Kinematic aberration of gravitational waveforms

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Based on a work in preparation in collaboration with
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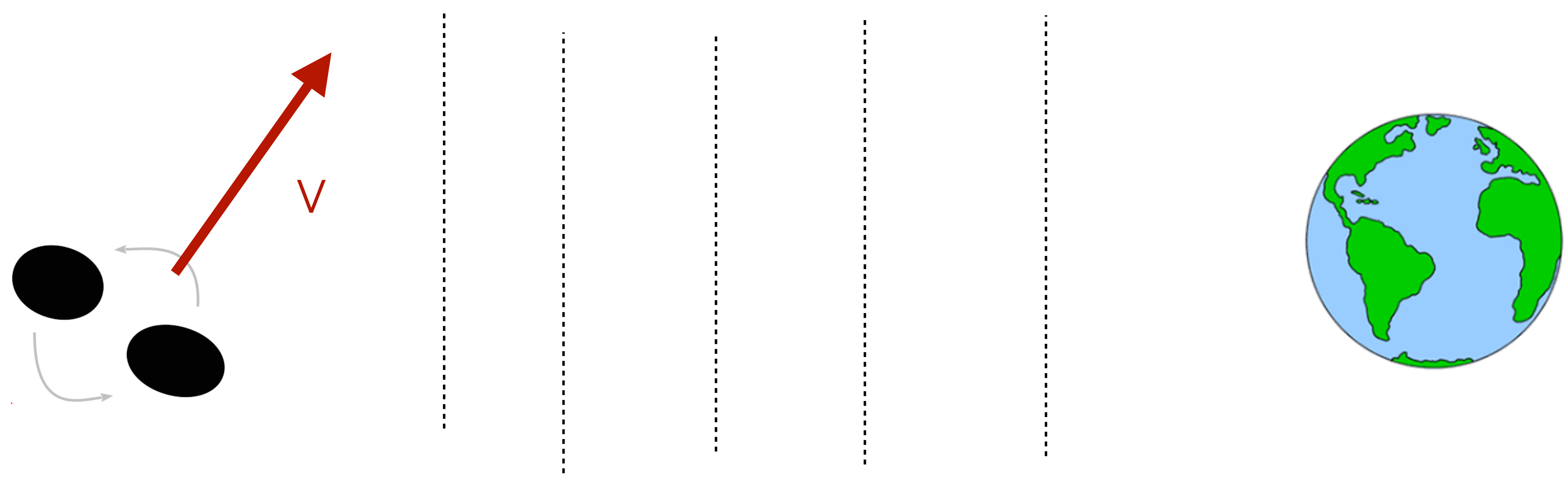
and G. Congedo, J. Gair, N. Tamanini

Outline

How are the two polarisations of a GW affected by the presence of a peculiar motion source-observer?



GW is a spin-2 object, it transforms as a **tensor under boosts**: non-transverse components are generated by the presence of peculiar velocities

In the observer frame, **spin-1 quantities** are generated

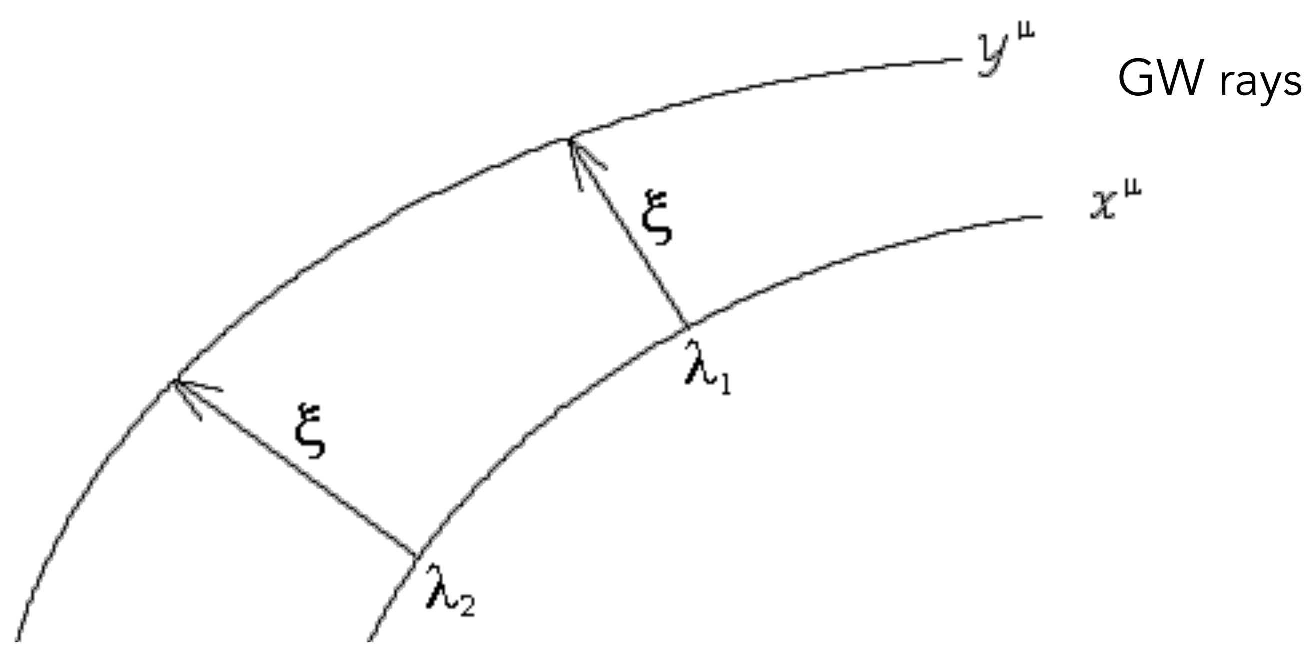
How does this effect manifest itself on observable quantities? Which are the **observational implications**?

Polarizations of a GW: general concepts

Effect of GW on test particles can be described by

$$\frac{d^2\xi^i}{dt^2} = \mathcal{R}_{0i0j}\xi^j$$ **geodesic deviation equation** (ξ^i vector between two nearby rays)

$$P_{ij}(t) \equiv \mathcal{R}_{0i0j}$$ **driving force matrix** (proportional to the GW in TT gauge)



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driving force matrix (proportional to the GW in TT gauge)

In a generic theory of gravity: 6 polarisations. For a wave propagating along z

$$P_{ij}(t) = \begin{pmatrix} -\mathrm{Re}\Psi_4 - \Phi_{22} & \mathrm{Im}\Psi_4 & -2\sqrt{2}\mathrm{Re}\Psi_3 \\ \mathrm{Im}\Psi_4 & \mathrm{Re}\Psi_4 - \Phi_{22} & 2\sqrt{2}\mathrm{Im}\Psi_3 \\ -2\sqrt{2}\mathrm{Re}\Psi_3 & 2\sqrt{2}\mathrm{Im}\Psi_3 & -6\Psi_2 \end{pmatrix}$$

general relativity: plus and cross polarisations

Polarisations transverse to the polarisation plane (modified gravity)

