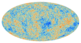
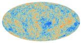
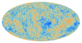
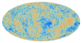
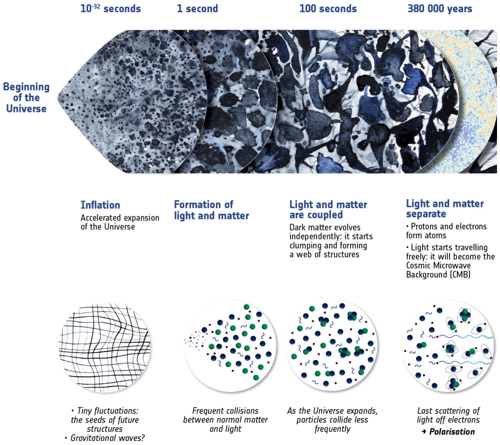
Cosmological master equations

$\frac{\mathrm{d}}{\mathrm{d}t}\Big|$  $\Big\rangle\Big\langle$  $\Big| = \mathcal{V}\Big(\Big|$  $\Big\rangle\Big\langle$  $\Big|\Big)$

Thomas Colas

SW14

Quantum origin of cosmic inhomogeneities

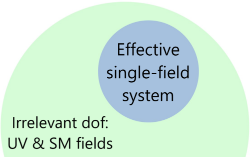


Quantum fluctuations of the primordial vacuum *seed all the structures of the Universe* $\Rightarrow$ Understanding this mechanism is crucial.

A minimal approach

Three observations:

1. **Single-field slow-roll inflation** provides an excellent fit of the data.
2. At some point, inflation must end: couple to **SM fields**.
3. **UV-completions** of inflation often introduce new degrees of freedom.



What is the *quantum description* *of the* *effective single-field system*?

An example of extra ingredient: spectator field

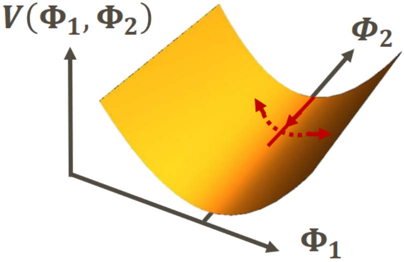
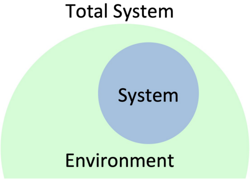


Figure: Adiabatic and entropic perturbations [Credits: L. Pinol]

- *WEFT result*: field stabilised, just a speed of sound at linear order.
- *OQS result*: curvature perturbations decohere while interacting with isocurvature modes [Prokopec & Rigopoulos, 2007].

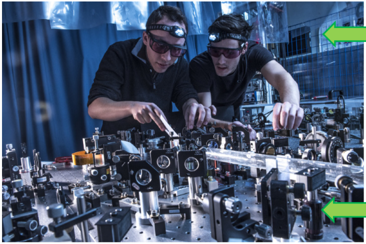
The early universe as an Open Quantum System (OQS)



- By integrating out the environment, the system dynamics becomes non-unitary.
- Cosmological perturbations are described by an OQS with dissipation and decoherence.
- They experience energy exchange and information loss into the environment.

Can we build an effective formalism which encompasses WEFT unitary results and OQS non-unitary evolution ?

The lab-based experiments wisdom



Credits: LKB

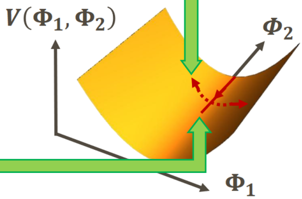
Environment:

- weakly specified

Interactions

System:

- measurable



Modelled by Master Equations

Assessing cosmological master equations

- Fundamental observables are correlators

$$\left\langle \widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \right\rangle (t) \equiv \mathrm{Tr} \left[\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \widehat{\rho}_{\mathrm{red}}(t) \right]$$

  Master Equations (ME) are dynamical equations for $\widehat{\rho}_{\mathrm{red}}(t)$.

- ME have already been applied in cosmology, see [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Hollowood & McDonald, 2017], [Martin & Vennin, 2018], [Brahma, Berera & Calderón-Figueroa, 2021], $\cdots$

- ME were **designed in a specific context** and **need some adaptations** $\Rightarrow$ Working in curved-space implies to reassess approximation schemes and regimes of validity.

- We **benchmark the ME program** on an exactly solvable model:
  - We have analytic control on the system dynamics;
  - We compare exact and ME results.

The curved-space Caldeira-Leggett model

- Action for the field sector:

$$S = -\int \mathrm{d}^4 x \sqrt{-\det g} \Bigg(\overbrace{\left[\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + \frac{1}{2} m^2 \varphi^2\right]}^{\text{System}} + \underbrace{\left[\frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi + \frac{1}{2} M^2 \chi^2\right]}_{\text{Environment}} + \underbrace{\lambda^2 \varphi \chi}_{\text{Interaction}} \Bigg)$$

- **Field redefinition**: rotation in field space of

$$\theta = -\frac{1}{2} \arctan\left(\frac{2\lambda^2}{M^2 - m^2}\right)$$

decouple the two sectors: fully integrable model.

