Solvable model of Genesis

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Avoiding cosmological singularity

Emergent universe

G.F.R. Ellis and R. Maartens, The emergent universe: Inflationary cosmology with no singularity, CQG 21 (2004)

Minkowski start: NEC violation

P. Creminelli, M.A. Luty, A. Nicolis and L. Senatore, Starting the Universe: Stable Violation of the Null Energy Condition and Non-standard Cosmologies, JHEP 12 (2006).

Galileon Genesis

P. Creminelli, A. Nicolis and E. Trincherini, Galilean Genesis: An alternative to inflation, JCAP 11 (2010) 021

Further work

Rubakov, Mironov, Libanov,Volkova, Ageeva, Petrov, NIshi, Kobayashi, Zhu, Zhang...

Genesis

In Genesis scenario the spacetime at some early stage, $t \sim 0$, is Minkowskian, $H = 0$ (possibly with an earlier contracting phase), but some NEC-violating matter then generates a quickly growing acceleration such that

$$\frac{\dot{H}}{H^2} \to \infty \quad \text{as} \quad t \to 0$$

During this acceleration period the energy density is generated and the Hubble parameter grows sufficiently large. As a result the energy of the exotic matter gets transformed into heat, and the Universe transits to the standard hot stage. For this to be realised, the Null Energy Condition

$$T_{\mu\nu} l^\mu l^\nu \geq 0, \Rightarrow \dot{H} = -4\pi G(p + \rho) \leq 0,$$

has to be strongly violated. NEC is violated in non-minimal scalar-tensor theories with derivative coupling such as Horndeski and DHOST. But there is a superluminality problem.

Ricci-kinetic theory

We consider the action with two independent couplings of the derivatives of the scalar field $\phi_\mu \equiv \phi_{,\mu}$ to the Ricci tensor and the Ricci scalar (Amendola:1993, Capozziello:1999, Sushkov:2009, Granda:2010)

$$S = \int d^4x\sqrt{-g}\,[R - (g_{\mu\nu} + \kappa_1 g_{\mu\nu} R + \kappa_2 R_{\mu\nu})\,\phi^\mu\phi^\nu]\,,$$

where $\phi^\mu = \phi_\nu g^{\mu\nu}$, $\phi_\nu = \partial_\nu\phi$. Einstein tensor $G_{\mu\nu}$ can be represented as $G_{\mu\nu} = \Theta_{\mu\nu}$, where the right hand side contains the third derivative terms

$$\Theta^{(3)}_{\mu\nu} = (\kappa_2 + 2\kappa_1)\,(g_{\mu\nu}\phi^\alpha\nabla_\alpha\Box\phi - \phi^\alpha\phi_{\alpha\mu\nu})\,,$$

where ∇_λ is covariant derivative with respect to the Levi-Civita connection of $g_{\mu\nu}$,

$$\Box = \nabla_\lambda\nabla^\lambda, \quad \phi_{\mu\nu} = \nabla_\mu\phi_\nu, \quad \phi_{\alpha\mu\nu} = \nabla_\alpha\phi_{\mu\nu}$$

.

Horndeski subclass $\kappa_2 + 2\kappa_1 = 0$

Similarly, the scalar equation

$$g^{\mu\nu}\phi_{\mu\nu} + \nabla_\mu\left[\phi_\nu(\kappa_1 g^{\mu\nu}R + \kappa_2 R^{\mu\nu})\right] = 0,$$

in the general case contains the third derivatives of the metric. But for $-2\kappa_1 = \kappa_2$ the Ricci-terms are combined into the Einstein tensor satisfying $\nabla_\mu G^{\mu\nu} = 0$. Then the scalar equations becomes the second order

$$\left(g^{\mu\nu} + \kappa G^{\mu\nu}\right)\phi_{\mu\nu} = 0,$$

while the third derivative terms in the Einstein equations dissapear. This particular case belongs to ghost-free Horndeski class.

