \phi_0 - \frac{t^2}{2}\Lambda$:

$$|\phi_{cl}\rangle = e^{-i\hat{H}t} \times e^{-i\phi_0 \int d^3x\hat{\pi}}|0\rangle = phase \times e^{-i\int d^3x\left(\phi_{cl}(t)\hat{\pi} - \dot{\phi}_{cl}(t)\hat{\phi}\right)}|0\rangle$$

with

$$\langle 0|\hat{\phi}|0\rangle = 0$$

Notice: $|0\rangle$ is a legitimate initial state!

Gauge theories

LB, Dvali, Sakhelashvili '21

Physicality conditions!

Source-free vacuum is not a consistent state in the presence of the source.

Yet, a consistent coherent state can be built over the source-free vacuum.

BRST quantization of QED

$$\mathcal{L} = -\frac{1}{4}\hat{F}_{\mu\nu}^2 + |D_\mu \hat{\Phi}|^2 - m^2 |\hat{\Phi}|^2 - \partial_\mu \hat{B} \hat{A}^\mu + \frac{1}{2}\xi \hat{B}^2 + \partial_\mu \hat{\bar{c}} \partial^\mu \hat{c}$$

There is a conserved charge:

$$\hat{Q}_B = \int d^3x \left[\hat{c} \left(g\hat{\rho} - \partial_j \hat{E}_j \right) + \hat{B}\hat{\Pi}_{\bar{c}} + \partial_j \left(\hat{c}\hat{E}_j \right) \right]$$

Physicality condition:

$$\hat{Q}_B |phys\rangle = 0$$

Coherent state of photons

$$|A\rangle = e^{-i\int d^3x \left(A_j^c \hat{E}_j - E_j^c \hat{A}_j + A_0^c \hat{B}\right)} |\Omega\rangle$$

$$\hat{Q}_B |phys\rangle = 0 \qquad \Longleftrightarrow \qquad \partial_j E_j^c = 0$$

Matter coherent states

In the absence of the gauge field:

$$|C\rangle = e^{-i\int d^3x\left(\Phi_c\hat{\Pi} - \Pi_c\hat{\Phi} + h.c.\right)}|\Omega\rangle$$

A naive expectation in QED:

$$|A\rangle \otimes |C\rangle$$

does not work

$$\hat{Q}_B\{|A\rangle \otimes |C\rangle\} \neq 0$$

Dirac operators

Instead

$$|C\rangle_g = e^{-i\int d^3x\left(\Phi_c\hat{\Pi}_g - \Pi_c\hat{\Phi}_g + h.c.\right)}|\Omega\rangle$$

with

$$\hat{\Phi}_g = \hat{\Phi}\cdot\exp\left(-ig\frac{1}{\nabla^2}\partial_j\hat{A}_j\right)\ ,$$
$$\hat{\Pi}_g = \hat{\Pi}\cdot\exp\left(+ig\frac{1}{\nabla^2}\partial_j\hat{A}_j\right)\ ;$$

So that

$$\hat{Q}_B|C\rangle_g = 0$$

Classical limit

$$g \to 0, \qquad (\Phi_c, \Pi_c) \to \infty, \qquad g\rho_c = ig\left(\Phi_c \Pi_c - \Phi_c^* \Pi_c^*\right) \to \textit{fixed}$$

Leads to

$$|C\rangle_g \to e^{-i\int d^3x \left(\Phi_c \hat{\Pi} - \Pi_c \hat{\Phi} + h.c.\right)} \cdot e^{i\int d^3x \; g\left[\frac{1}{\nabla^2}\partial_j \rho_c\right]\hat{A}_j} |\Omega\rangle$$

Classical source

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$$\hat{H}_J = \hat{H} + \int d^3x \hat{A}_\mu J^\mu_{\rm cl}$$

For a coherent state of the electromagnetic field:

$$|J\rangle = e^{-i\int d^3x \left(A^c_j \hat{E}_j - E^c_j \hat{A}_j + A^c_0 \hat{B}\right)}|\Omega\rangle \qquad \text{with} \qquad \hat{H}|\Omega\rangle = 0$$

Due to the fact that $\hat{Q}^J_B = \hat{Q}_B + \int d^3x\ \hat{c} J^0_{\rm cl}$:

$$\hat{Q}^J_B|J\rangle = 0 \qquad \Longleftrightarrow \qquad \partial_j E^c_j = J^0_{\rm cl}$$

Notice: $|\Omega\rangle$ is an unphysical state, in the presence of the source.

Schwinger pair creation would lead to the evolution of $|J\rangle$!

de Sitter in linearized gravity

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The same fundamental questions, regarding the consistency of the coherent state description, as in Einstein's theory arise:

- Vacuum?
- Longitudinal polarizations?

Linearized Gravity

Follows from GR in $M_{\rm pl} \to \infty$ limit, for $g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}}$.