- **Gaussian system**:
  - All information contained in the system covariance $\boldsymbol{\Sigma}_{\varphi\varphi}(\eta)$;
  - Quantum information properties: $\gamma(\eta) = \det\left[\boldsymbol{\Sigma}_{\varphi\varphi}(\eta)\right]^{-1}/4$.

An effective master equation

$$\underbrace{\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = \overbrace{-i\left[\widehat{H}_0(\eta) + \widehat{H}^{(\mathrm{LS})}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right]}^{\text{Unitary evolution}} + \sum_{i,j}\mathcal{D}_{ij}(\eta)\left[\widehat{\boldsymbol{z}}_i\widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{z}}_j - \frac{1}{2}\left\{\widehat{\boldsymbol{z}}_j\widehat{\boldsymbol{z}}_i, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]}_{\text{Non-unitary evolution}}$$

with $\widehat{\boldsymbol{z}} = (\widehat{v}_\varphi, \widehat{p}_\varphi)^{\mathrm{T}}$.

- **Unitary evolution**: *Lamb-shift Hamiltonian*

$$\widehat{H}_0(\eta) + \widehat{H}^{(\mathrm{LS})}(\eta) = \frac{1}{2}\left[\widehat{p}_\varphi\widehat{p}_\varphi + \left(k^2 + m^2a^2 + \boldsymbol{\Delta}_{11}\right)\widehat{v}_\varphi\widehat{v}_\varphi + \left(\frac{a'}{a} + \boldsymbol{\Delta}_{12}\right)\left\{\widehat{v}_\varphi, \widehat{p}_\varphi\right\}\right]$$

- **Non-unitary evolution**: *diffusion and dissipation*

$$\mathcal{D}(\eta) = \begin{pmatrix} \boldsymbol{D}_{11} & \boldsymbol{D}_{12} - i\boldsymbol{\Delta}_{12} \\ \boldsymbol{D}_{12} + i\boldsymbol{\Delta}_{12} & 0 \end{pmatrix}$$

Transport equation and non-perturbative resummation

- ME generates an effective transport equation:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \underbrace{\boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega}}_{\text{Unitary evolution}} \underbrace{- 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}}_{\text{Non-unitary evolution}}$$

- ME studied in cosmology for its ability to resum late-time secular effects

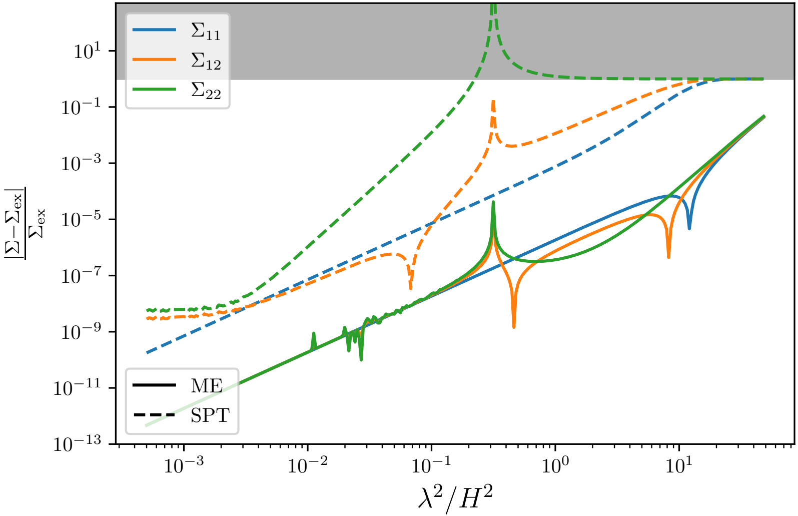
  [Boyanovsky, 2015], [Burgess, Holman & Tasinato, 2015], [Brahma, Berera & Calderón-Figueroa, 2021]

- Resumation obtained when solving the transport equation **non-perturbatively**, considering ME as a *bona fide* dynamical map.