Ψ_4

2 dofs

Ψ_3

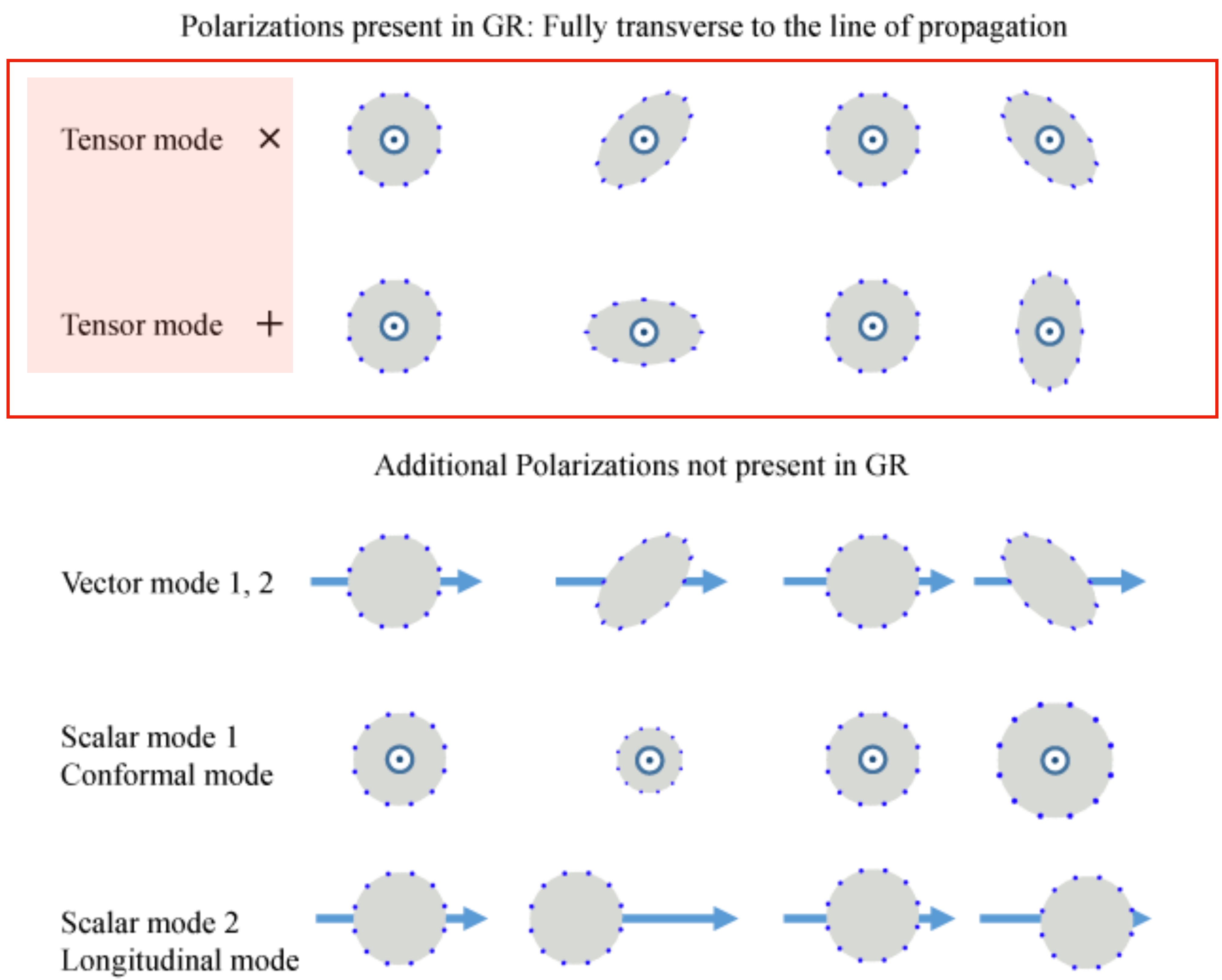
2 dofs

Φ_{22}

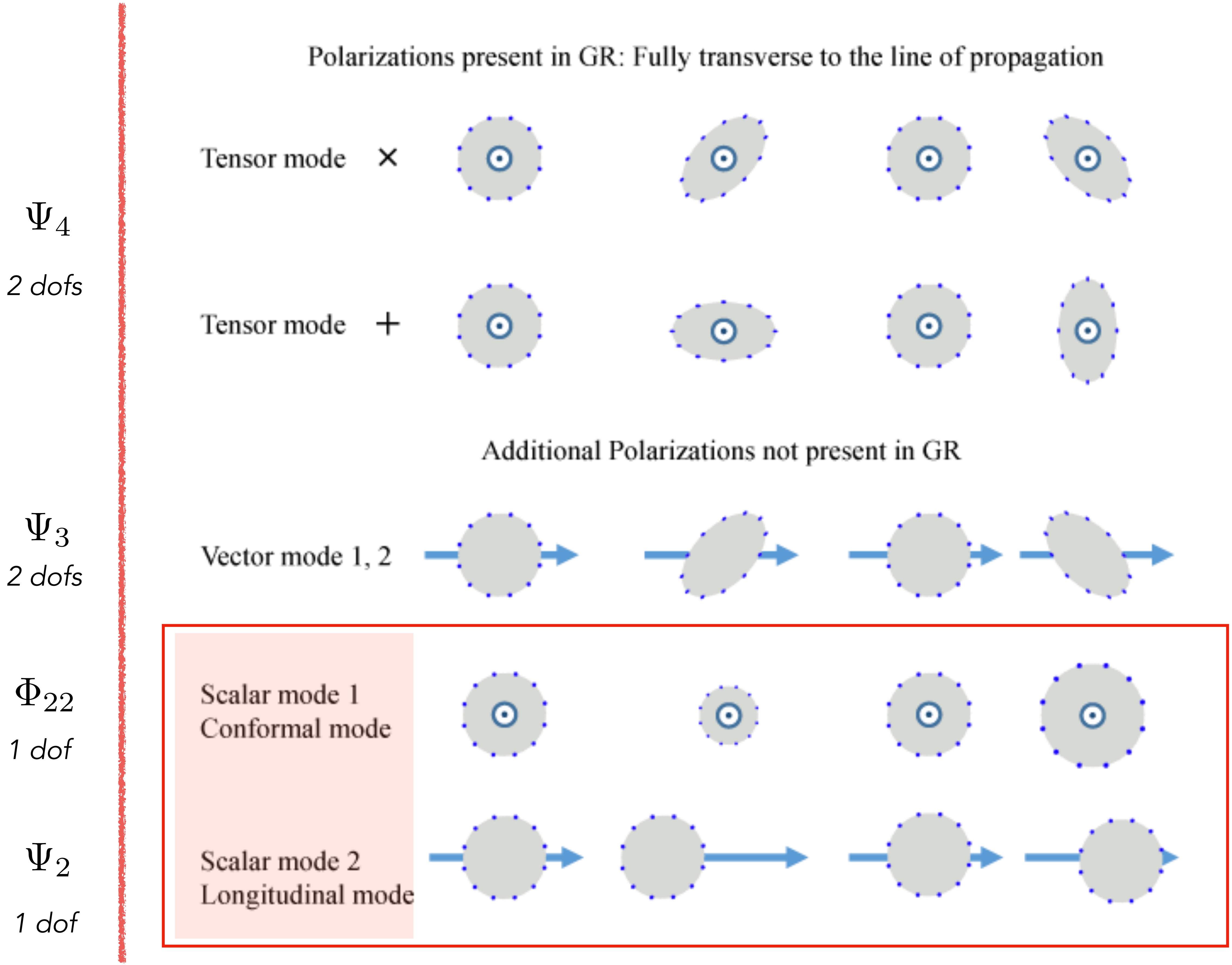
1 dof

Ψ_2

1 dof



C de Rham "Massive gravity"



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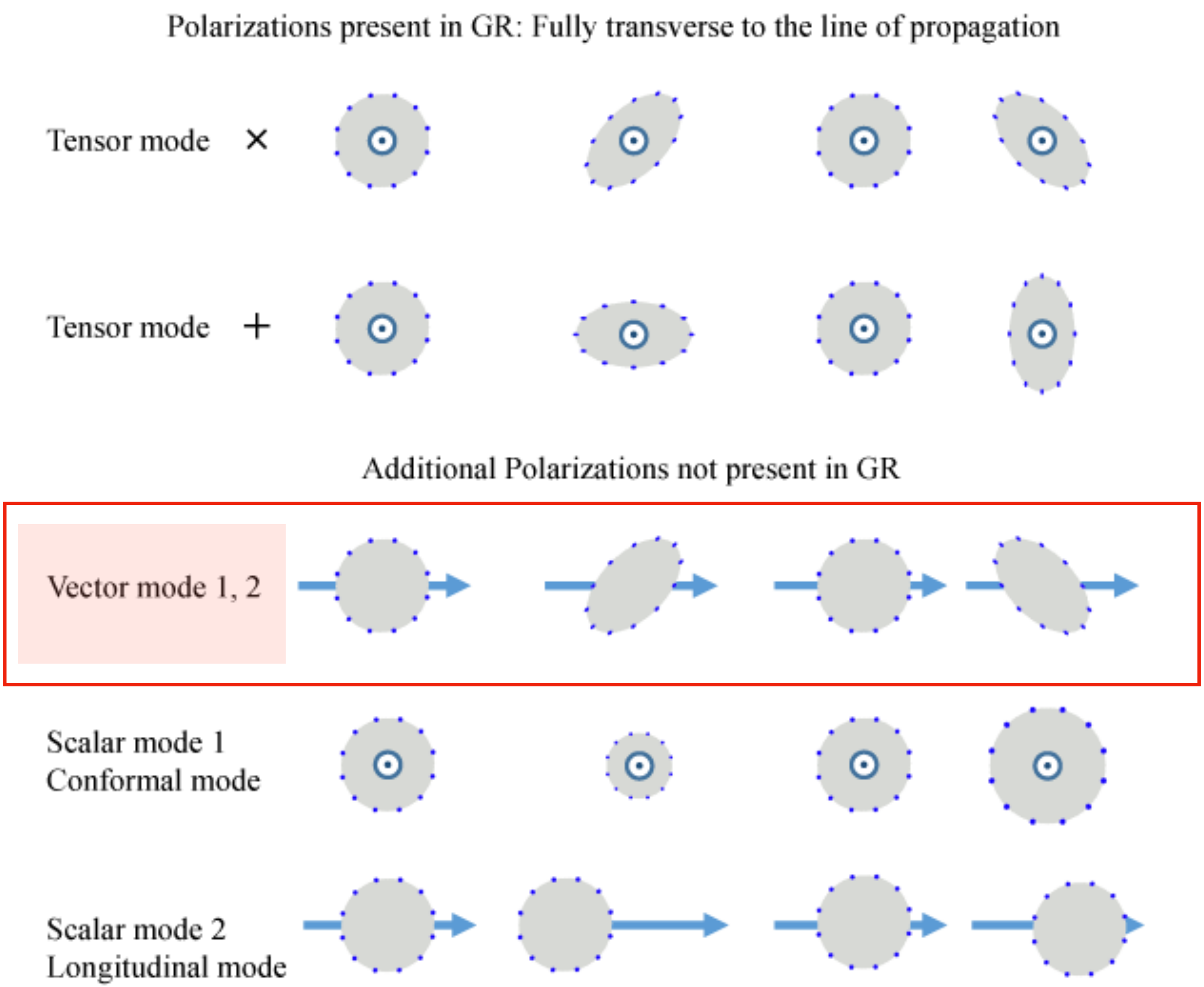

