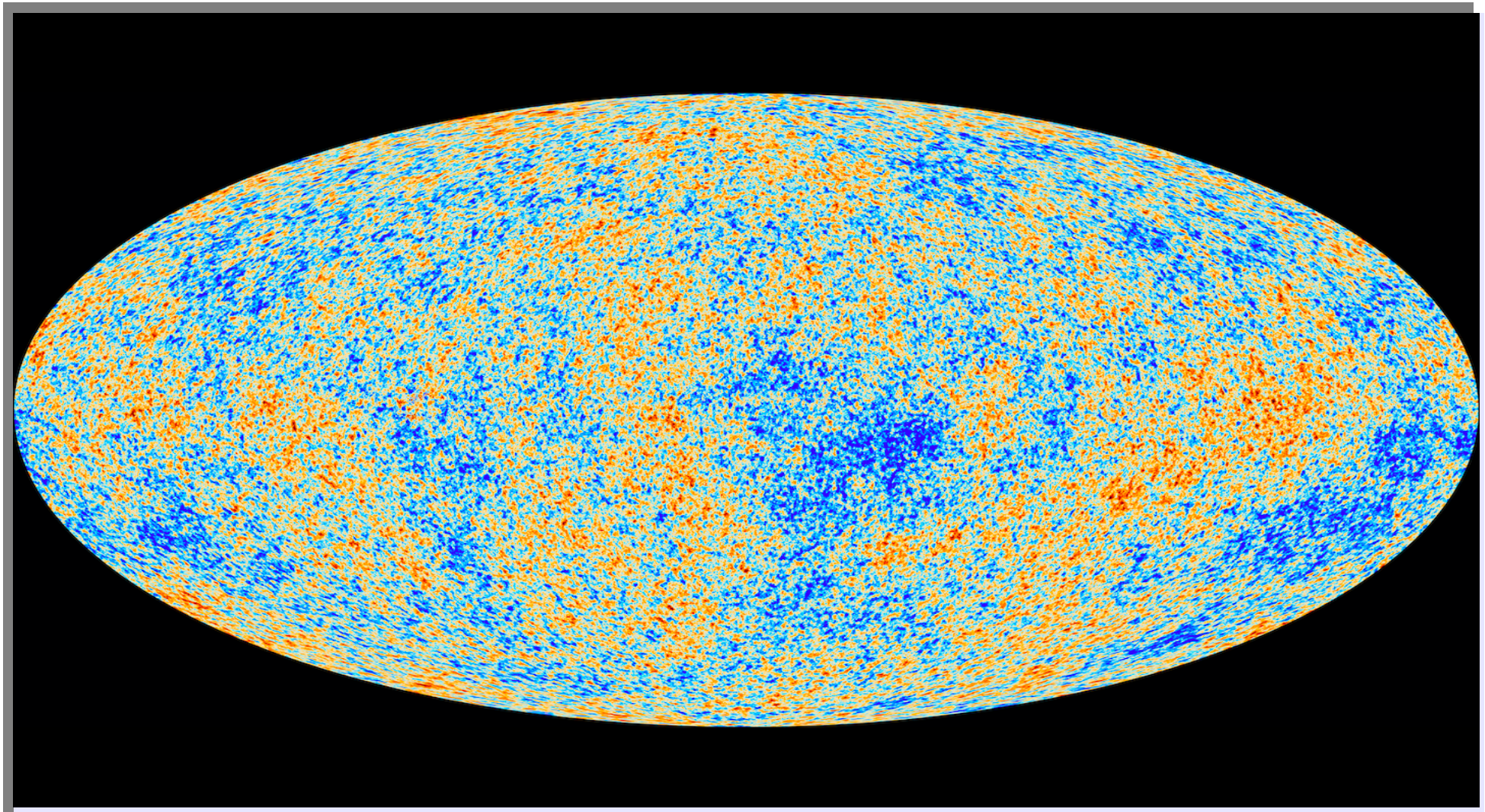

A brief introduction to Cosmic Microwave Background properties in a Universe with multiply connected topology

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In the current Concordance model, the ΛCDM model, Universe is implicitly of infinite spatial extent with space-time structure described by

$$M^3 \times R$$

where R describes cosmic time and M^3 the 3-dimensional comoving spatial section of constant Gaussian curvature.