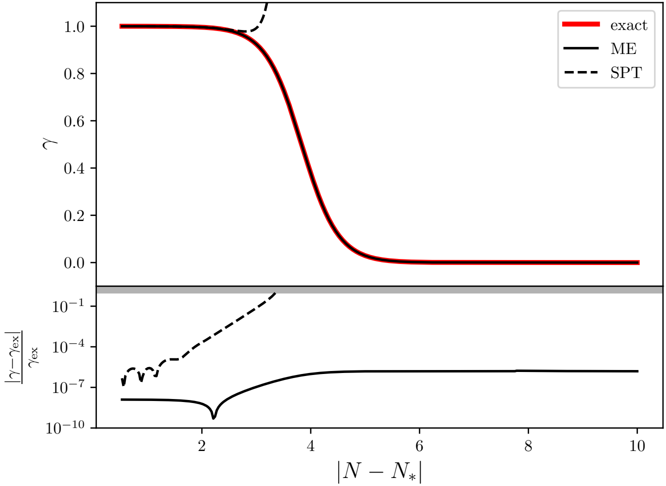
- Benchmark against standard perturbation theory (SPT) results:

  1. Accuracy on the **system covariance** $\boldsymbol{\Sigma}_{\varphi\varphi}(\eta)$;
  2. Ability to recover the **purity** $\gamma(\eta) = \det\left[\boldsymbol{\Sigma}_{\varphi\varphi}(\eta)\right]^{-1}/4$.

Results on the power spectra



Results on the purity



Summary

- In cosmology, we need to deal with elusive environments.
    - **EFT** and **OQS** separate **things we know** from **things we don't**.
- ME may allow us to go beyond standard tools.
    - Approximation schemes et regime of validity must be **reassessed**.
- We benchmarked cosmological ME on an integrable model.
    1. **Improved precision** on the system covariance;
    2. **Recovery of the quantum information properties**.

Thank you for your attention !

Outline

Flat vs curved-space Caldeira-Leggett model

Flat space

- System: a harmonic oscillator of frequency

$$\omega^2 = k^2 + m^2$$

- Environment : large number of harmonic oscillators
- Linear interaction:

$$\widehat{H}^{\text{int}}_{\boldsymbol{k}} = \sum_{\boldsymbol{q}} \lambda^2_{\boldsymbol{q}} \widehat{v}^{(S)}_{\boldsymbol{k}} \widehat{v}^{(E)}_{\boldsymbol{q}}$$

$\Rightarrow$ system interacts with infinitely many dof.

Curved space

- System: a parametric oscillator of frequency

$$\omega^2 = k^2 + m^2 a^2 - a''/a$$

- Environment : large number of parametric oscillators BUT
- Linear interaction + symmetries

$$\widehat{H}^{\text{int}}_{\boldsymbol{k}} = \lambda^2 a^2 \widehat{v}^{(S)}_{\boldsymbol{k}} \widehat{v}^{(E)}_{-\boldsymbol{k}}$$

$\Rightarrow$ system only interacts with ONE environmental dof.

TCL_2 coefficients

$$\boldsymbol{D}_{11}(\eta) = -4\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \,\mathrm{Im}\left\{p_\varphi(\eta) v_\varphi^*(\eta')\right\} \mathrm{Re}\left\{v_\chi(\eta) v_\chi^*(\eta')\right\}$$

$$\boldsymbol{D}_{12}(\eta) = 2\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \,\mathrm{Im}\left\{v_\varphi(\eta) v_\varphi^*(\eta')\right\} \mathrm{Re}\left\{v_\chi(\eta) v_\chi^*(\eta')\right\}$$

$$\boldsymbol{\Delta}_{11}(\eta) = -4\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \,\mathrm{Im}\left\{p_\varphi(\eta) v_\varphi^*(\eta')\right\} \mathrm{Im}\left\{v_\chi(\eta) v_\chi^*(\eta')\right\}$$

$$\boldsymbol{\Delta}_{12}(\eta) = 2\lambda^4 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta' a^2(\eta') \,\mathrm{Im}\left\{v_\varphi(\eta) v_\varphi^*(\eta')\right\} \mathrm{Im}\left\{v_\chi(\eta) v_\chi^*(\eta')\right\}$$

- Can we compare them with exact counterparts ?
  - Fundamental object: system covariance $\boldsymbol{\Sigma}_{\varphi\varphi}$
  - Look at the *exact* and *effective* **transport equation**:

$$\frac{\mathrm{d}\boldsymbol{\Sigma}_{\varphi\varphi}}{\mathrm{d}\eta} = \boldsymbol{\omega}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\Sigma}_{\varphi\varphi} - \boldsymbol{\Sigma}_{\varphi\varphi}\left(\boldsymbol{H}^{(\varphi)} + \boldsymbol{\Delta}\right)\boldsymbol{\omega} - \boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega} - 2\boldsymbol{\Delta}_{12}\boldsymbol{\Sigma}_{\varphi\varphi}$$

Comparison of the coefficients

- Exact coefficients:

$$\boldsymbol{\Delta}_{\mathrm{ex},11} = -\frac{\lambda^4}{M^2 - m^2} a^2, \qquad \boldsymbol{\Delta}_{\mathrm{ex},12} = 0$$
$$\boldsymbol{D}_{\mathrm{ex},11} = -\frac{2\lambda^4}{M^2 - m^2} a^2 \boldsymbol{\Sigma}_{\chi\chi,12}, \qquad \boldsymbol{D}_{\mathrm{ex},12} = \frac{\lambda^4}{M^2 - m^2} a^2 \boldsymbol{\Sigma}_{\chi\chi,11}$$