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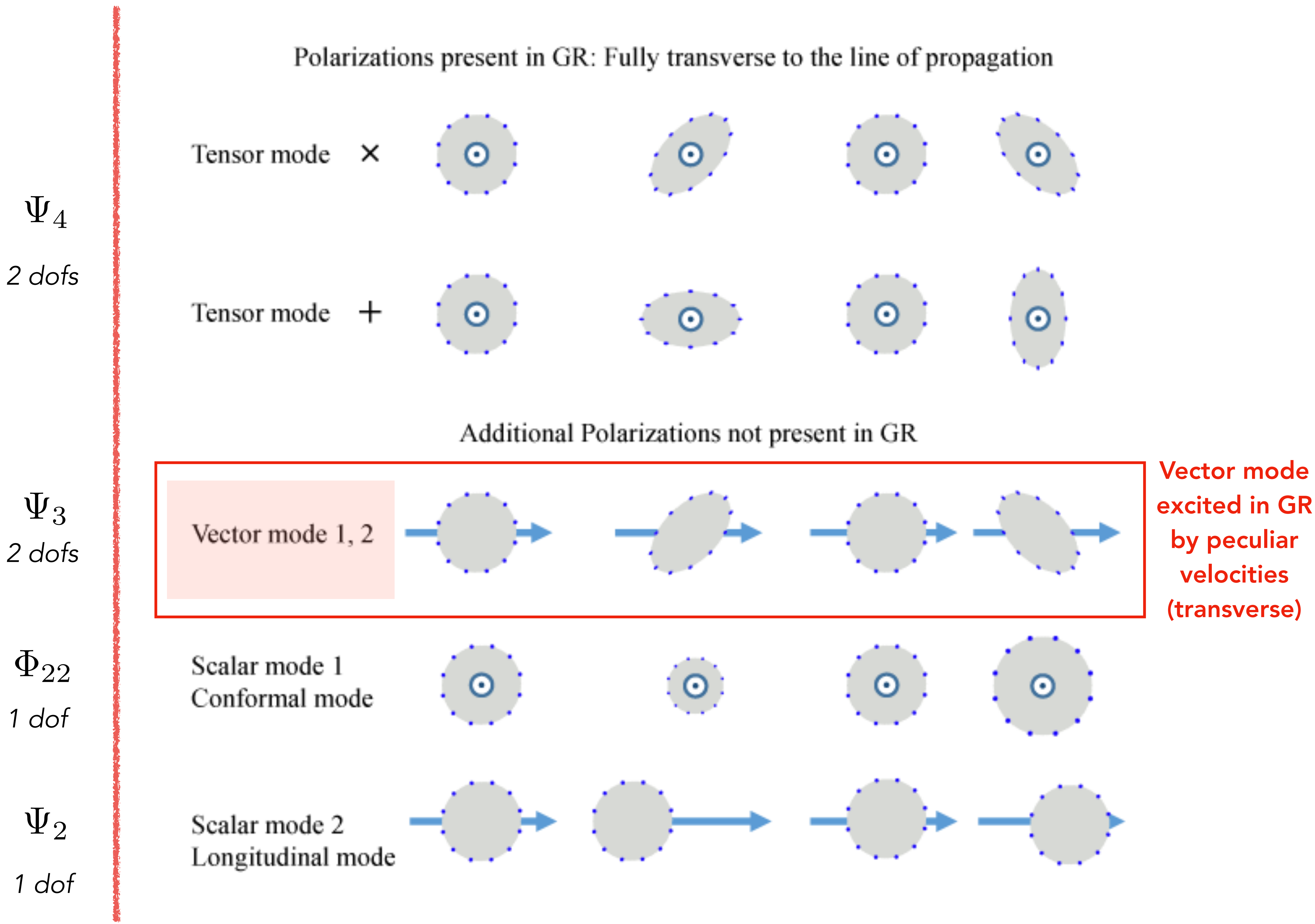

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Polarizations of a GW: general concepts

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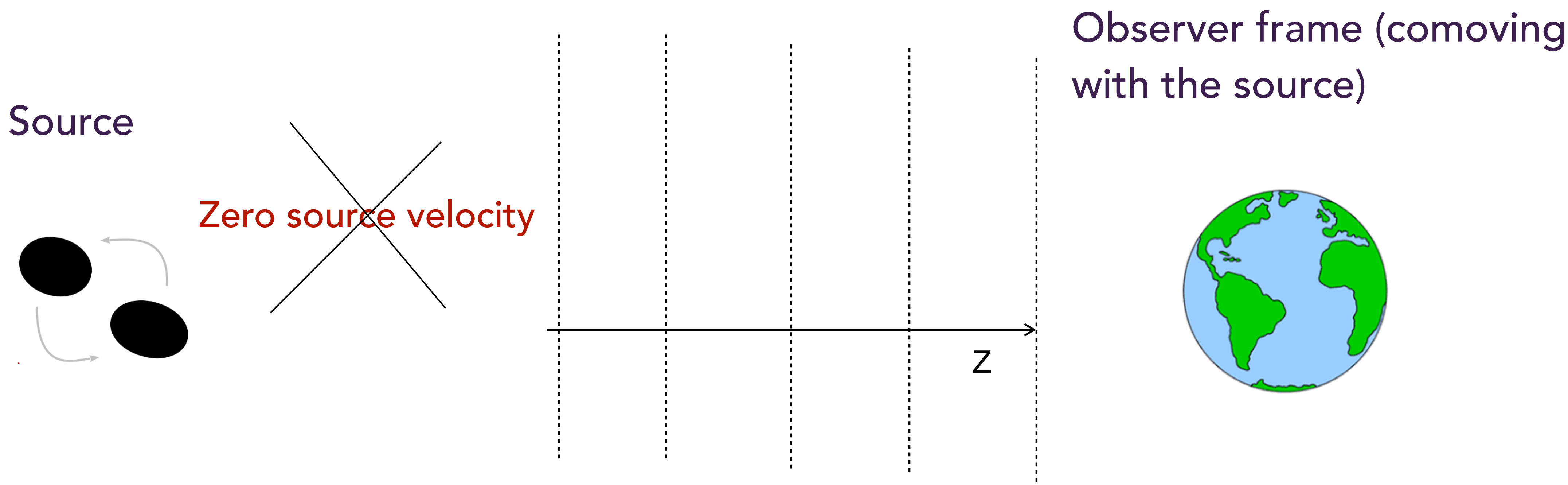
For a wave propagating along the z direction

$$P_{ij}(t) = \begin{pmatrix} -\mathrm{Re}\Psi_4 - \Phi_{22} & \mathrm{Im}\Psi_4 & -2\sqrt{2}\mathrm{Re}\Psi_3 \\ \mathrm{Im}\Psi_4 & \mathrm{Re}\Psi_4 - \Phi_{22} & 2\sqrt{2}\mathrm{Im}\Psi_3 \\ -2\sqrt{2}\mathrm{Re}\Psi_3 & 2\sqrt{2}\mathrm{Im}\Psi_3 & -6\Psi_2 \end{pmatrix}$$

Apparent transverse polarisations appear in general relativity in the presence of a relative motion source-observer

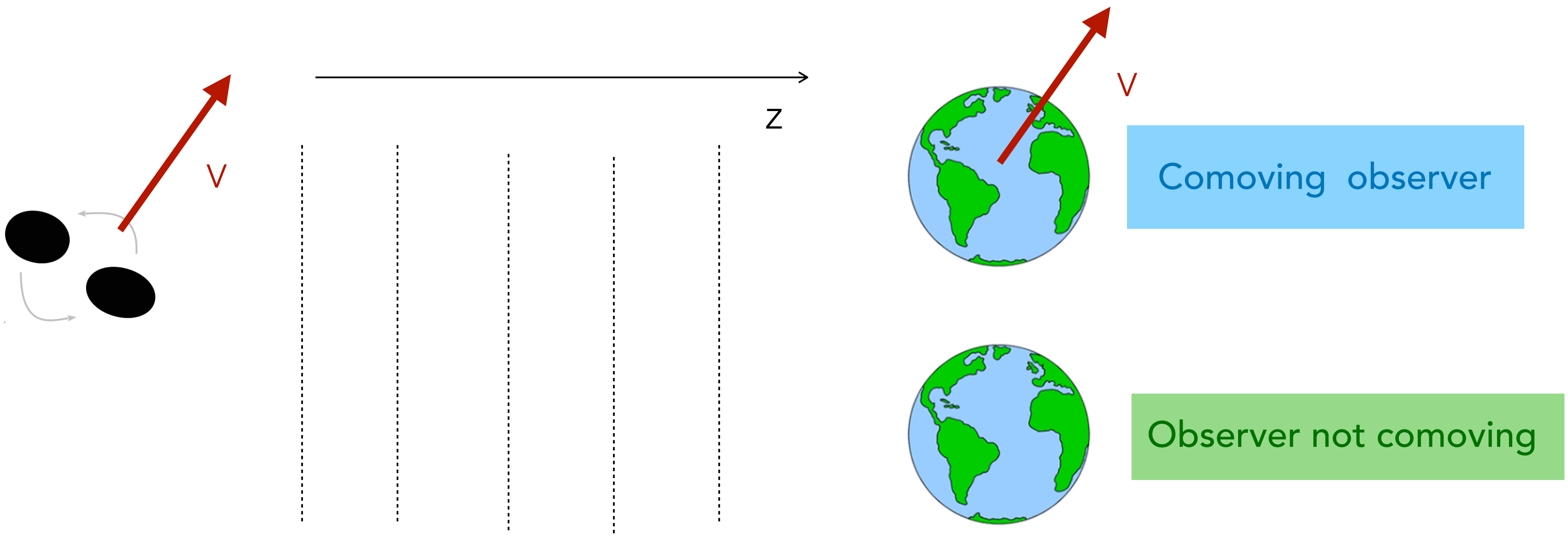
Observed GW signal: no relative velocity observer-source