This theory, in the metric version, was found to provide inflationary solutions without a potential Sushkov:2009. It also contains wormhole solutions (Sushkov and Korolev:2011).

Palatini kinetically coupled theories

The same action gives rise to a new class of ghost-free theories for any values of two coupling constants, provided that the same kinetic action is considered as the Palatini action (DG and S.Zhidkova, Phys.Lett.B 790 (2019) 453).

A particular horizonless solution in the static sector was found, as well as a nonsingular cosmological solution, and a nonsingular pp-wave solution. There is no superluminality problem.

Subsequently it was found (DG, Eur.Phys.J.C 80 (2020) 5, 443) that the theory is disformally dual to the conformally coupled theory, also violating NEC, but in our case violation of NEC is stronger.

Here we show that this theory provide an simple analytically solvable models of Genesis

In the static sector there are no black holes: possible solutions are either wormholes or naked singularities, separated by a particular regular soliton-like solution

Einstein-Palatini equations

Variation of the action with respect to the metric leads to the Einstein-Palatini equation

$$\lambda \hat{R}_{\mu\nu} - \phi_\mu \phi_\nu (1 + \kappa_1 \hat{R}) - 2\kappa_2 \hat{R}_{\alpha(\mu}\phi_{\nu)}\phi^\alpha - g_{\mu\nu} L/2 = 0,$$

where the connection has to be specified.

Variation over ϕ gives rise to a scalar equation

$$\partial_\mu \left[\sqrt{-g} \left(\phi^\mu + \kappa_1 \hat{R} \phi^\mu + \kappa_2 \hat{R}_{\alpha\beta} g^{\beta\mu} \phi^\alpha \right) \right] = 0,$$

which, in principle, can contain third derivatives of the metric and the fourth derivatives of the scalar field, so one could suspect that the theory has Ostrogradski ghosts.

But after determination of the connection via independent variation of the action, the theory can be shown to admit the Einstein frame, which does not contain higher derivatives.

Palatini variation

In absence of fermions, the Ricci tensor is contracted with symmetric tensors, and the theory is projective-invariant. Thus, the torsion can be consistently set equal to zero the variation over the connection gives

$$\hat{\nabla}_\lambda \left(\sqrt{-g} W^{\mu\nu}\right) = 0, \quad W^{\mu\nu} = \lambda g^{\mu\nu} - \kappa_2 \phi^\mu \phi^\nu,$$

$$\lambda = (1 - \kappa_1 X), \quad X = \phi_\alpha \phi^\alpha.$$

This can be transformed into an equation

$$\hat{\nabla}_\lambda \hat{g}_{\mu\nu} = 0$$

for some second metric $\hat{g}_{\mu\nu}$, in which case $\hat{\Gamma}$ can be identified with the Levi-Civita connection of the latter:

$$\hat{\Gamma}^\lambda_{\mu\nu} = \frac{1}{2} \hat{g}^{\lambda\tau} \left(\partial_\mu \hat{g}_{\lambda\nu} + \partial_\mu \hat{g}_{\mu\lambda} - \partial_\lambda \hat{g}_{\mu\nu}\right).$$

Disformal transformation

The connection generating metric $\hat{g}_{\mu\nu}$ is related to the physical metric $g_{\mu\nu}$ (to which presumably couples the matter), by a disformal transformation:

$$\hat{g}_{\mu\nu} = \sqrt{\Lambda\lambda}\left(g_{\mu\nu} + \kappa_2\Lambda^{-1}\phi_\mu\phi_\nu\right),$$

where $\Lambda = 1 - (\kappa_1 + \kappa_2)X, \; X = \phi_\mu\phi_\nu g^{\mu\nu}$. This transformation is invertible (the proof is non-trivial, since X is related to $\hat{X}$ in a complicated way). Remarkably, not only the Palatini connection is expressed in terms of the second metric $\hat{g}_{\mu\nu}$, but the full theory in terms of this metric has the Einstein form:

$$S = \int \sqrt{-\hat{g}}\left[R_{\mu\nu}(\hat{g}) - \phi_\mu\phi_\nu\,\hat{\Lambda}^{-1}\right]\hat{g}^{\mu\nu}d^4x$$

In spite of the non-standard scalar part is, the Einstein frame theory does not contain higher derivatives, so our theory is ghost-free.