- TCL_2 coefficients
  1. In the super-Hubble regime $|-k\eta| \ll 1$;
  2. When the environment is heavy $M \gg H$;

$$\boldsymbol{\Delta}_{11} = \boldsymbol{\Delta}_{\mathrm{ex},11} + \boldsymbol{\Delta}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}, \qquad \boldsymbol{\Delta}_{12} = \boldsymbol{\Delta}_{\mathrm{ex},11} + \boldsymbol{\Delta}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}$$
$$\boldsymbol{D}_{11} = \boldsymbol{D}_{\mathrm{ex},11} + \boldsymbol{D}_{11}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}, \qquad \boldsymbol{D}_{12} = \boldsymbol{D}_{\mathrm{ex},12} + \boldsymbol{D}_{12}^{\mathrm{spur}}(\eta_0) + \mathrm{h.o.}$$

where the matching is at order $\mathcal{O}(\lambda^4)$.

Spurious terms in the super-Hubble regime

When $M \gg H$,

$$\boldsymbol{D}_{11}^{\mathrm{spur}} = \frac{1}{2\mu_\chi}\frac{1}{\nu_\varphi^2+\mu_\chi^2}\frac{\lambda^4}{H^4}\frac{k^2}{z^2}\left(\frac{z_0}{z}\right)^{3/2}\left(\nu_\varphi - \frac{3}{2}\right)$$

$$\boldsymbol{D}_{12}^{\mathrm{spur}} = -\frac{1}{4\mu_\chi}\frac{1}{\nu_\varphi^2+\mu_\chi^2}\frac{\lambda^4}{H^4}\frac{k}{z}\left(\frac{z_0}{z}\right)^{3/2}$$

$$\boldsymbol{\Delta}_{11}^{\mathrm{spur}} = \frac{1}{2\nu_\varphi}\frac{1}{\nu_\varphi^2+\mu_\chi^2}\frac{\lambda^4}{H^4}\frac{k^2}{z^2}\left(\frac{z_0}{z}\right)^{3/2}\left(\nu_\varphi - \frac{3}{2}\right)$$

$$\boldsymbol{\Delta}_{12}^{\mathrm{spur}} = -\frac{1}{4\nu_\varphi}\frac{1}{\nu_\varphi^2+\mu_\chi^2}\frac{\lambda^4}{H^4}\frac{k}{z}\left(\frac{z_0}{z}\right)^{3/2}$$

with $z = -k\eta$ and

$$\nu_\varphi \equiv \frac{3}{2}\sqrt{1-\left(\frac{2m}{3H}\right)^2}, \quad \text{and} \quad \mu_\chi = \frac{3}{2}\sqrt{\left(\frac{2M}{3H}\right)^2 - 1}$$

.

Analytic results on the covariance

Integrating the transport equation:

$$\begin{aligned}\boldsymbol{\Sigma}_{\varphi\varphi}(\eta) = &\, e^{-2\int_{\eta_0}^{\eta}\mathrm{d}\eta' \boldsymbol{\Delta}_{12}(\eta')}\boldsymbol{g}_{\mathsf{LS}}(\eta,\eta_0)\boldsymbol{\Sigma}_{\varphi\varphi}(\eta_0)\boldsymbol{g}^{\mathrm{T}}_{\mathsf{LS}}(\eta,\eta_0) \\ &- \int_{\eta_0}^{\eta}\mathrm{d}\eta' e^{-2\int_{\eta'}^{\eta}\mathrm{d}\eta'' \boldsymbol{\Delta}_{12}(\eta'')}\boldsymbol{g}_{\mathsf{LS}}(\eta,\eta')\left[\boldsymbol{\omega}\boldsymbol{D}(\eta')\boldsymbol{\omega}\right]\boldsymbol{g}^{\mathrm{T}}_{\mathsf{LS}}(\eta,\eta').\end{aligned}$$

SPT results (1)

Mode function decomposition

$$\widehat{v}_\varphi(\eta) = v_{\varphi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\varphi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\varphi\chi}(\eta)\widehat{a}_\chi + v^*_{\varphi\chi}(\eta)\widehat{a}^\dagger_\chi$$
$$\widehat{v}_\chi(\eta) = v_{\chi\varphi}(\eta)\widehat{a}_\varphi + v^*_{\chi\varphi}(\eta)\widehat{a}^\dagger_\varphi + v_{\chi\chi}(\eta)\widehat{a}_\chi + v^*_{\chi\chi}(\eta)\widehat{a}^\dagger_\chi$$

which obey **equations of motion**

$$v''_{\varphi\varphi} + \omega^2_\varphi(\eta)v_{\varphi\varphi} = -\lambda^2 a^2(\eta)v_{\chi\varphi}$$
$$v''_{\chi\varphi} + \omega^2_\chi(\eta)v_{\chi\varphi} = -\lambda^2 a^2(\eta)v_{\varphi\varphi}$$

and

$$v''_{\chi\chi} + \omega^2_\chi(\eta)v_{\chi\chi} = -\lambda^2 a^2(\eta)v_{\varphi\chi}$$
$$v''_{\varphi\chi} + \omega^2_\varphi(\eta)v_{\varphi\chi} = -\lambda^2 a^2(\eta)v_{\chi\chi}$$