Observed GW signal for wave propagating along z

$$P_{ij} = \frac{1}{2} \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Observed strain: relative velocity source-observer



GW for observer non-comoving

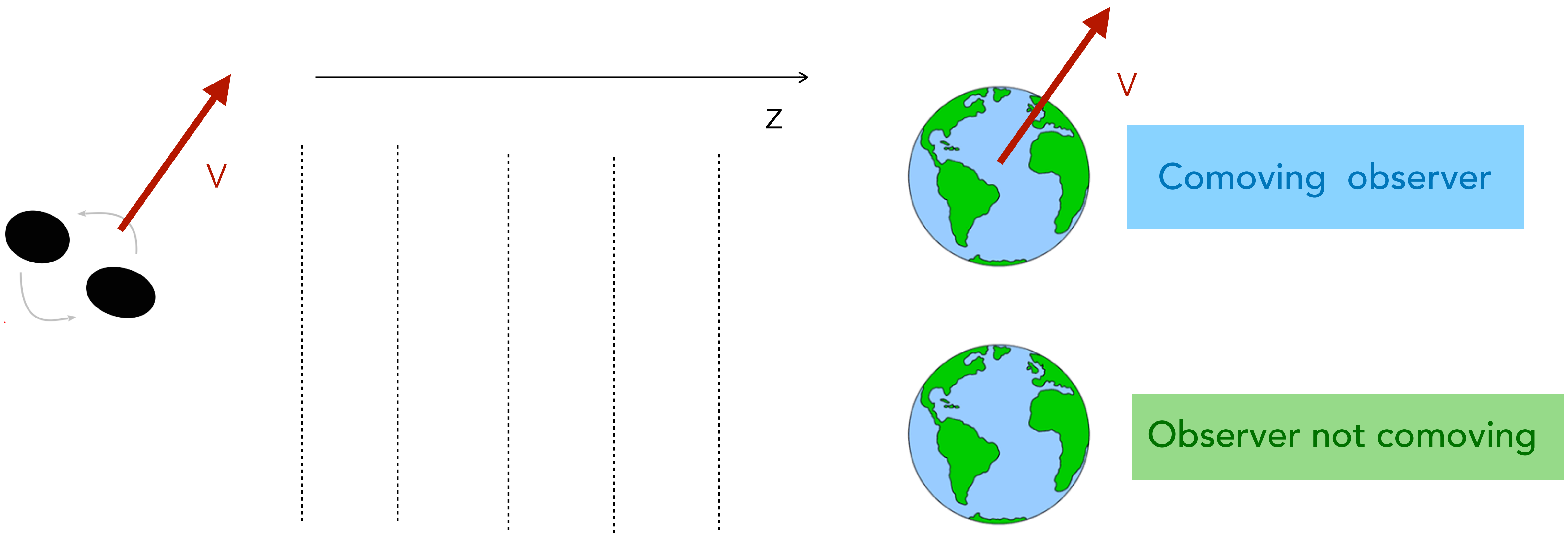
GW for observer comoving

Boost transformation

$$h_{\mu\nu} = \Lambda_{\mu}{}^{\alpha} \Lambda_{\nu}{}^{\beta} \tilde{h}_{\alpha\beta}$$

Lorentz Matrix

Observed strain: relative velocity source-observer



Observed gravitational wave propagating along z (in TT gauge)

$$P_{ij} = \begin{pmatrix} \tilde{h}_+ & \tilde{h}_\times & 0 \\ \tilde{h}_\times & -\tilde{h}_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Comoving observer

$$P_{ij} = \begin{pmatrix} \tilde{h}_+ & \tilde{h}_\times & v_x\tilde{h}_+ + v_y\tilde{h}_\times \\ \tilde{h}_\times & -\tilde{h}_+ & v_x\tilde{h}_\times - v_y\tilde{h}_+ \\ v_x\tilde{h}_+ + v_y\tilde{h}_\times & v_x\tilde{h}_\times - v_y\tilde{h}_+ & 0 \end{pmatrix},$$

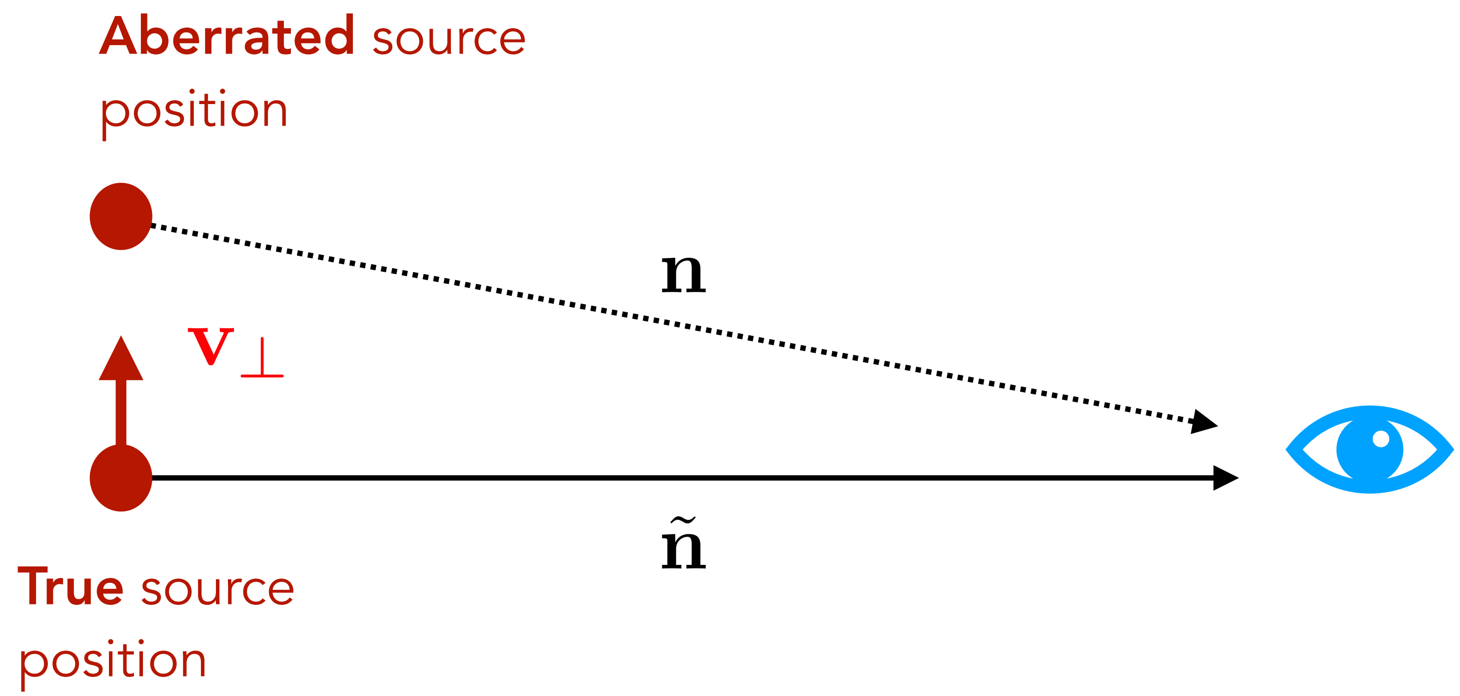
Relative velocity source-observer: spin-1 modes excited as an effect of aberration

Observationally, what do we actually see?

In the presence of a peculiar motion, the **direction of propagation is aberrated**

$$\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|} = \tilde{\mathbf{n}} - \boxed{\mathbf{v}_\perp}$$