BRST invariant formulation (Kugo, Ojima '78):

$$\mathcal{L} = \frac{1}{2}(\partial_\alpha \hat{h}_{\mu\nu})^2 - \frac{1}{2}(\partial_\alpha \hat{h})^2 + \partial_\alpha \hat{h} \partial_\mu \hat{h}^{\mu\alpha} - \partial_\mu \hat{h}^{\mu\alpha} \partial_\nu \hat{h}^\nu_\alpha$$
$$- \partial_\mu \hat{B}_\nu \left(\hat{h}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu} \hat{h} \right) + \frac{1}{2}\xi \hat{B}_\mu^2 + \partial_\alpha \hat{\bar{C}}_\mu \partial^\alpha \hat{C}^\mu$$

with

$$\delta \hat{h}_{\mu\nu} = \theta \left(\partial_\mu \hat{C}_\nu + \partial_\nu \hat{C}_\mu \right) ,$$
$$\delta \hat{\bar{C}}_\mu = \theta \hat{B}_\mu ,$$

$$\delta \hat{\pi}_{ij} = 2\theta \left(\nabla^2 \delta_{ij} - \partial_i \partial_j \right) \hat{C}_0 .$$

Coherent state of gravitons

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$$|h\rangle = e^{-i\int d^3x\left(h^c_{ij}\hat{\pi}_{ij} - \pi^c_{ij}\hat{h}_{ij} - \Pi^\mu_c \hat{B}_\mu\right)}|\Omega\rangle\,, \quad \text{with} \quad \Pi^\mu \equiv -h^{0\mu} + \frac{1}{2}\eta^{0\mu}h\,.$$

BRST invariance:

$$\hat{Q}_B|h\rangle = e^{-i\int d^3x\left(h^c_{ij}\hat{\pi}_{ij} - \pi^c_{ij}\hat{h}_{ij} - \Pi^\mu_c \hat{B}_\mu\right)} \times \int d^3x \left(-2h^c_{ij}(\nabla^2\delta_{ij} - \partial_i\partial_j)\hat{C}_0 + 2\pi^c_{ij}\partial_i\hat{C}_j\right)|\Omega\rangle = 0\,.$$

This leads to the classical constraints:

$$\left(\nabla^2\delta_{ij} - \partial_i\partial_j\right) h^c_{ij} = 0\,, \qquad \text{and} \qquad \partial_i\pi^c_{ij} = 0\,.$$

de Sitter in Linearized Gravity

Cosmological Constant:

$$\Delta\mathcal{L} = -\lambda\hat{h}\,.$$

Classically:

$$h_{ij} = -\frac{\lambda}{6}\left(t^2\delta_{ij} + x_i x_j\right)\,, \quad h_{00} = h_{0j} = 0\,.$$

Corresponding to de Sitter background in the limit

$$M_{\rm pl} \to \infty\,, \qquad \lambda = \mathit{fixed}\,, \qquad H^2 \simeq \frac{\lambda}{M_{\rm pl}} \to 0\,.$$

In this limit

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm pl}} \qquad \to \qquad \eta_{\mu\nu}$$

de Sitter as a BRST invariant coherent state

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A coherent state over Minkowski, even in the presence of CC?

In analogy with QED:

$$|dS\rangle = e^{-i\int d^3x\,\left(h^c_{ij}\hat{\pi}_{ij}+\dots\right)}|\Omega\rangle$$

The theory at hand is described by

$$\hat{H} = \hat{H}_0 + \int \lambda \hat{h}$$

Minkowski state $|\Omega\rangle$ is defined as the vacuum of $\hat{H}_0$.

Hence, it's no longer physical: $\hat{Q}|\Omega\rangle \neq 0\,,\quad$ since $\quad \hat{Q} = \hat{Q}_0 + \Delta\hat{Q}$.

Nevertheless:

$$\hat{Q}|dS\rangle = 0\,, \qquad \text{as long as} \qquad \left(\nabla^2\delta_{ij} - \partial_i\partial_j\right) h^c_{ij} - \lambda = 0\,.$$

The state that satisfies this constraint and gives the desired expectation values is

$$|dS\rangle = e^{-i\int d^3x\,\left(h^c_{ij}\hat{\pi}_{ij} + \frac{1}{2}h^c_{kk}\hat{B}_0\right)}|\Omega\rangle$$

with

$$h^c_{ij} = -\frac{\lambda}{6}x_i x_j$$

Concluding Remarks:

We have discussed the BRST consistency of introducing fundamentally classical sources in gauge theories.

The coherent states sourced by such sources were shown to satisfy the physicality conditions.

These states are expected to exhibit a gradual loss of coherence.

Despite limitations of our dS construction, the state exhibits some of the properties of de Sitter spacetime upon reintroducing interactions with spectator fields softly.

Next obvious step would be to see if our construction can be extended to Einstein's gravity without obstruction.