SPT results (2)

Solution **order by order**

- Zeroth order:

$$v^{(0)}_{\varphi\varphi}(\eta) = v_\varphi(\eta)$$
$$v^{(0)}_{\chi\chi}(\eta) = v_\chi(\eta)$$

  and $v^{(0)}_{\varphi\chi}(\eta) = v^{(0)}_{\chi\varphi}(\eta) = 0$.

- First order:

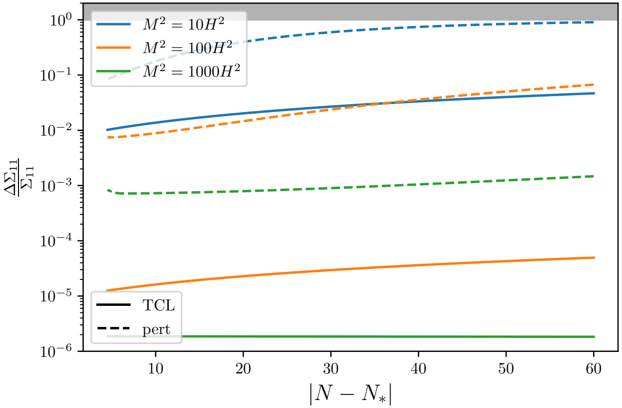
$$v^{(1)}_{\varphi\chi}(\eta) = -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\varphi(\eta) v_\varphi^*(\eta_1) \right\} v_\chi(\eta_1)$$
$$v^{(1)}_{\chi\varphi}(\eta) = -2\lambda^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \, a^2(\eta_1) \, \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_1) \right\} v_\varphi(\eta_1).$$

- ...

Correlators are evaluated in the Heisenberg picture

$$\boldsymbol{\Sigma}(\eta) = \frac{1}{2} \mathrm{Tr}\left[\left\{ \widehat{\boldsymbol{z}}(\eta), \widehat{\boldsymbol{z}}^{\mathrm{T}}(\eta) \right\} \widehat{\rho}_0 \right]$$

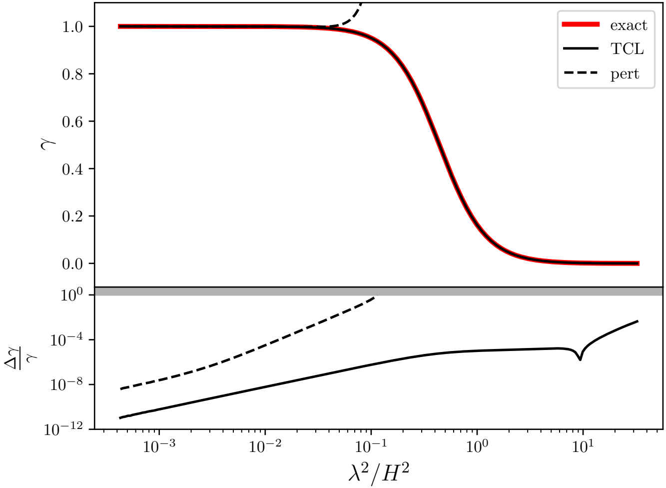
Power spectra and late-time resummation



Late-time resummation:

$$\boldsymbol{\Sigma}^{\mathrm{TCL}}_{\varphi\varphi} \supset e^{\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}|N-N_*|}\boldsymbol{\Sigma}^{(0)}_{\varphi\varphi}$$

Purity and coupling



Outline

The master equation zoo

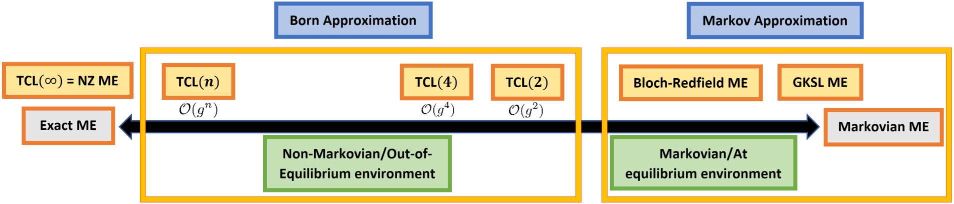
- Fundamental observables are correlators

$$\left\langle \widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \right\rangle (t) \equiv \mathrm{Tr}\left[\widehat{\mathcal{O}}_1 \widehat{\mathcal{O}}_2 \cdots \widehat{\mathcal{O}}_n \widehat{\rho}_{\mathrm{red}}(t) \right]$$

- **Master Equations** (ME) are dynamical equations for $\widehat{\rho}_{\mathrm{red}}(t)$

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}\left(\widehat{\rho}_{\mathrm{red}}\right)$$

- There exists a **whole bestiary** of MEs



At which level should we work in cosmology ?