Reversibility conditions

Here $\hat{\Lambda}$ is Λ with $X = \phi_\mu \phi_\nu g^{\mu\nu}$ expressed through $\hat{X} = \phi_\mu \phi_\nu \hat{g}^{\mu\nu}$:

$$\hat{X} = X(1 - \kappa_1 X)^{1/2}([1 - (\kappa_1 + \kappa_2)X]^{-3/2}.$$

This relation becomes singular when one of the scale factors reaches zero. We, therefore, demand $\lambda > 0, \Lambda > 0$ in the physical region of spacetime.

By the inverse function theorem, the solution $X(\hat{X})$ exists at any point where the derivative $d\hat{X}/dX \neq 0$. In our case this derivative is zero at $X = X_{\rm cr} = 2/(2\kappa_1 + 3\kappa_2)$. But then $\hat{X}(X_{\rm cr}) = 2/(\sqrt{3}\kappa_2)$. To the right and to the left of this point, the function $\hat{X}(X)$ is monotonous. The cubic equation has three solutions $X(\hat{X})$, from which one is real in the physical domain, and we choose it. Thus, the disformal transformation is one-to-one and reversible indeed within the physical region. The existence of a physical region restricts the range of possible parameters.

Non-Horndeski coupling $\kappa_1 + \kappa_2 = 0$

The case $\kappa_2 = -\kappa_1 \equiv \kappa$ is exceptional. Then $\Lambda = 1$, and the EF theory reduces to the Einstein theory, minimally coupled to a massless scalar:

$$S = \int \sqrt{-\hat{g}}\,[R_{\mu\nu}(\hat{g}) - \phi_\mu \phi_\nu]\,\hat{g}^{\mu\nu} d^4x.$$

The Einstein equation reads

$$R_{\mu\nu} = \phi_\mu \phi_\nu,$$

while the scalar obeys the covariant D'Alembert equation $\hat{\Box}\phi = 0$. On shell, the following conditions hold $L = 0,\ \hat{R} = X$, implying that the Palatini-Einstein equation is valid.

Therefore, the disformal transformation form the initial metric to the one, defining the Palatini connection, can be considered as the transformation form the Jordan frame to the Einstein frame.

Einstein frame cosmology

$$d\hat{s}^2 = -\hat{N}^2 dt^2 + \hat{a}^2 dl_k^2, \quad dl_k^2 = d\chi^2 + f_k d\Omega^2, \tag{1}$$

where $k = -1, 0, 1$, with $f_1 = \sin^2\chi$, $f_0 = \chi^2$, $f_{-1} = \cosh^2\chi$ for spatially closed, flat and open universes respectively. The relevant components of the Ricci tensor are

$$\hat{R}_{tt} = \frac{3\dot{\hat{N}}\dot{\hat{a}}}{\hat{N}\hat{a}} - \frac{3\ddot{\hat{a}}}{\hat{a}}, \quad \hat{R}_{\chi\chi} = \frac{\hat{a}\ddot{\hat{a}}}{\hat{N}^2} - \frac{\hat{a}\dot{\hat{N}}\dot{\hat{a}}}{\hat{N}^3} + \frac{2\dot{\hat{a}}^2}{\hat{N}^2} + 2k. \tag{2}$$