Transverse velocity



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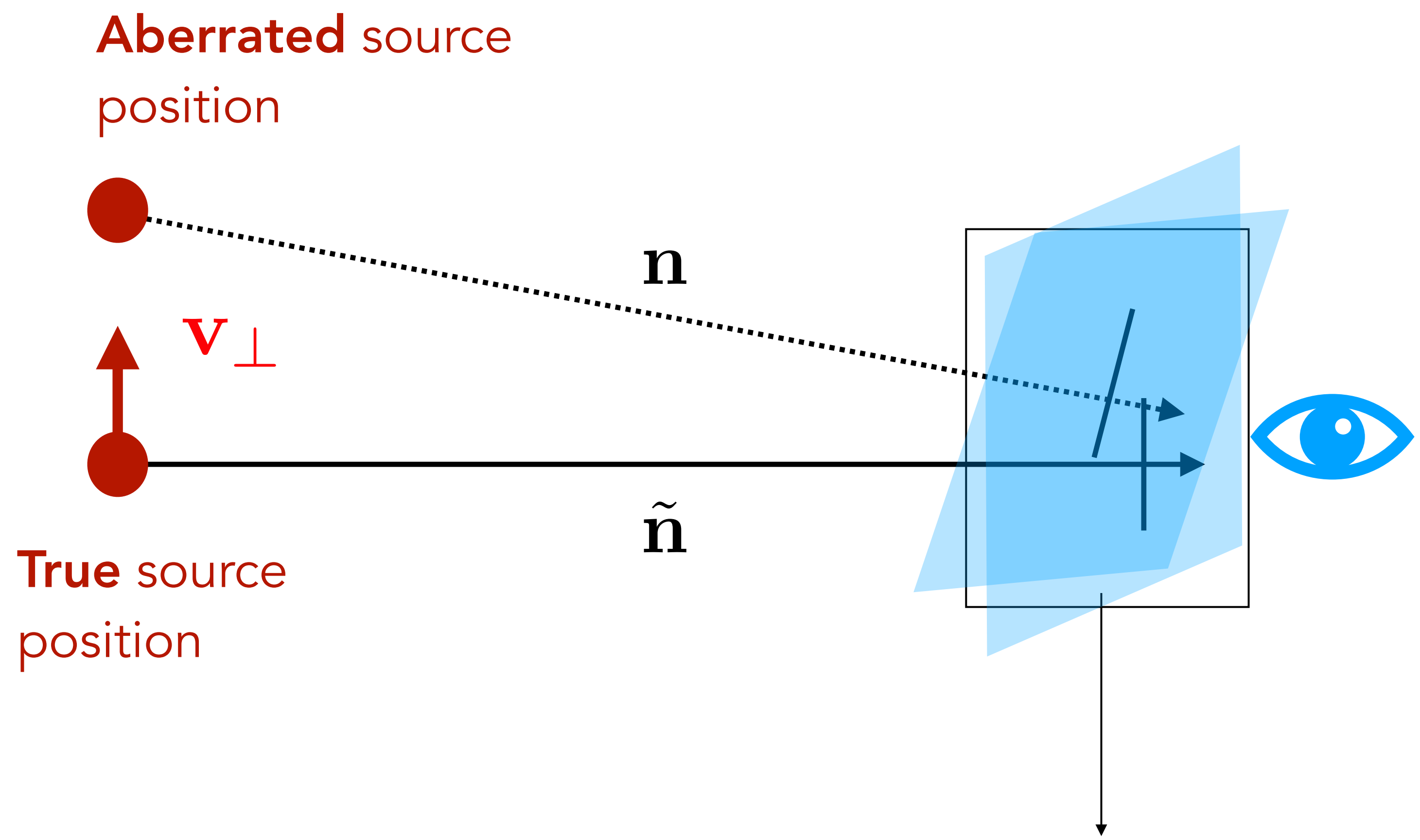
Transverse velocity



Polarisation plane is orthogonal to the observed direction, hence is aberrated as well

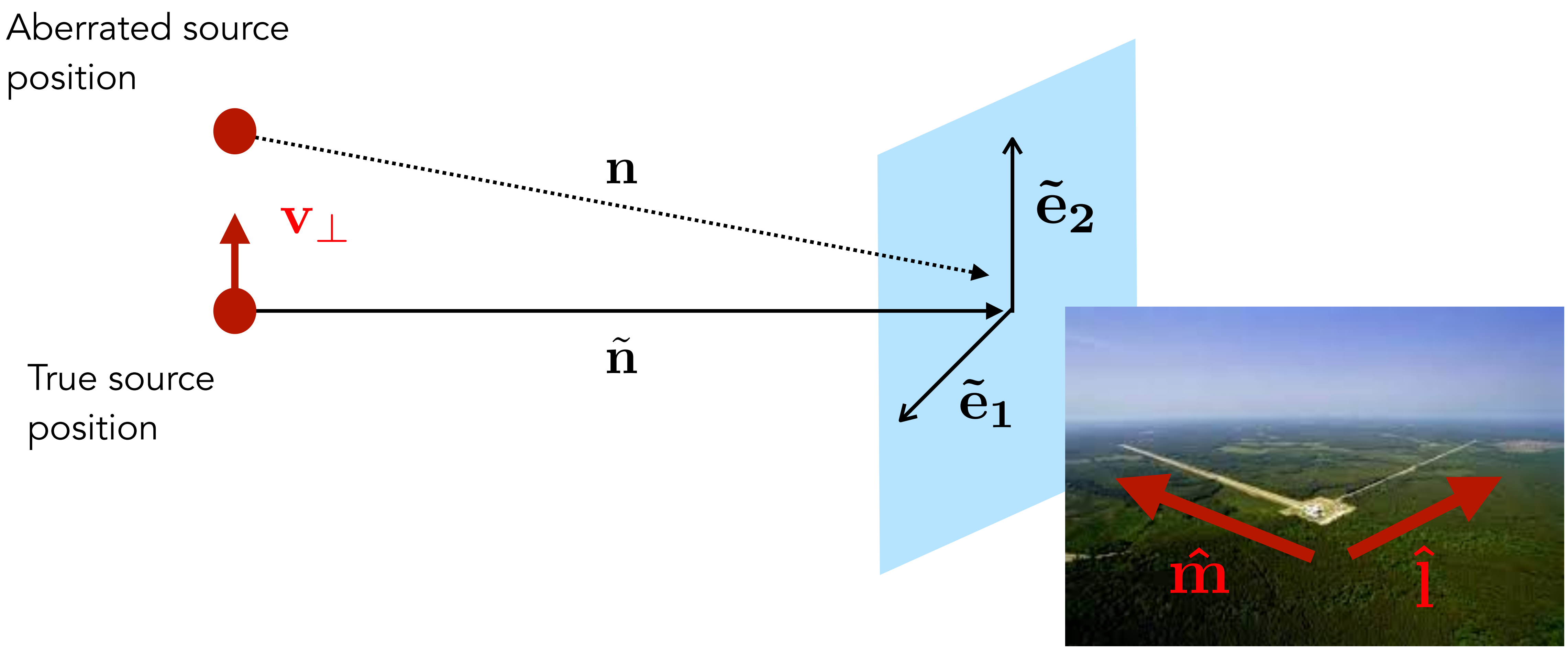
Observed strain with respect to non-aberrated polarisation basis

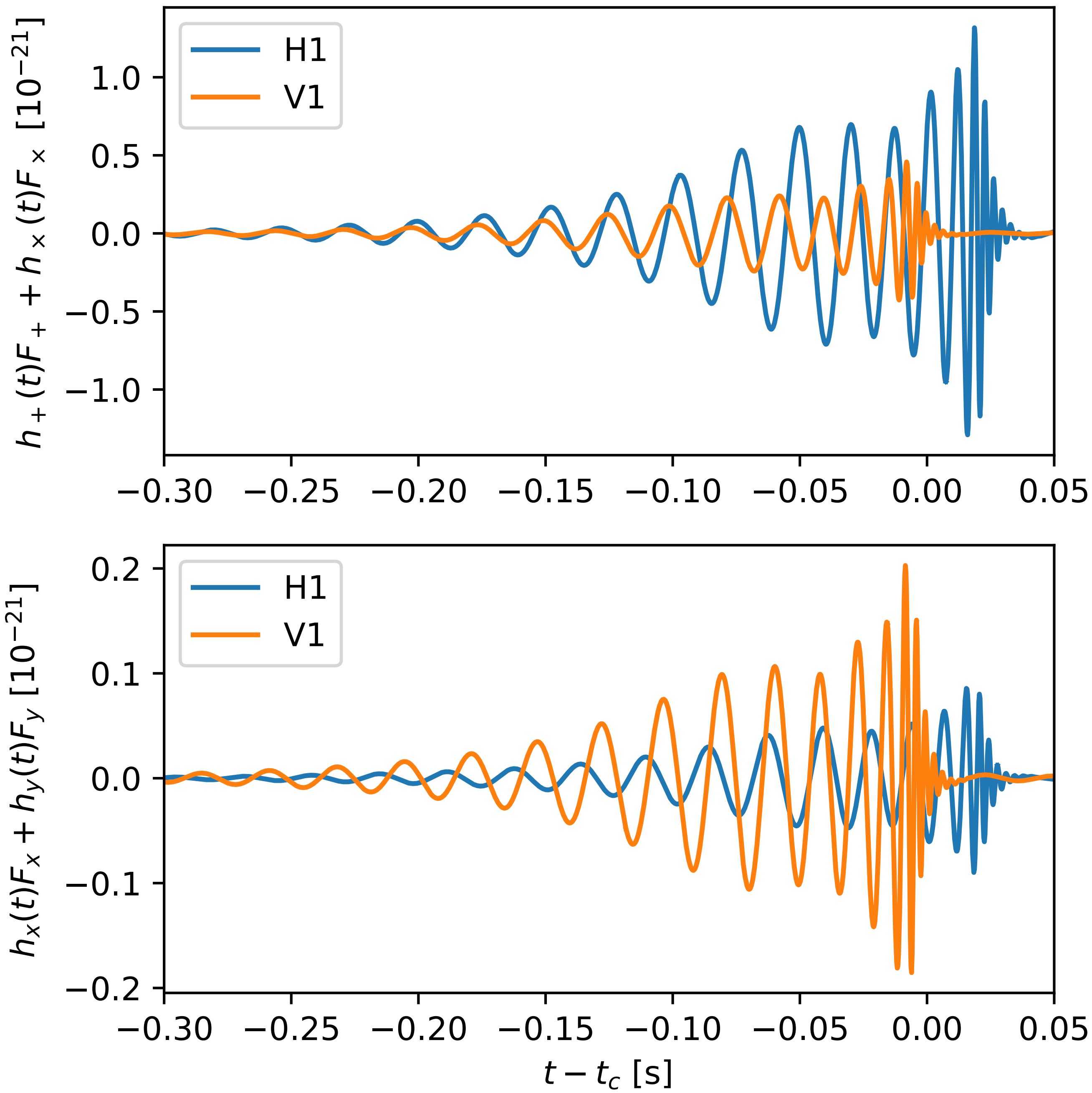
If we could access the non-aberrated (true) source position $\tilde{\mathbf{n}}$

$$P_{ij}(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j) = F_+(\tilde{\mathbf{n}})h_+ + F_\times(\tilde{\mathbf{n}})h_\times + F_1(\tilde{\mathbf{n}})h_1 + F_2(\tilde{\mathbf{n}})h_2$$