Equipped with the FLRW metric, this space-time is either of vanishing curvature or negative or strictly positive curvature. But General Relativity does not prescribe the global structure and topology of our Universe.

Our Universe may as well be compact..

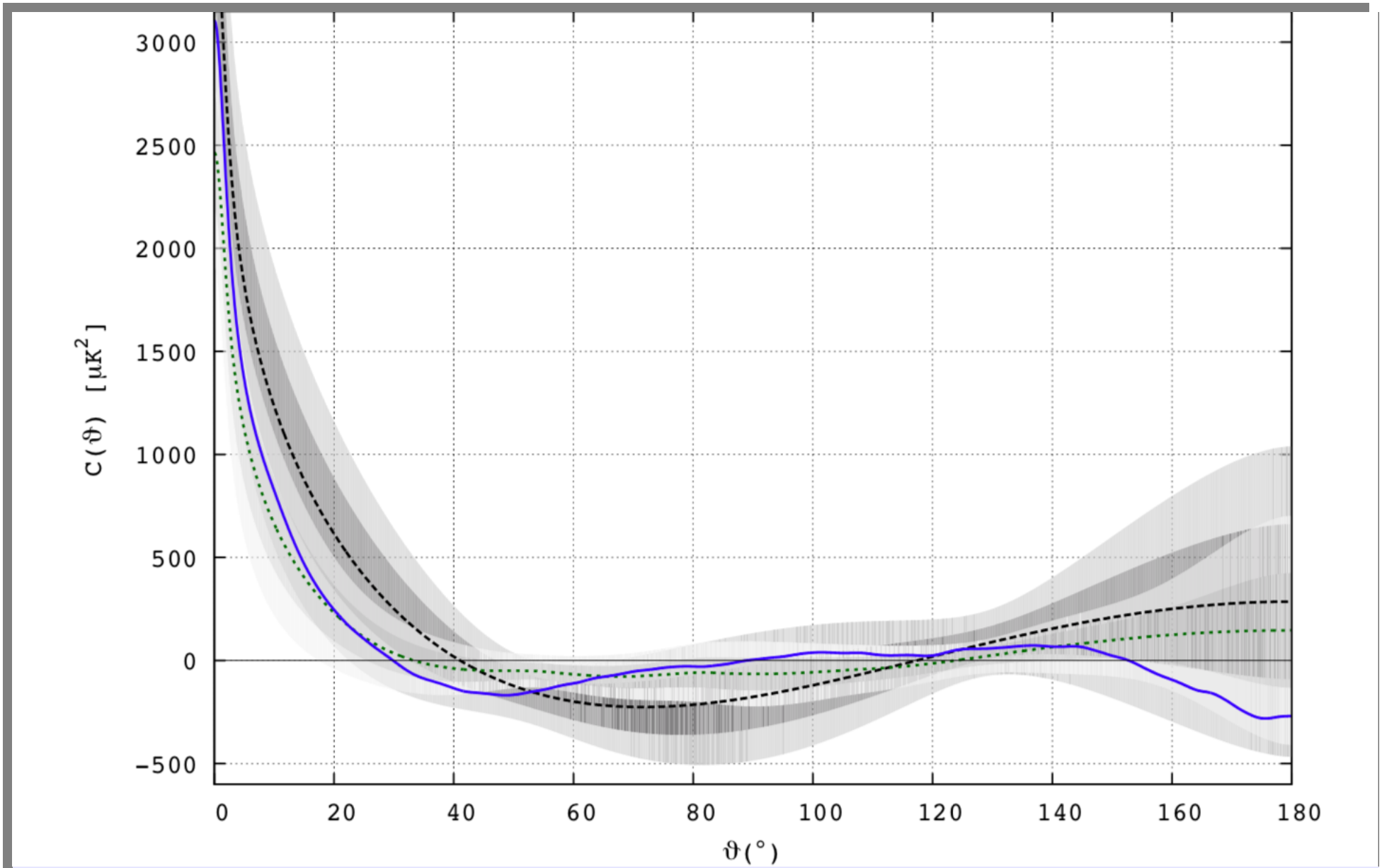
Cosmic Microwave Background (CMB) temperature anisotropy measurements by COBE, WMAP and Planck unveil an interesting property.