The component $(\chi\chi)$ of the Einstein equations $\hat{R}_{\mu\nu} = \Lambda\hat{g}_{\mu\nu} + \partial_\mu\hat{\varphi}\partial_\nu\hat{\varphi}$. admits the first integral $\frac{\hat{a}^4\dot{\hat{a}}^2}{\hat{N}^2} + k\hat{a}^4 + \frac{1}{3}\Lambda\hat{a}^6 = a_0^4$, using which we find in the gauge $\hat{a} = 2a_0 t$

$$\hat{N}^2 = \frac{(2a_0)^6 t^4}{a_0^4 - kt^4 + \Lambda t^6/3}, \quad \dot{\hat{\varphi}}^2 = \frac{6a_0^4}{t^2\left(a_0^4 - kt^4 + \Lambda t^6/3\right)}. \tag{3}$$

In the cosmological case the relevant sign of the coupling constant κ is positive. We will be interested by behavior of the scale factor near the singularity of the MES solution. Then both the cosmological constant and curvature terms are negligible, so we start with $k = 0$, $\Lambda = 0$ in the synchronous gauge

$$d\hat{s}^2 = \hat{g}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + \hat{a}^2 \delta_{ij} dx^i dx^j, \tag{4}$$

where

$$\hat{a} = a_0 t^{1/3}, \qquad \phi = \sqrt{2} \ln t / \sqrt{3}, \tag{5}$$

as was found by Zel'dovich in 1972. Obviously, this metric is singular at $t = 0$ and describes a decelerating expansion.
Now we transform to the Jordan frame of the new kinetic theory. One obtains an algebraic equation for N:

$$\left(N - 2z/(3\sqrt{3}) \right)^3 = N^2, \quad z = \kappa\sqrt{3}/t^2. \tag{6}$$

Its real solution is smooth, although in terms of real functions it looks piecewise:

$$N^2 = \frac{2z}{3\sqrt{3}} + \frac{1}{\sqrt{3}} \begin{cases} 2\cos\left(\frac{1}{3}\arccos(x)\right), & z < 1, \\ A^{1/3} + A^{-1/3}, & z > 1, \end{cases} \tag{7}$$

where $A = \left(z + \sqrt{z^2 - 1}\right)^{1/3}$. For large z (small t) one has:

$$g_{tt} = (\alpha t)^{-2}\left(1 + (\alpha t)^{4/3}\right), \qquad \alpha = (3/2\kappa)^{1/2}. \tag{8}$$

The scale factor is $a^2 = \hat{a}^2 N^{2/3}$. We need to go to the synchronous time $t \to \tau(t)$ solving the equation $Ndt = d\tau$. For small t, keeping the leading term, one finds:

$$dt/d\tau = \alpha\, t \;\to\; t = \mathrm{e}^{\alpha\tau}, \tag{9}$$

so that $t \to 0$ corresponds $\tau \to -\infty$.

Now compute the Hubble parameter differentiating with respect to synchronous time in the vicinity of $t = 0$:

$$H = \frac{1}{a}\frac{da}{dt}\frac{dt}{d\tau} = 2\sqrt{\alpha}(\alpha t)^{4/3}. \tag{10}$$

Its derivative reads

$$\dot{H} = \frac{dH}{d\tau} = \frac{2\alpha}{9}(\alpha t)^{4/3}, \tag{11}$$

and satisfies condition of strong NEC violation: the ratio

$$\frac{\dot{H}}{H^2} = \frac{9}{2}(\alpha t)^{-4/3} = \frac{9}{2\alpha^{4/3}}\mathrm{e}^{-4\alpha\tau/3}, \tag{12}$$

diverges exponentially in terms of the synchronous time as $\tau \to -\infty$. Such behavior is typical for Genesis scenario So, NEC violation is sufficient for Genesis. But a more detailed calculation shows that the full amount of expansion is insufficient for a realistic scenario .

