The Scale Invariant Vacuum Paradigm

main results and current progress [1]

Andre Maeder[1] and **Vesselin G. Gueorguiev**[2,3]

[1] *Geneva Observatory, Geneva, Switzerland,*
[2] *Institute for Advanced Physical Studies (IAPS), Sofia, Bulgaria,*
[3] *Ronin Institute for Independent Scholarship, Montclair, NJ, USA.*

Hot topics in Modern Cosmology,
Spontaneous Workshop XIV,
IESC Cargese, France, 8 – 14 May 2022

Outline

Scale Invariance and Physical Reality.

The presence of a scale is related to the existence of physical connection and causality.

- The laws of physics (formulae) change upon change of scale!
  - Consistent units is paramount for dimensional estimates.
  - Presence of a scale is closely related to the material content.
- Nevertheless, an empty Universe would be scale invariant!
  - Maxwell equations are scale invariant in vacuum.
  - The field equations of General Relativity are scale invariant for empty space and zero cosmological constant.
- What amount of matter is sufficient to kill scale invariance?
  - How about Cosmology and the evolution of the Universe?
  - The problem of scales is related to the existence of physical connection and causality.

Scale Invariance and Physical Reality.

The presence of a scale is related to the existence of physical connection and causality.

- The laws of physics (formulae) change upon change of scale!
  - Consistent units is paramount for dimensional estimates.
  - Presence of a scale is closely related to the material content.
- Nevertheless, an empty Universe would be scale invariant!
  - Maxwell equations are scale invariant in vacuum.
  - The field equations of General Relativity are scale invariant for empty space and zero cosmological constant.
- What amount of matter is sufficient to kill scale invariance?
  - How about Cosmology and the evolution of the Universe?
  - The problem of scales is related to the existence of physical connection and causality.

Scale Invariance and Physical Reality.

The presence of a scale is related to the existence of physical connection and causality.

- The laws of physics (formulae) change upon change of scale!
  - Consistent units is paramount for dimensional estimates.
  - Presence of a scale is closely related to the material content.
- Nevertheless, an empty Universe would be scale invariant!
  - Maxwell equations are scale invariant in vacuum.
  - The field equations of General Relativity are scale invariant for empty space and zero cosmological constant.
- What amount of matter is sufficient to kill scale invariance?
  - How about Cosmology and the evolution of the Universe?
  - The problem of scales is related to the existence of physical connection and causality.

Einstein GR and Weyl Integrable Geometry.

- In GR metric-compatible connection guarantees $\delta\left\|\vec{v}\right\| = 0$ along a geodesic.
- Could the Dark Matter and Dark Energy phenomena be artifacts of non-zero $\delta\left\|\vec{v}\right\|$, but often negligible $\delta\left\|\vec{v}\right\| \approx 0$ and almost zero value, that accumulate over cosmic distances?
- In Weyl Integrable Geometry $\oint \delta\left\|\vec{v}\right\| = 0$ along a closed loop - this defeats the Einstein objection!
- Not implementing re-parametrization invariance in a model could lead to un-proper time parametrization [14] that seems to induce "fictitious forces" in the equations of motion similar to the forces derived in the weak field SIV regime.

Einstein GR and Weyl Integrable Geometry.

- In GR metric-compatible connection guarantees $\delta \left\|\vec{v}\right\| = 0$ along a geodesic.
- Could the Dark Matter and Dark Energy phenomena be artifacts of non-zero $\delta \left\|\vec{v}\right\|$, but often negligible $\delta \left\|\vec{v}\right\| \approx 0$ and almost zero value, that accumulate over cosmic distances?
- In Weyl Integrable Geometry $\oint \delta \left\|\vec{v}\right\| = 0$ along a closed loop - this defeats the Einstein objection!
- Not implementing re-parametrization invariance in a model could lead to un-proper time parametrization [14] that seems to induce "fictitious forces" in the equations of motion similar to the forces derived in the weak field SIV regime.

Einstein GR and Weyl Integrable Geometry.

- In GR metric-compatible connection guarantees $\delta\left\|\vec{v}\right\| = 0$ along a geodesic.
- Could the Dark Matter and Dark Energy phenomena be artifacts of non-zero $\delta\left\|\vec{v}\right\|$, but often negligible $\delta\left\|\vec{v}\right\| \approx 0$ and almost zero value, that accumulate over cosmic distances?
- In Weyl Integrable Geometry $\oint \delta\left\|\vec{v}\right\| = 0$ along a closed loop - this defeats the Einstein objection!
- Not implementing re-parametrization invariance in a model could lead to un-proper time parametrization [14] that seems to induce "fictitious forces" in the equations of motion similar to the forces derived in the weak field SIV regime.

Einstein GR and Weyl Integrable Geometry.

- In GR metric-compatible connection guarantees $\delta\left\|\overrightarrow{v}\right\|=0$ along a geodesic.
- Could the Dark Matter and Dark Energy phenomena be artifacts of non-zero $\delta\left\|\overrightarrow{v}\right\|$, but often negligible $\delta\left\|\overrightarrow{v}\right\|\approx 0$ and almost zero value, that accumulate over cosmic distances?
- In Weyl Integrable Geometry $\oint\delta\left\|\overrightarrow{v}\right\|=0$ along a closed loop - this defeats the Einstein objection!
- Not implementing re-parametrization invariance in a model could lead to un-proper time parametrization [14] that seems to induce “fictitious forces” in the equations of motion similar to the forces derived in the weak field SIV regime.