Given the angular 2-point correlation function (2-pcf) defined by

$$C(\vartheta) := <\delta T(\hat{a})\ \delta T(\hat{a}')> \text{ with } \hat{a}.\hat{a}' = \cos\vartheta,$$

where the brackets denote averaging over all directions $\hat{a}$ and $\hat{a}'$ separated by an angle ϑ, on the full CMB sky.

The 2-pcf of the observed CMB temperature map shows no correlations for angles ϑ >60arc-degrees


3000
2500
2000
1500
1000
500
0
−500
C(ϑ) [μK²]
0
20
40
60
80
100
120
140
160
180
ϑ(°)

As the 2-pcf plots shows, in addition, the 2-pcf of the CMB temperature map in the implicitly infinite ΛCDM Universe model displays correlations at any angle ϑ except the passage by zero near 40 and 120 arc-degrees.

Another important result comes from the CMB temperature map in the 3-toroidal universe:

For a size of $L=3$ Hubble lengths, the 2-pcf presents a lack of correlation at any angle beyond 40arc-degrees.

All these 2-pcfs are calculated over the whole CMB foreground corrected temperature maps.
But the results with a mask hidding the foreground regions are similar.

What the 3-toroidal universe is?

The 3-torus is a multiply connected topological manifold having for fundamental domain a parallelepiped rectangle or a cube.

3-torus T^3

– is the quotient of E^3 by a symmetry group Γ such that T^3

- is orientable
- is finite without boundary,
- is of zero intrinsic curvature,
- presents periodic boundary conditions,
- is globally homogeneous,
- breaks the $SO(3)$-symmetry so is globally anisotropic.

A globally isotropic physical property in an infinite universe becomes triply periodic and therefore anisotropic in the **3-torus** which for this reason is globally anisotropic.

In a 3-toroidal universe the spectrum of spatial frequencies of the Laplacian is discrete, while it is a continuum in the infinite universe.

The infinite number of discrete vibrational modes of the **3-torus** has a first mode defined by the side length L of the cubic torus, it shows an infrared cut-off at $\lambda = L$.

The ensemble of discrete modes of the **3-torus**, determined by all the constructive and destructive interferences, affects the physics of the primordial CMB.

To illustrate the numerical issues:
- for the CMB spherical harmonics limited to $l = 1000$ there are 61 556 892 vibrational modes to compute in a 3-torus universe.

Calculation of the CMB temperature anisotropies in such finite universe demands long computing sessions. Furthemore, for sake of statistical validity, 10^5 CMB maps are calculated for each 3-torus variant and ΛCDM models.

A signature of CIRCLES IN THE SKY (CITS) is expected to confirm the 2-pcf result, however this signature is not observed on the primordial CMB.
CITS Signal is detected over CMB simulations in the 3-toroidal universe...

Thus, initially, was missing a confirmation of the possible finiteness of our Universe detected with the 2-pcf...