The environment as noises

Classical Brownian motion

- Langevin equation

$$\mathrm{d}\mathcal{O} = \{H, O\}\mathrm{d}t + \mathrm{d}\xi$$

- Fokker-Planck equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \mathcal{L}_{\mathrm{FP}}[P]$$

- Wiener path integral [Wiener, 1923]

$$P = \int \mathcal{D}q e^{-S_0(q)}$$

Quantum Brownian motion

- Stochastic Schrödinger equation

$$|\mathrm{d}\psi\rangle = -i\left[\widehat{H}, \widehat{\mathcal{O}}\right]\mathrm{d}t + \mathrm{d}\widehat{\xi}$$

- Master equation

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}t} = \mathcal{V}[\widehat{\rho}_{\mathrm{red}}]$$

- Influence functional [Vernon, 1959]

$$\widehat{\rho}_{\mathrm{red}} = \int \mathcal{D}\phi_0^{\pm}\mathcal{I}\left[\phi_0^{\pm}\right]\widehat{\rho}_{\mathrm{red},0}$$

TCL_2 Fokker-Planck equation

The reduced Wigner function evolves according to

$$\frac{\mathrm{d}W_{\text{red}}}{\mathrm{d}\eta} = \left\{\widetilde{H}_0 + \widetilde{H}^{(\mathrm{LS})}, W_{\text{red}}\right\}$$
$$+\boldsymbol{\Delta}_{12}\sum_i \frac{\partial}{\partial \boldsymbol{z}_i}\left(\boldsymbol{z}_i W_{\text{red}}\right) - \frac{1}{2}\sum_{i,j}\left[\boldsymbol{\omega}\boldsymbol{D}\boldsymbol{\omega}\right]_{ij}\frac{\partial^2 W_{\text{red}}}{\partial \boldsymbol{z}_i \partial \boldsymbol{z}_j},$$

Exploring the hidden universe

How do we explore extensions to single-field slow-roll inflation?

- Already exist very successful approaches, eg.
  1. Stochastic inflation [Starobinsky & Yokoyama, 1994];
  2. EFT of inflation [Cheung *et al.*, 2007];
  3. Cosmological bootstrap [Arkani-Hamed *et al.*, 2018], [Pajer *et al.*, 2020];
- How do we build synergies/connections between these methods ?
  - **EFT**: systematic expansion of the Lagrangian including all higher order operators allowed by symmetries.
    $\Rightarrow$ Prescriptions to construct ME non-unitary dissipator ?
  - **Cosmological bootstrap**: constrained shape of environmental correlators based on the underlying symmetries.
    $\Rightarrow$ Constraints on the memory kernel ?

Characterizing imprints of the hidden universe on observable cosmology may reveal its nature.

Outline

Example 1: an exact ME

Master equation: dynamical equation for the quantum state of the system.

1. Start with Liouville-von Neumann equation

$$\frac{\mathrm{d}\widetilde{\rho}}{\mathrm{d}\eta} = -ig\left[\widetilde{\mathcal{H}}_{\mathsf{int}}(\eta), \widetilde{\rho}(\eta)\right] \equiv g\mathcal{L}(\eta)\widetilde{\rho}(\eta)$$

2. Introduce projectors $\widetilde{\rho} \mapsto \mathcal{P}\widetilde{\rho} = \widetilde{\rho}_{\mathsf{red}} \otimes \rho_{\mathrm{E}}$ and $\mathcal{Q}\widetilde{\rho} = \widetilde{\rho} - \mathcal{P}\widetilde{\rho}$

3. Rewrite dynamics as

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \mathcal{K}(\eta, \eta')\mathcal{P}\widetilde{\rho}(\eta)$$

$\mathcal{K}(\eta, \eta')$: **memory kernel** which depends on **the coupling** and **the environment**.

Example 2: an effective ME

1. Expand in powers of **the coupling** constant

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\mathcal{P}\widetilde{\rho}(\eta) = \sum_{n=0}^{\infty} g^n \mathcal{K}_n(\eta)\mathcal{P}\widetilde{\rho}(\eta)$$

2. Lowest order leads to the non-Markovian ME

$$\frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -g^2 \int_{\eta_0}^{\eta} \mathrm{d}\eta' \, \mathrm{Tr}_E \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta), \left[\widetilde{\mathcal{H}}_{\mathrm{int}}(\eta'), \widetilde{\rho}_{\mathrm{red}}(\eta) \otimes \rho_{\mathrm{E}}\right]\right]$$

3. Error function $e_r^{(2)} \sim g^2 ||\mathcal{K}_4(\eta)||/||\mathcal{K}_2(\eta)||$.

Example 3: a Markovian ME

1. When the environment is a **bath** (large number of dof, thermal equilibrium), the dynamics is **Markovian**, the system admits a **semi-group evolution**

$$V(\eta_1)V(\eta_2) = V(\eta_1 + \eta_2)$$

.