Detector tensor

Spin-1 modes (longitudinal to polarisation plane)





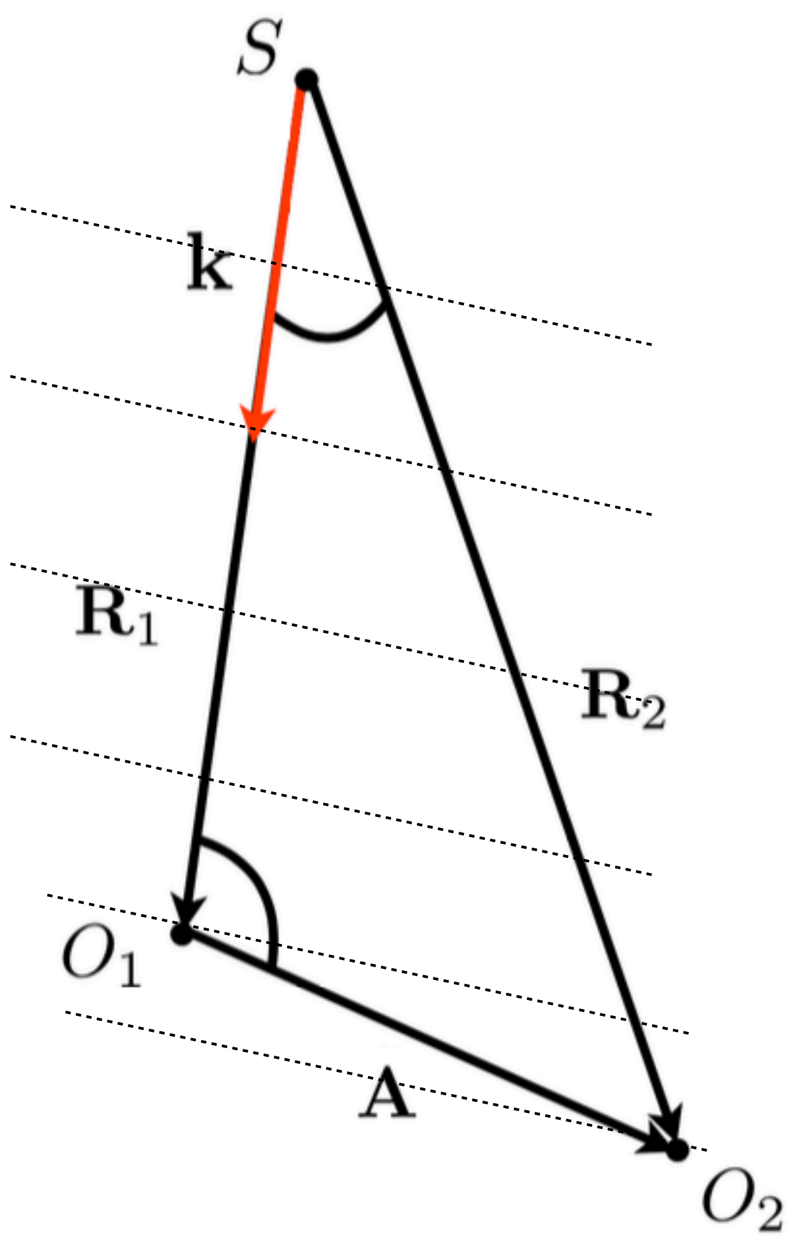
Tensor modes

Vector modes

Binary with $30M_\odot - 30M_\odot$

At 500 Mpc and transverse velocity **0.1 c** (sky position of GW170817)

Is there a way to reconstruct the true position of the source



What about time-delay information

$$\Phi(t, \mathbf{R}_1) = -k^\mu x_{\mu 1} = E\left(t - \mathbf{R}_1 \cdot \mathbf{n}\right)$$
$$\Phi(t, \mathbf{R}_2) = -k^\mu x_{\mu 2} = E\left(t - \mathbf{R}_2 \cdot \mathbf{n}\right)$$

Phase shift

$$\Delta\Phi = -E\,\mathbf{A} \cdot \mathbf{n}$$

Aberrated direction

$$\mathbf{n} = \frac{\mathbf{k}}{|\mathbf{k}|} = \tilde{\mathbf{n}} - \mathbf{v}_\perp$$

If I have multiple interferometers in a network, from **phase shift** I can only reconstruct **aberrated direction**

Observed strain with respect to aberrated polarisation basis

However, we can only access the **aberrated direction**: distorted spin-2

Kinematic mixing

$$P_{ij}(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j) = \hat{h}_+ F_+(\mathbf{n}) + \hat{h}_\times F_\times(\mathbf{n})$$

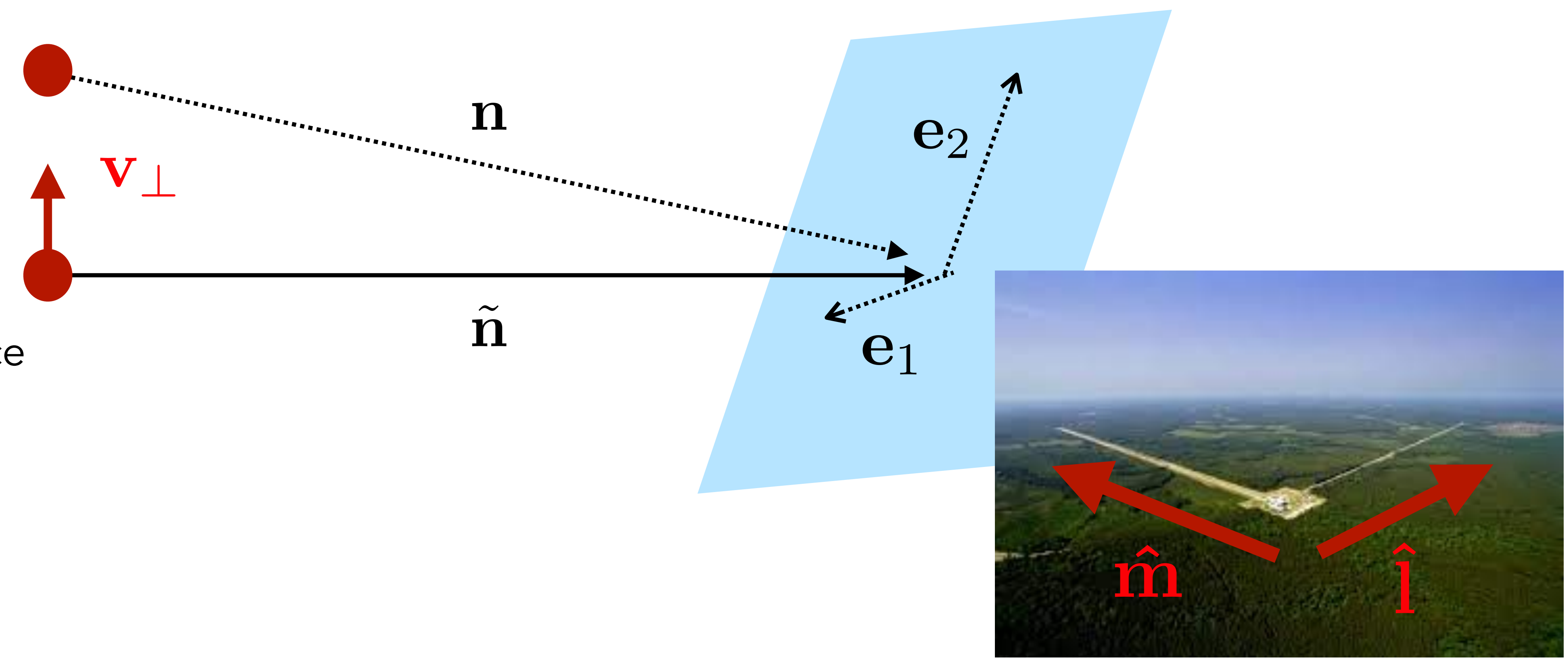
Detector tensor

$$\hat{h}_+ \equiv \tilde{h}_+ \cos(2\delta\psi) + \tilde{h}_\times \sin(2\delta\psi)$$
$$\hat{h}_\times \equiv \tilde{h}_\times \cos(2\delta\psi) - \tilde{h}_+ \sin(2\delta\psi)$$

$$\delta\psi \propto v_\perp$$

Aberrated source position

True source position



Observationally: distorted spin-2 polarizations

Kinematic mixing

$$P_{ij}(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j) = \hat{h}_+ F_+(\mathbf{n}) + \hat{h}_\times F_\times(\mathbf{n})$$

$$\hat{h}_+ \equiv \tilde{h}_+ \cos(2\delta\psi) + \tilde{h}_\times \sin(2\delta\psi)$$
$$\hat{h}_\times \equiv \tilde{h}_\times \cos(2\delta\psi) - \tilde{h}_+ \sin(2\delta\psi)$$

From an **observational point of view**, I will see only spin-2 fields but
— from aberrated direction
— with mixed polarisations (with respect to the emitted ones) $\delta\psi \propto v_\perp$

Transverse velocities induce a bias in the reconstruction of orbital parameters
How important is this bias for cosmology (luminosity distance and sky localisation)?