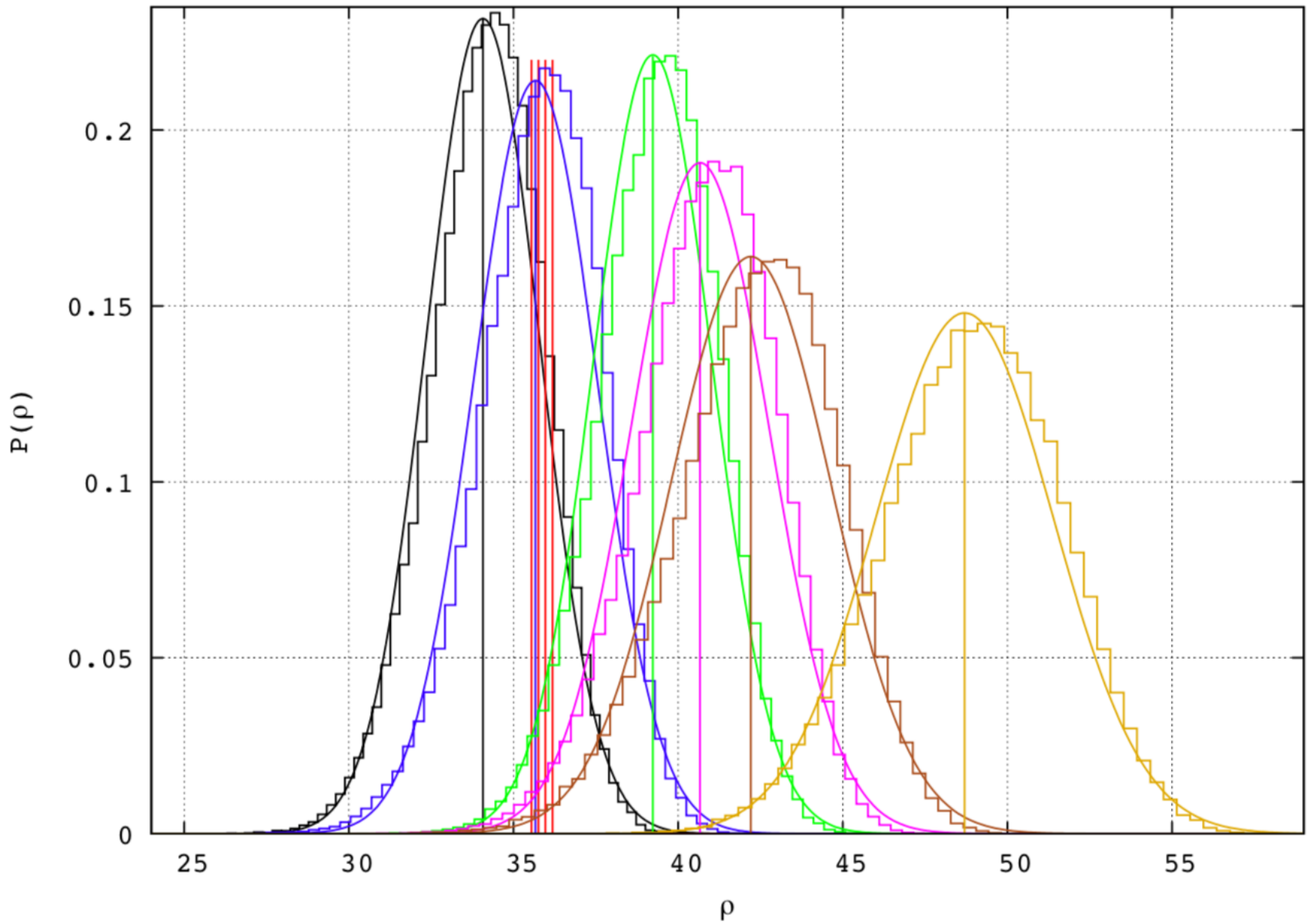
We recently show that the normalized standard deviation of the CMB temperature gradient ρ is another signature of the finiteness of our Universe defined by:

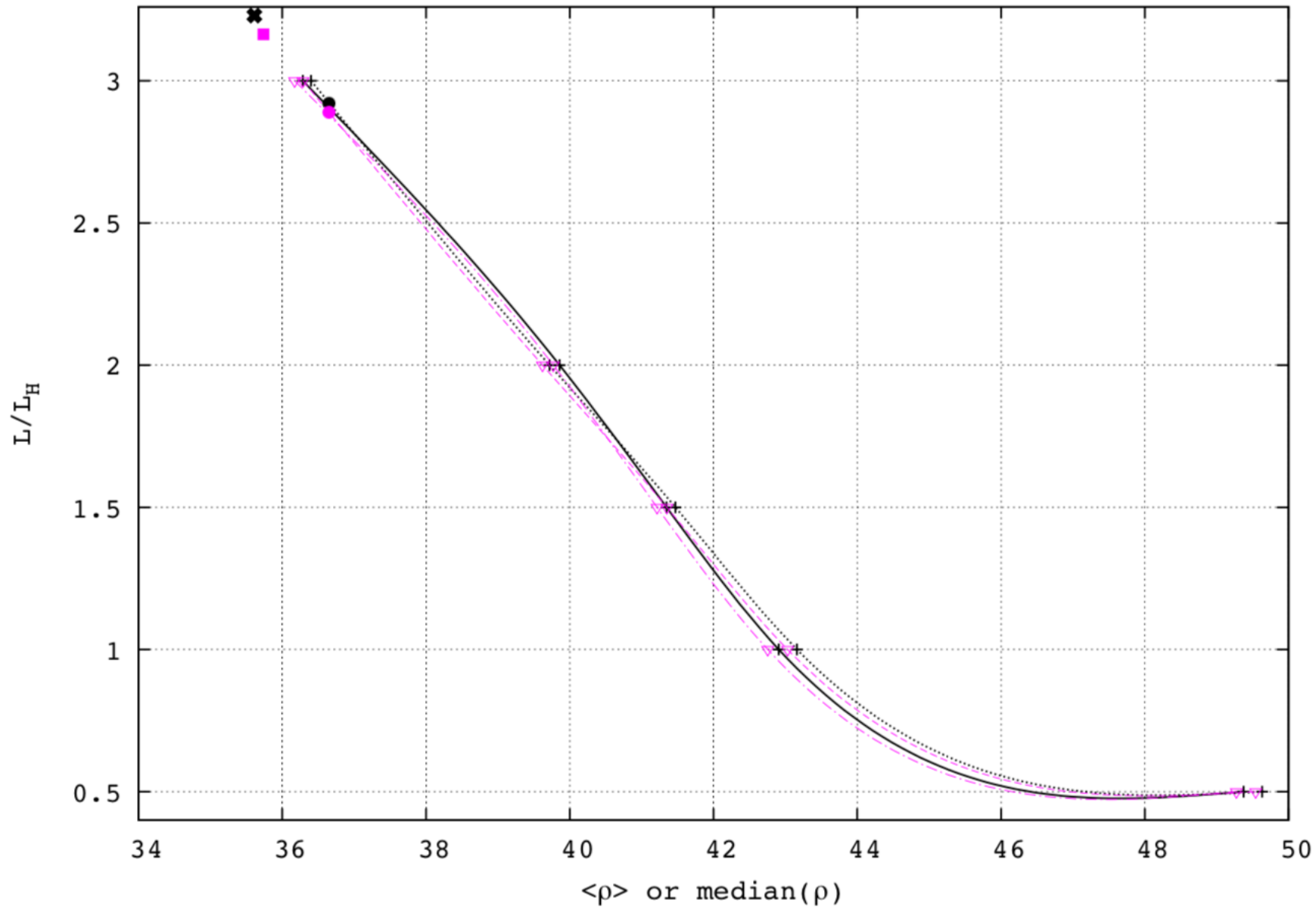
$$\rho^2 := \sigma_1^2 / \sigma_0^2$$

arXiv:2106.13205 Aurich, Buchert, France and Steiner
'The variance of the CMB temperature gradient: a new signature of a multiply connected Universe' 2021

Thereafter, are the Probability Distribution Functions for the CMB in 5 tori of size L=0.5, 1.0, 1.5, 2.0 and 3.0 Hubble lengths, and the CMB in the infinite ΛCDM. Also, the vertical red bars give the ρ values for the 4 Planck maps.

See **arXiv:2106.13205**


0.2
0.15
0.1
0.05
0
P(ρ)
25
30
35
40
45
50
55
ρ


3
2.5
2
1.5
1
0.5
L/L_H
34
36
38
40
42
44
46
48
50
<ρ> or median(ρ)

Concluding Remarks and Outlook questions

In addition we check that CMB isotropy and homogeneity in a toroidal universe are preserved in the sense of ρ at the level of the ΛCDM. Detection of the global 3-torus anisotropy should be made a different way..

– ρ is a new signature of the multiply connected nature of our Universe
– 2-pcf result therefore confirmed
– Minkowski functionals analysis of the CMB should also confirm the result
– so does another ansatz of the CMB analysis (see next talk!)

– Are other forms of Universe MC topology in better agreement with the observations in the sense of the signatures of 2-pcf and ρ ?
– How to connect the signature ρ and the volume or typical side length of the universe ?
– Do exact, or, approximated solutions to Einstein's equation in a 3-toroidal universe are available?

THANK YOU FOR YOUR ATTENTION