2. It implies a specific form for the ME [Lindblad 1976]

$$\frac{\mathrm{d}\widehat{\rho}_{\mathrm{red}}}{\mathrm{d}\eta} = -i\left[\widehat{H}(\eta), \widehat{\rho}_{\mathrm{red}}(\eta)\right] + \sum_k \gamma_k \left[\widehat{\boldsymbol{L}}_k \widehat{\rho}_{\mathrm{red}}(\eta)\widehat{\boldsymbol{L}}_k^\dagger - \frac{1}{2}\left\{\widehat{\boldsymbol{L}}_k^\dagger \widehat{\boldsymbol{L}}_k, \widehat{\rho}_{\mathrm{red}}(\eta)\right\}\right]$$

3. It relies on a fast decay of temporal correlations in the environment.

Question: At which level should we work in cosmology ?

The emergence of Markovianity

- Fast decay of environmental correlations

$$\mathcal{K}^{>}(\eta, \eta') \xrightarrow[\text{graining}]{\text{coarse-}} \delta(\eta - \eta')$$

ME reduces to a GKSL equation for which the dynamical map reads

$$\mathcal{L}\left[\widehat{\rho}_{\text{red}}\right] = -i\left[\widehat{\mathcal{H}}, \widehat{\rho}_{\text{red}}\right] + \gamma\left(\widehat{L}\widehat{\rho}_{\text{red}}\widehat{L}^{\dagger} - \frac{1}{2}\left\{\widehat{L}^{\dagger}\widehat{L}, \widehat{\rho}_{\text{red}}\right\}\right)$$

GKSL equation is CPTP: physical consistency of the solutions ensured.

- Non-Markovian evolution/non-semigroup dynamical map implies dissipator matrix non-positive semi-definite.

- Non-positive semi-definite dissipator matrix is a generic feature of Non-Markovian OQS: not directly related to CPTP properties.

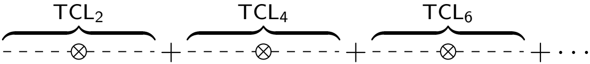
- Curved-space Caldeira-Leggett model ME belongs to the class of Gaussian non-Markovian ME $\Rightarrow$ CPTP ensured by [Diósi & Ferialdi, 2014].

Outline

What has been resummed ?

- In the **exact theory**, there is only one 1PI:

- In the **effective theory**, there is an infinite tower of 1PI:



  one for each of the TCL cumulant.

- Moreover, there are **non-unitary contributions** from diffusion and dissipation which do not have diagrammatic representation.
- Hence, the question of knowing **which diagram has been resumed** is **ill-posed**. This feature is shared with WEFT and the DRG.

Late-time resummation technique

Following [Boyanovsky, 2015], [Brahma *et al.*, 2021],

$$\langle \widetilde{v}_\varphi(\eta)\widetilde{v}_\varphi(\eta)\rangle = v_-(\eta)v_-(\eta)\left\langle \widehat{P}_\varphi^2 \right\rangle + v_-(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi\widehat{P}_\varphi + \widehat{P}_\varphi\widehat{Q}_\varphi \right\rangle$$
$$+v_+(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi^2 \right\rangle \to v_+(\eta)v_+(\eta)\left\langle \widehat{Q}_\varphi^2 \right\rangle$$

with

$$\frac{\mathrm{d}\left\langle \widehat{Q}_\varphi^2 \right\rangle}{\mathrm{d}\eta} = \Gamma(\eta)\left\langle \widehat{Q}_\varphi^2 \right\rangle$$

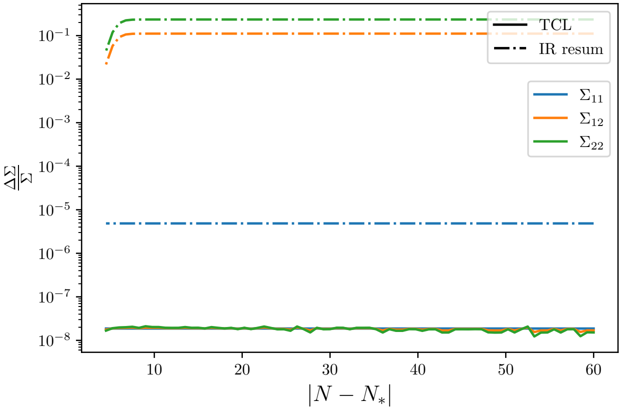
obtained from the TCL_2 ME.