How important is this kinematic induced bias

We simulate **three different populations** in detector frame masses

1) neutron star binaries with $1.4M_{\odot} - 1.4M_{\odot}$

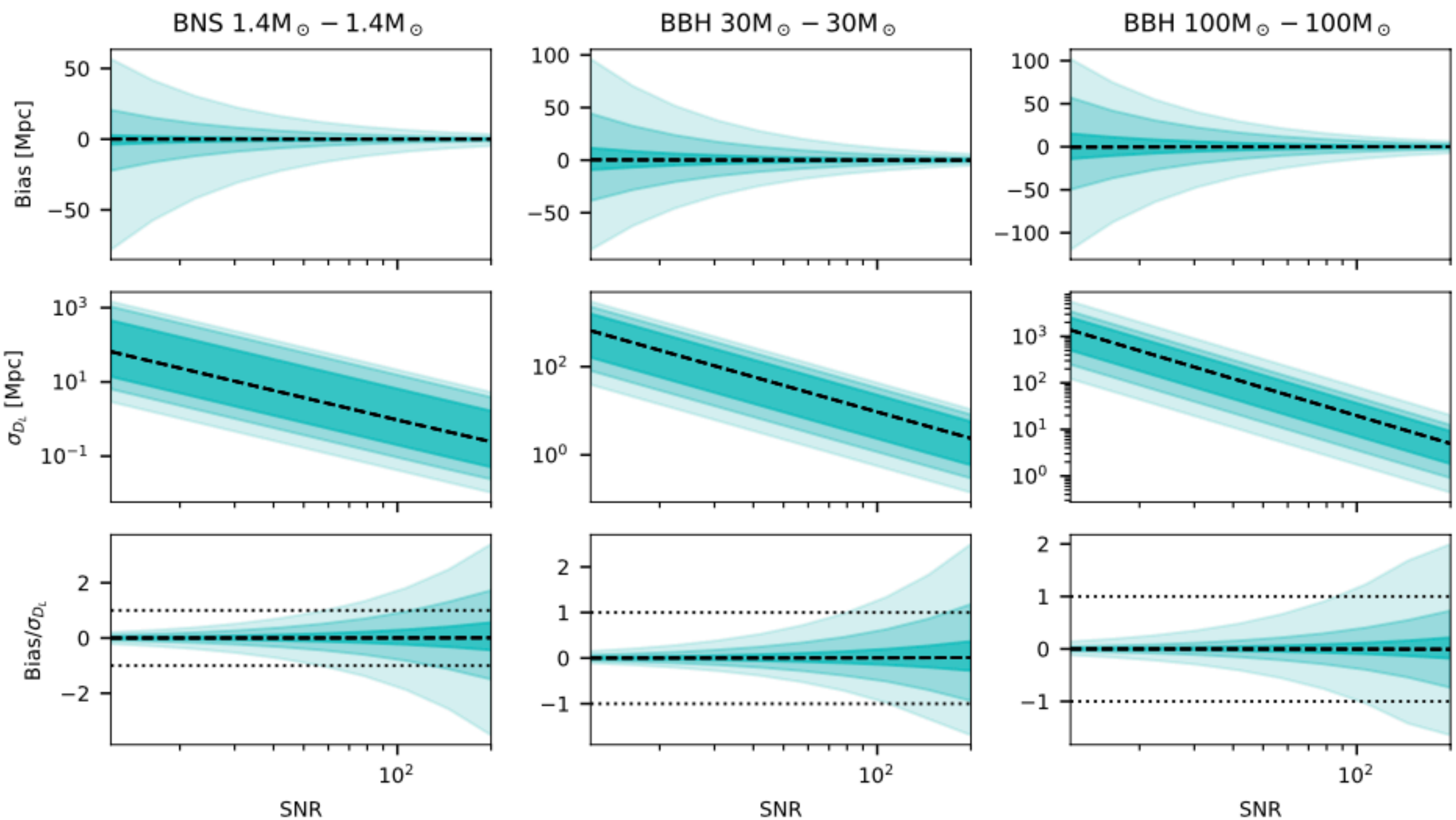
2) black hole-black hole binaries with $30M_{\odot} - 30M_{\odot}$

3) black hole-black hole binaries with $100M_{\odot} - 100M_{\odot}$

Assumptions: isotropic sky distribution and orbital orientation, aligned spins, time of arrival uniform in one year

Peculiar motion: isotropic with modulus from Maxwellian distribution with mean 500 km/s. (in agreement with galaxy observations)

Kinematic induced bias on luminosity distance (2LIGO+Virgo)



Bias

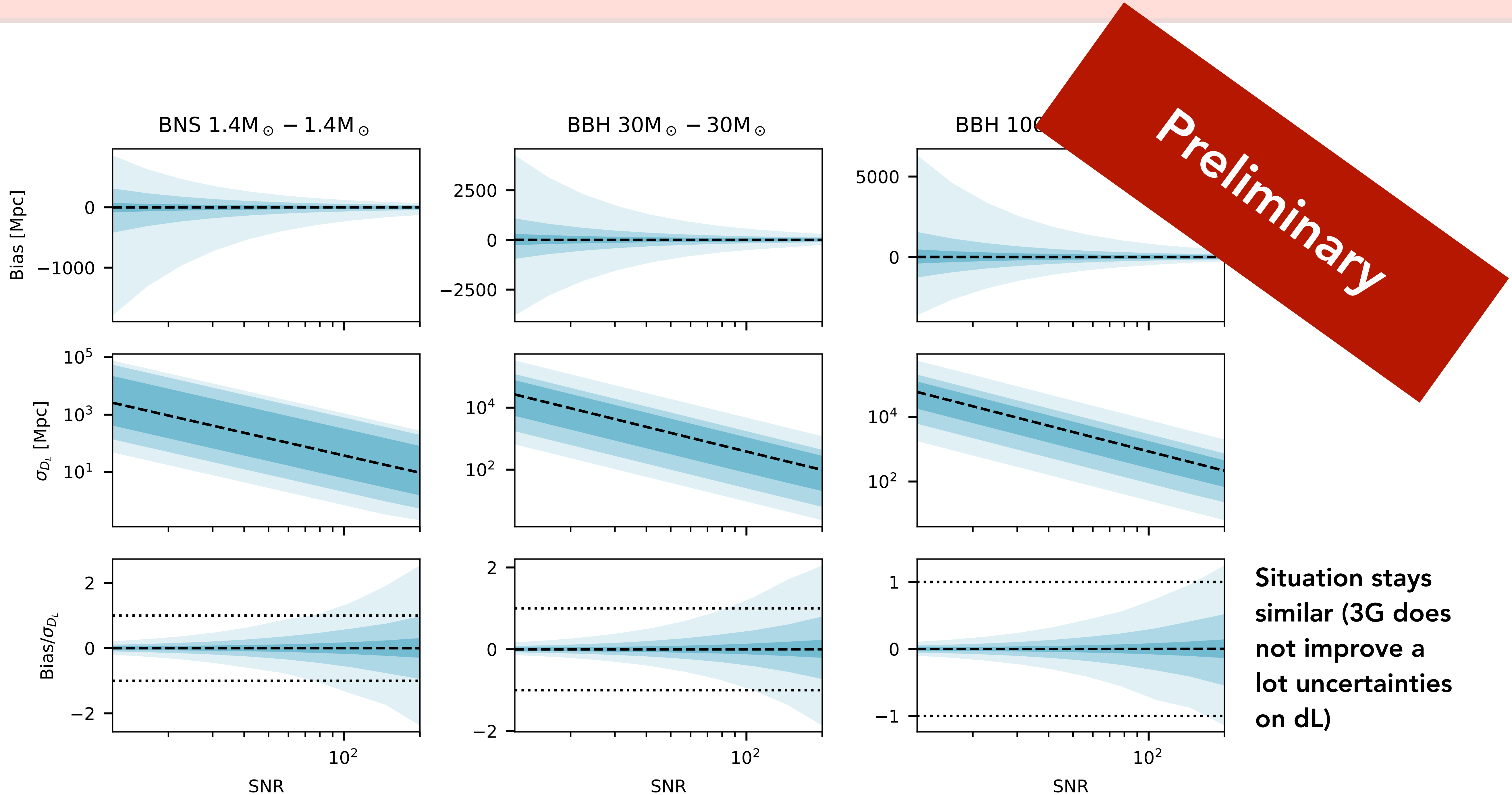
Statistical error

Bias/statistical error

5% chance of getting bias for SNR>100

Typical bias scales as $|v|d_L$ hence are larger for low SNR (low distance)
However for these events, the statistical error is also larger: bias important at high SNR