Static sector

General static spherically symmetric solution of the minimal Einstein-massless scalar theory is presented by the Fisher metric (1948) rediscovered by Janis, Newman and Winicour (1968) and many others, now called FJNW or gamma-metric

$$ds^2 = -\left(1 - \frac{b}{r}\right)^{\gamma} dt^2 + \left(1 - \frac{b}{r}\right)^{-\gamma} dr^2 + r^2 \left(1 - \frac{b}{r}\right)^{1-\gamma} d\Omega,$$

where the range of the radial variable is v $r \in (b, \infty)$, where q is the scalar charge and $0 < \gamma < 1,\ \gamma = \left(1 - 4q^2/b^2\right)^{1/2}$.
The scalar field reads

$$\phi = \frac{q}{b} \ln \left(1 - \frac{b}{r}\right).$$

The would-be horizon $r = b$ is a naked singularity. Note that the region $r < b$ corresponds to cosmological solution (Abdolrahimi and Shoom, 2010).

Jordan frame

To pass to Jordan frame (physical metric of the Palatini kinetic theory) one has to perform disformal transformations

$$g_{\mu\nu} = \hat{g}_{\mu\nu}\lambda^{-1/2} - \kappa\phi_\mu\phi_\nu,$$

where $\lambda = (g_{rr}/w)^{-2/3}, \quad w = A = (1 - \frac{b}{r})^{-\gamma}$ and g_{rr} obeys the following equation:

$$(g_{rr} - \frac{2x}{3\sqrt{3}})^3 = wg_{rr}, \quad x = \frac{3\sqrt{3}\kappa q^2}{2r^2(r-b)^2}.$$

A real solution of this equation is

$$g_{rr} = \frac{2x}{3\sqrt{3}} + \frac{1}{\sqrt{3}} \begin{cases} 2w\cos\left(\frac{1}{3}\arccos x/w\right), & x < w, \\ w^{2/3}B + w^{1/3}B^{-1}, & x > w, \end{cases}$$

where $B = (x + \sqrt{x^2 - w^2})^{1/3}$. Two other Jordan frame metric components are

$$g_{tt} = \hat{g}_{tt}\lambda^{-1/2};\; g_{\theta\theta} \equiv Q = \hat{g}_{\theta\theta}\lambda^{-1/2} = R\lambda^{-1/2}.$$

Physical interpretation: Wormholes

It is easy to see that the transformation to the Jordan frame preserve asymptotic flatness. Indeed, when $r \to \infty$, the variables $x \to 0$, $w \to 1$, so that

$$g_{rr} \sim 1 + x/\sqrt{3} \Rightarrow \lambda = 1 + O(1/r^4) \tag{13}$$

and we get as $r \to \infty$:

$$g_{tt} \simeq -1 + \frac{\gamma b}{r} \quad g_{rr} \simeq 1 - \frac{\gamma b}{r} \quad g_{\theta\theta} \sim r^2.$$

Now we would like to see whether the Jordan metric function $g_{\theta\theta} \equiv Q(r)$ may have a local minimum (a turning point) at some point $r_t > b$:

$$\left.\frac{dQ}{dr}\right|_{r=r_t} = 0, \quad \left.\frac{d^2Q}{dr^2}\right|_{r=r_t} > 0.$$

It could correspond to a wormhole throat, provided that the space $b < r < r_t$ has infinite volume (we will see that it is the case)

Approach to $r = b$ in Jordan frame

Though an analytic dependence $Q(r)$ can be found by several successive operations, an analytic solution to this equation is not possible, so we check the behavior $Q(r)$ numerically, varying the coupling constant κ and the solution parameter γ.
Consider first an asymptotic behavior of the Jordan metric as $r \to b$. Performing expansions for $x \to \infty, w \to 0$ careful analysis of various expansion terms for the values $0 < \gamma < 1$ of the power index, we obtain in the leading approximation

$$ds^2 = -\mu u^{2(2\gamma-1)/3} dt^2 + \frac{\mu^3 du^2}{u^2} + \mu b^2 u^{(1-2\gamma)/3} d\Omega$$

with $\mu = (\kappa q^2/b^4)^{1/3}$, $u = (r/b - 1)$. Clearly $g_{\theta\theta}$ explodes as $r \to b$ if $\gamma > 1/2$, but goes to zero when $\gamma < 1/2$. Numerical analysis shows that for $\gamma < 1/2$, $g_{\theta\theta}$ does not have a local minimum for all positive values of the coupling constant κ . For $\gamma > 1/2$ a local minimum arises, with r_t depending on γ and κ.