In the *curved-space Caldeira-Leggett model*, leads to

$$\boldsymbol{\Sigma}_{\varphi\varphi}^{\mathrm{TCL}} \supset e^{-\frac{1}{\nu_\varphi}\frac{H^2}{M^2-m^2}\frac{\lambda^4}{H^4}\ln(-k\eta)}\boldsymbol{\Sigma}_{\varphi\varphi}^{(0)}$$

where late-time secular effects have been resummed.

Late-time resummation and the DRG



This resummation technique shares many features with the DRG [Burgess *et al.*, 2009].

Are they equivalent ?

Outline

TCL$_4$ generator

$$
\begin{aligned}
\mathcal{K}_4(\eta) = \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 \int_{\eta_0}^{\eta_2} \mathrm{d}\eta_3 \\
\Big[\mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_1)\mathcal{P}\mathcal{L}(\eta_2)\mathcal{L}(\eta_3)\mathcal{P} \\
-\mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_2)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_3)\mathcal{P} - \mathcal{P}\mathcal{L}(\eta)\mathcal{L}(\eta_3)\mathcal{P}\mathcal{L}(\eta_1)\mathcal{L}(\eta_2)\mathcal{P}\Big]
\end{aligned}
$$

TCL$_4$ master equation

$$
\begin{aligned}
\frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_4}}{\mathrm{d}\eta} = \frac{\mathrm{d}\widetilde{\rho}_{\mathrm{red}}^{\mathrm{TCL}_2}}{\mathrm{d}\eta} - 4\lambda^8 a^2(\eta) \int_{\eta_0}^{\eta} \mathrm{d}\eta_1 a^2(\eta_1) \int_{\eta_0}^{\eta_1} \mathrm{d}\eta_2 a^2(\eta_2) \int_{\eta_0}^{\eta_2} \mathrm{d}\eta_3 a^2(\eta_3) \\
\times \Big\{ \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_2) \right\} \mathrm{Re}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_3) \right\} \mathrm{Im}\left\{ v_\varphi(\eta_1) v_\varphi^*(\eta_2) \right\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_3), \widetilde{\rho}_{\mathrm{red}}(\eta)]] \\
+ i\, \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_2) \right\} \mathrm{Im}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_3) \right\} \mathrm{Im}\left\{ v_\varphi(\eta_1) v_\varphi^*(\eta_2) \right\} [\widetilde{v}_\varphi(\eta), \{\widetilde{v}_\varphi(\eta_3), \widetilde{\rho}_{\mathrm{red}}(\eta)\}] \\
+ \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_3) \right\} \mathrm{Re}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \right\} \mathrm{Im}\left\{ v_\varphi(\eta_1) v_\varphi^*(\eta_3) \right\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_2), \widetilde{\rho}_{\mathrm{red}}(\eta)]] \\
+ i\, \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_3) \right\} \mathrm{Im}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \right\} \mathrm{Im}\left\{ v_\varphi(\eta_1) v_\varphi^*(\eta_3) \right\} [\widetilde{v}_\varphi(\eta), \{\widetilde{v}_\varphi(\eta_2), \widetilde{\rho}_{\mathrm{red}}(\eta)\}] \\
+ \Big[- \mathrm{Re}\left\{ v_\chi(\eta) v_\chi^*(\eta_3) \right\} \mathrm{Im}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \right\} + \mathrm{Im}\left\{ v_\chi(\eta) v_\chi^*(\eta_3) \right\} \mathrm{Re}\left\{ v_\chi(\eta_1) v_\chi^*(\eta_2) \right\} \Big] \\
\mathrm{Im}\left\{ v_\varphi(\eta_2) v_\varphi^*(\eta_3) \right\} [\widetilde{v}_\varphi(\eta), [\widetilde{v}_\varphi(\eta_1), \widetilde{\rho}_{\mathrm{red}}(\eta)]] \Big\}.
\end{aligned}
$$

Equivalence between perturbative TCL and in-in formalism

Cosmologists are used to compute correlators using the **in-in formalism**.

- At linear order, it is similar to the perturbative results presented above.

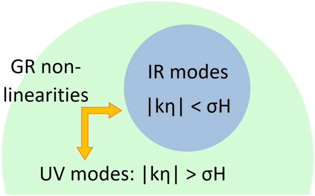
We have shown that:

- Perturbative TCL_2 is equivalent to $\mathcal{O}(\lambda^4)$ in-in.
- Perturbative TCL_4 is equivalent to $\mathcal{O}(\lambda^8)$ in-in.

Probably the proof **extend at all order**. Indeed, from the TCL cumulant expansion, all terms at a given order are included. It should ensure the matching with the in-in formalism at a given order.

Outline

The OpenEFT formalism



In [Brahma *et al.*, 2020], the leading cubic contribution is

$$H_{\mathrm{int}} = \frac{M_{\mathrm{Pl}}^2}{2} \int \mathrm{d}^3 x \varepsilon_H^2 a \zeta^2 \partial^2 \zeta$$

- UV modes backreact on the IR dynamics.
- They induce decoherence of the IR sector.