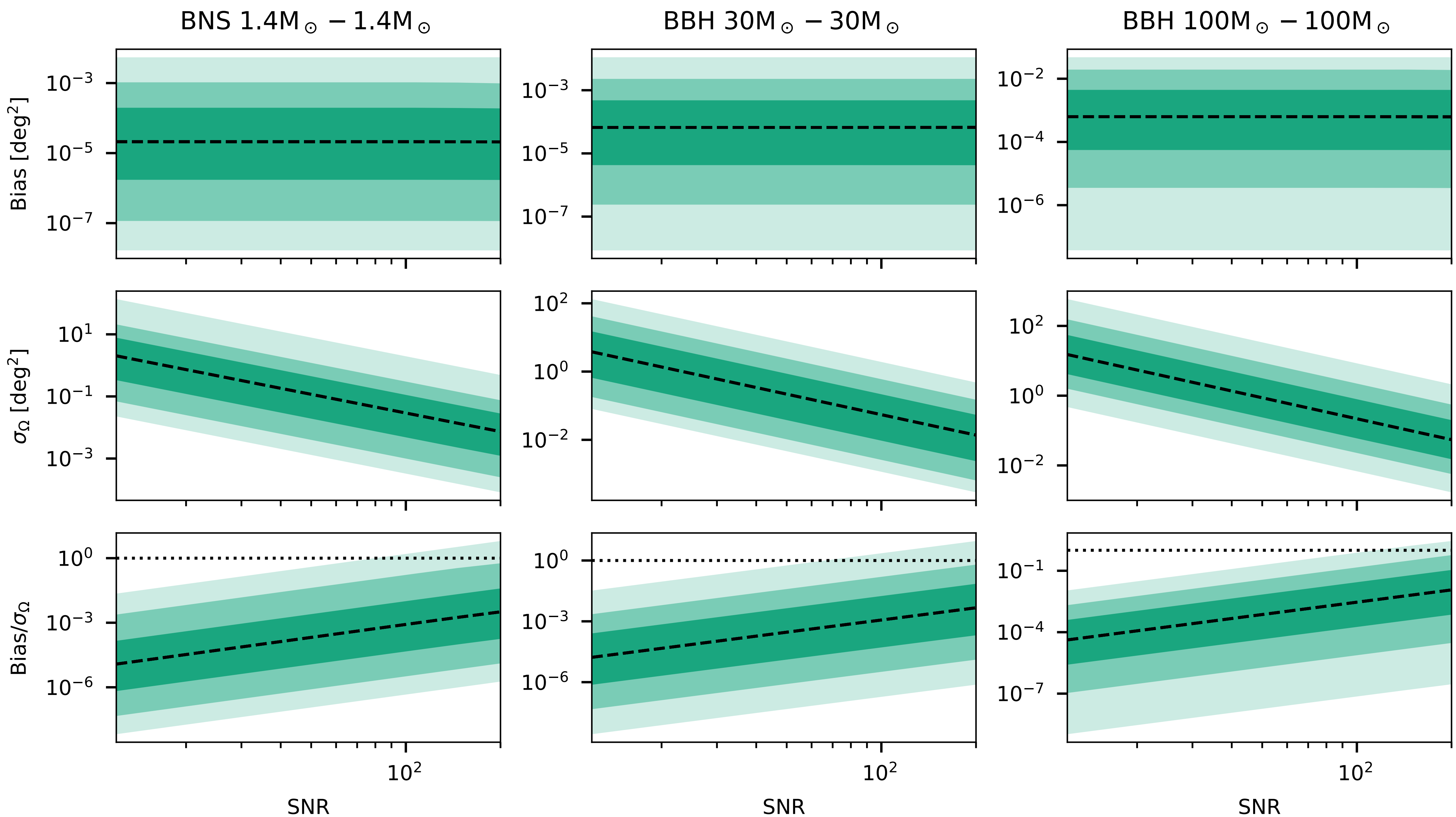
Kinematic induced bias on luminosity distance (ET+CE)



Situation stays similar (3G does not improve a lot uncertainties on dL)

High number of observable events: every year O(1) binary neutron star merger and O(10) binary black holes with bias on reconstruction dL larger than 1-sigma

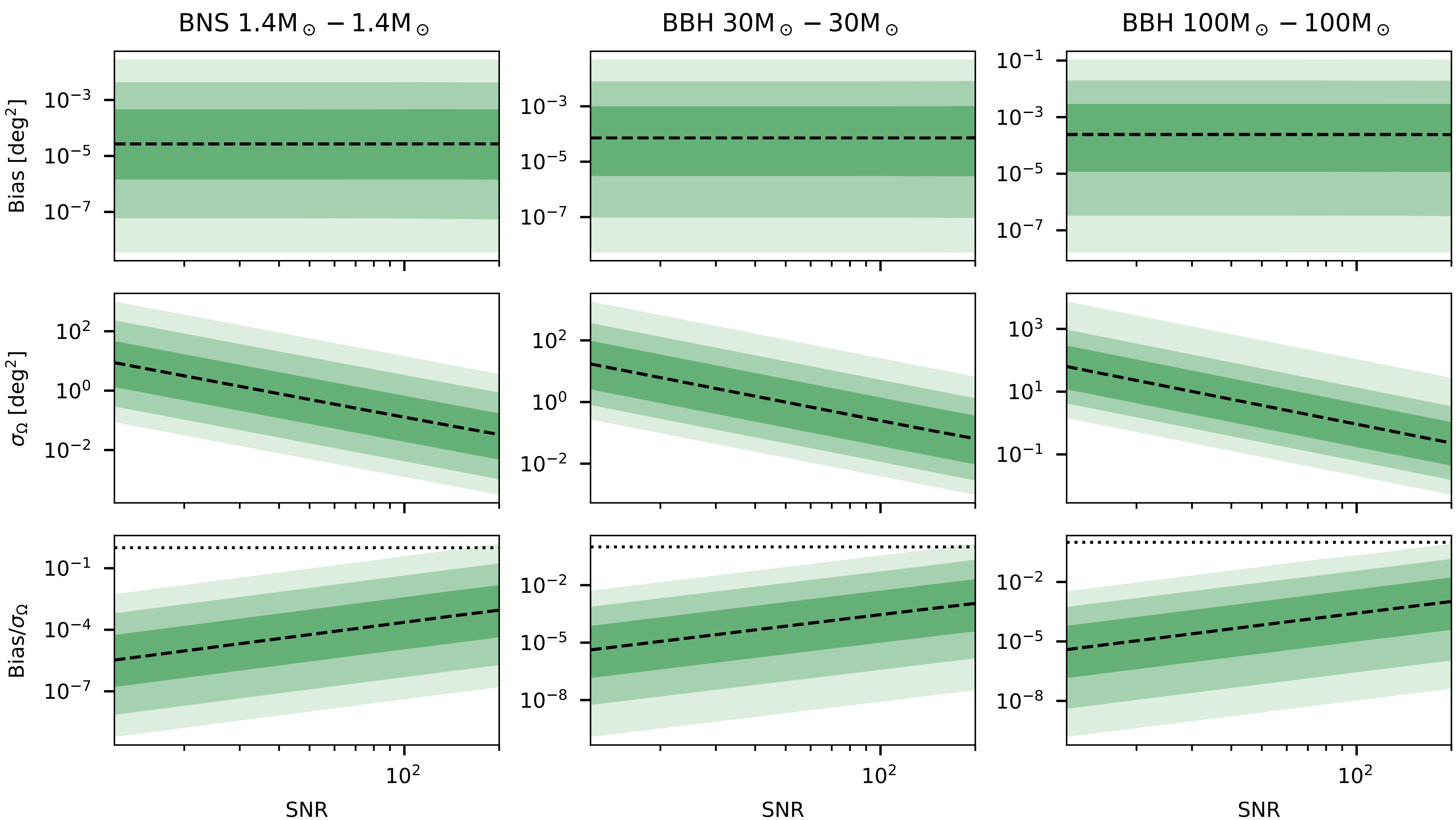
Kinematic induced bias on sky localisation (sky areas) - LVC case



Bias on localisation area is flat in SNR: it depends on $|v_\perp|^2$ which does not significantly affect SNR

Sky position bias not represent problem as statistical errors are very large

Kinematic induced bias on sky localisation (sky areas) - ET+CE case



Statistical errors larger of a factor 10 with respect to the LVC case (we only have 2 detectors). Bias on sky localisation even less relevant.

Take home message

—Source velocity transverse to the line of sight: **spin-1** appearing in the observer frame
These are not new degrees of freedom, as they are proportional to spin-2 polarisations

—They appear because of **aberration**: polarisation plane observed is not the true one

—**Observationally** we only have access to aberrated direction: the antenna pattern functions are defined wrt this direction. We reconstruct spin-2 modes aberrated, with a **kinematic mixing**

—This gives an **irreducible bias** in the reconstruction of orbital parameters

—Relevant when we use binaries as standard sirens: for ET+CE we expect order 10 events to have **bias larger than 1-sigma** in the reconstruction of luminosity distance

Thank you

$$
\begin{aligned}
F_{+}(\tilde{\mathbf{n}}) &= \frac{1}{2}\left(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j\right)\left(\tilde{e}_{1i}\tilde{e}_{1j} - \tilde{e}_{2i}\tilde{e}_{2j}\right), \\
F_{\times}(\tilde{\mathbf{n}}) &= \frac{1}{2}\left(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j\right)\left(\tilde{e}_{1i}\tilde{e}_{2j} + \tilde{e}_{2i}\tilde{e}_{1j}\right), \\
F_{1}(\tilde{\mathbf{n}}) &= \frac{1}{2}\left(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j\right)\left(\tilde{n}_i\tilde{e}_{1j} + \tilde{e}_{1i}\tilde{n}_j\right), \\
F_{2}(\tilde{\mathbf{n}}) &= \frac{1}{2}\left(\hat{l}_i\hat{l}_j - \hat{m}_i\hat{m}_j\right)\left(\tilde{n}_i\tilde{e}_{2j} + \tilde{e}_{2i}\tilde{n}_j\right).
\end{aligned}
$$