Numerical solutions

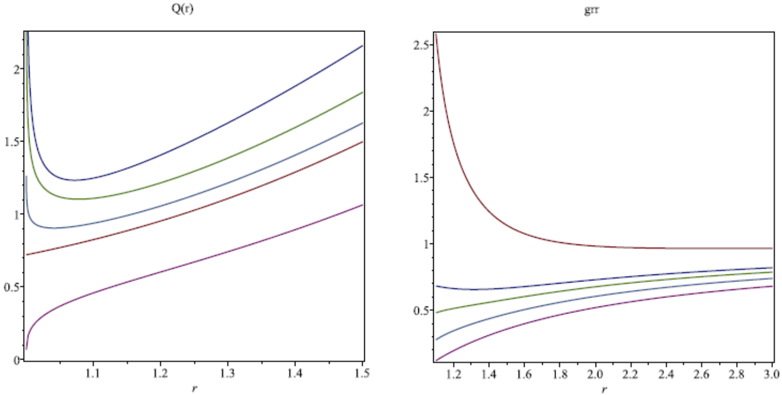


Figure 1: Metric functions $g_{\theta\theta}$ and g_{tt} for various values of γ: $\gamma = 0.1$ (purple), $\gamma = 0.5$ (red), $\gamma = 0.6$ (light blue), $\gamma = 0.75$ (green), $\gamma = 0.95$ (blue).

Numerical solutions

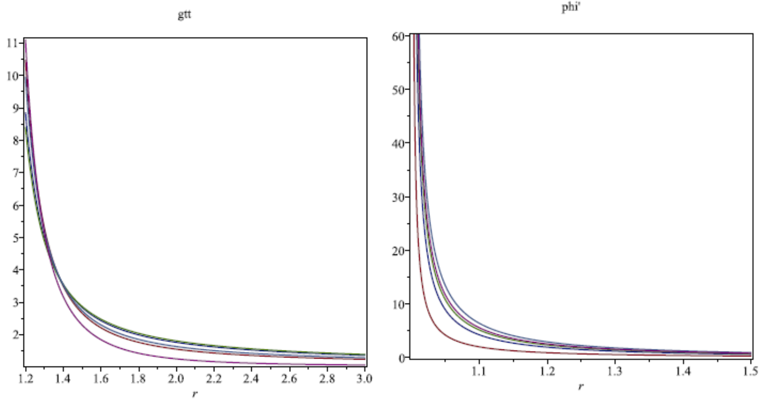


Figure 2: Metric function g_{rr} and the derivative of the scalar field (ϕ') for various values of γ : $\gamma = 0.1$ (purple), $\gamma = 0.5$ (red), $\gamma = 0.6$ (light blue), $\gamma = 0.75$ (green), $\gamma = 0.95$ (blue).

Numerical solutions

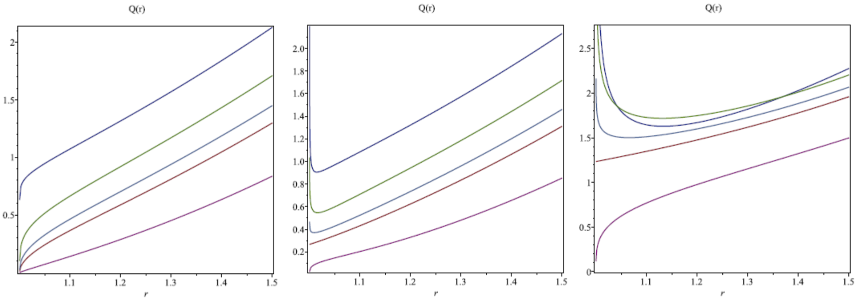


Figure 3: Metric function ($g_{\theta\theta}$) for various values of κ: $\kappa = 0$ (left figure), $\kappa = 0.01$ (center), $\kappa = 10$ (right); and γ: $\gamma = 0.1$ (purple), $\gamma = 0.5$ (red), $\gamma = 0.6$ (light blue), $\gamma = 0.75$ (green), $\gamma = 0.95$ (blue).

For $\kappa = 2$,

γ	3/5	3/4	0,95	0,99
r_t/b	1.041	1.078	1.072	1.041

With growing κ, the position of the local minimum moves to the right, as it seen in Fig.3.

Left to the minimum, the radial function Q increases again and tends to infinity as $r \to b$, revealing existence of infinite volume 3-space between the surfaces $r = r_t$ and $r = b - \delta$ for $\gamma > 1/2$:

$$V_\delta = \int_{r_t}^{b-\delta} \sqrt{h} dr \oint d\Omega = 6\pi\mu b^3 \left(\frac{\delta}{b}\right)^{(1-2\gamma)/3} .$$

From Fig. 1 (right panel) one can also see that $g_{tt} > 0$ for all $r > b$, so the solutions do not have horizons.

The Ricci tensor and the Ricci scalar of this metric are

$$R_{\mu\nu}(g)dx^\mu dx^\nu = -\frac{(1-2\gamma)^2}{6u^2}du^2 + d\Omega,$$

$$R(g) = \frac{2u^{(2\gamma-1)/3}}{b^2} - \frac{(1-2\gamma)^2}{6\mu^2}.$$

Second sheet of the wormhole

Clearly our womrholes are asymmetric, with AF sheet $r_t < r < \infty$, and the second sheet $b < r < r_t$. The asymptotic of the second sheet corresponds to $r \to b$. Computing the mixed components of the Einstein tensor of the asymptotic metric, one find the effective energy density and the principal pressures in the Riemmanian interpretation of this geometry:

$$\epsilon = \frac{u^{(2\gamma-1)/3}}{b^2} - \frac{(2\gamma-1)^2}{12\mu^2}, \quad p_r = -\frac{u^{(2\gamma-1)/3}}{b^2} - \frac{(2\gamma-1)^2}{12\mu^2},$$

Their sum is negative, demonstrating violation of NEC in our theory. The tangential pressure is positive and finite $p_\theta = (2\gamma-1)^2/12\mu^2$. So in Riemannian geometry view one detects on the second sheet a negative radial pressure and constant positive tangential pressure. At the asymptotic of the second sheet one has constant negative and equal energy density and the radial pressure, $\epsilon = p_r < 0$. The Ricci and Kretschmann scalars are finite.

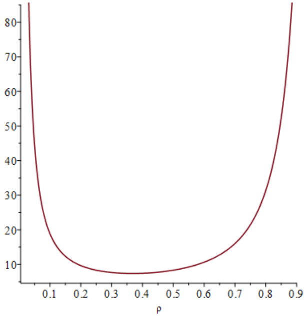
Isotropic coordinates

The asymptotic metric of the second sheet can be presented in isotropic coordinates t, ρ, θ, ϕ changing the radial variable as

$$\rho = \exp\left(g u^{(2\gamma-1)/6}\right), \quad g = b(2\gamma - 1)/6\mu,$$

$$ds^2 = -(g \ln \rho)^4 dt^2 + \frac{1}{g^2} \frac{1}{(\rho \ln \rho)^2} dl_3^2 = d\rho^2 + \rho^2 d\Omega, \quad \rho \in [0, 1],$$

where $dl_3^2 = d\rho^2 + \rho^2 d\Omega$ is the flat metric on a 3-ball of unit radius, and the conformal factor $f^2 = 1/(\rho \ln \rho)^2$:



$\gamma = 1/2$: soliton

In the Einstein frame the FJNW solution for $\gamma = 1/2$ satisfies the condition $g_{rr} = 1/|g_{tt}|$:

$$ds^2 = -\sqrt{1-\frac{b}{r}}dt^2 + \frac{dr^2}{\sqrt{1-\frac{b}{r}}} + r^2\sqrt{1-\frac{b}{r}}d\Omega,$$

It has curvature invariants divergent as $r \to b$. But its Jordan frame counterpart turns out to be non-singular, namely

$$\mu^{-1}ds^2 = -dt^2 + \xi^{-2}dr^2 + b^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

Passing to the new radial coordinate $\rho = b\ln\xi$ extending the domain $r \in (b, \infty)$ to a complete real line, one finds that the manifold to be isomorphfic to the product of the two-dimensional Minkowski space and a sphere of radius b, i.e. $M_{1,1} \times S^2$ This manifold is geodesically complete. Therefore, our solution is a regular scalar-tensor soliton. Its striking feature is that, within the framework of our theory, it is supported by a singular scalar.

Naked singularities

The range of the power index $0 < \gamma < 1/2$ corresponds to solutions singular in the Jordan frame. This can be clearly seen from inspection of the Ricci scalar

$$R(g) = \frac{2u^{(2\gamma-1)/3}}{b^2} - \frac{(1-2\gamma)^2}{6\mu^2}.$$

The constant term is due to the two-sphere sector of the metris, while the first term tends to infinity for $\gamma < 1/2$.
Also, one can see that the three-volume of the region between $r = b - \delta$ and $r = b$ for $\gamma < 1/2$ tends to zero when $\delta \to 0$

$$V_\delta = 6\pi\mu b^3 \left(\delta/b\right)^{(1-2\gamma)/3},$$

where $u = r/b - 1$. The volume $V_\delta \to 0$ for $\gamma < 1/2$, while in the wormhole case $\gamma > 1/2$ it is infinite. Between these two regimes lies the soliton case. From our figures above one can see that in all three cases the metric has no horizons. Therefore, the static spherical sector of our theory does not contain black holes.

Classification of solutions

In the wormhole case the Kretschmann scalar of the second sheet is finite, so the spacetime is non-singular:

$$K = R_{\mu\nu\lambda\tau}(g)R^{\mu\nu\lambda\tau}(g) = \frac{1}{36\mu^4}(48(2\gamma-1)^2\gamma^2 + 24\gamma(\gamma-1)+3).$$

The wormhole is traversable: radial time-like and null geodesics freely pass throug the throat. The tidal forces remain finite.

For $\gamma < 1/2$ the asymptotic is singular: the Kretschmann scalar diverges as $(\frac{r}{b}-1)^{\frac{2(2\gamma-1)}{3}}$.

Since our solution in the Einstein frame was generic in its class, we see that the Jordan frame static spherically symmetric solution splits into three types:

$\gamma > 1/2$ — an asymmetric wormhole
$\gamma = 1/2$ — regular AF soluton with $M_{1,1} \times S_2$ at the center,
$\gamma < 1/2$ — nakedly singular AF spacetime.

No solutions contain horizons

Conclusions

- The Palatini derivatively coupled ST theory with the non-Horndeski ratio of coupling constants is an especially simple ghost-free NEC-violating theory, disformally related to pure Einstein with a minimally coupled scalar. The disformal transformation is reversible.
- The Jordan metric of the cosmological solution is not singular and exhibits Genesis-type behavior. However, the Palatini connection associated with the metric of the Einstein frame is singular, so bosonic and fermionic matter may have different expansion laws. Unfortunately, full expansion seems to be insufficient for a realistic model.
- In the static sector there are no black holes. There are wormholes, or naked singularities, between which lies a solution with a regular "center" of topology is $M_{1,1} \times S^2$. All these solutions are known analytically.

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