Weyl Integrable Geometry and Dirac co-Calculus [2, 3].

- Weyl Geometry uses $g_{\mu\nu}$ along with k_μ and a scalar field λ.
- Covariant derivatives based on Dirac co-calculus.
- In the Weyl Integrable Geometry $k_\mu = -\partial_\mu \ln(\lambda)$.

Gauge change & derivatives. Einstein GR frame' otherwise WIG.

- $l' \to \lambda(x) l \Leftrightarrow ds' = \lambda ds \Rightarrow g'_{\mu\nu} = \lambda^2 g_{\mu\nu}$,
- co-tensor of power n: $Y'_{\mu\nu} \to \lambda^n Y_{\mu\nu}$,
- derivative of co-scalar of power n: $S_{*\mu} = \partial_\mu S - n k_\mu S$,
- for co-vector of power n: $A_{\nu*\mu} = \partial_\mu A_\nu - n k_\nu A_\mu -^* \Gamma^\alpha_{\nu\mu} A_\alpha$,
- for co-co-vector of power n: $A^\nu_{*\mu} = \partial_\mu A^\nu - n k^\nu A_\mu +^* \Gamma^\nu_{\mu\alpha} A^\alpha$,
- where ${}^*\Gamma^\nu_{\mu\alpha} = \Gamma^\nu_{\mu\alpha} + g_{\mu\alpha} k^\nu - g^\nu_\mu k_\alpha - g^\nu_\alpha k_\mu$.

Weyl Integrable Geometry and Dirac co-Calculus [2, 3].

- Weyl Geometry uses $g_{\mu\nu}$ along with k_μ and a scalar field λ.
- Covariant derivatives based on Dirac co-calculus.
- In the Weyl Integrable Geometry $k_\mu = -\partial_\mu \ln(\lambda)$.

Gauge change & derivatives. Einstein GR frame' otherwise WIG.

- $l' \to \lambda(x) l \Leftrightarrow ds' = \lambda ds \Rightarrow g'_{\mu\nu} = \lambda^2 g_{\mu\nu}$,
- co-tensor of power n: $Y'_{\mu\nu} \to \lambda^n Y_{\mu\nu}$,
- derivative of co-scalar of power n: $S_{*\mu} = \partial_\mu S - n k_\mu S$,
- for co-vector of power n: $A_{\nu*\mu} = \partial_\mu A_\nu - n k_\nu A_\mu -^* \Gamma^\alpha_{\nu\mu} A_\alpha$,
- for co-co-vector of power n: $A^\nu_{*\mu} = \partial_\mu A^\nu - n k^\nu A_\mu +^* \Gamma^\nu_{\mu\alpha} A^\alpha$,
- where ${}^*\Gamma^\nu_{\mu\alpha} = \Gamma^\nu_{\mu\alpha} + g_{\mu\alpha} k^\nu - g^\nu_\mu k_\alpha - g^\nu_\alpha k_\mu$.

Ricci tensor and the Einstein Equation

First in 1973 by Dirac [3] and then in 1977 by Canuto et al. [4]

Ricci tensor and scalar in Weyl and Einstein' frames:

$$R_{\mu\nu} = R'_{\mu\nu} - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_\mu\kappa_\nu + 2g_{\mu\nu}\kappa^\alpha\kappa_\alpha - g_{\mu\nu}\kappa^\alpha_{;\alpha}, \quad (1)$$

$$R = R' + 6\kappa^\alpha\kappa_\alpha - 6\kappa^\alpha_{;\alpha}. \quad (2)$$

The Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}, \quad (3)$$

$$R'_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R' - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_\mu\kappa_\nu + 2g_{\mu\nu}\kappa^\alpha_{;\alpha} - g_{\mu\nu}\kappa^\alpha\kappa_\alpha = -8\pi G T_{\mu\nu} - \lambda^2\Lambda_{\mathrm{E}}g_{\mu\nu}. \quad (4)$$

Ricci tensor and the Einstein Equation

First in 1973 by Dirac [3] and then in 1977 by Canuto et al. [4]

Ricci tensor and scalar in Weyl and Einstein' frames:

$$R_{\mu\nu} = R'_{\mu\nu} - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_\mu \kappa_\nu + 2 g_{\mu\nu} \kappa^\alpha \kappa_\alpha - g_{\mu\nu} \kappa^\alpha_{;\alpha}\,, \qquad (1)$$

$$R = R' + 6\kappa^\alpha \kappa_\alpha - 6\kappa^\alpha_{;\alpha}\,. \qquad (2)$$

The Einstein equation:

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R = -8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}\,, \qquad (3)$$

$$R'_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu} R' - \kappa_{\mu;\nu} - \kappa_{\nu;\mu} - 2\kappa_\mu \kappa_\nu + 2 g_{\mu\nu} \kappa^\alpha_{;\alpha} - g_{\mu\nu} \kappa^\alpha \kappa_\alpha = -8\pi G T_{\mu\nu} - \lambda^2 \Lambda_{\mathrm{E}}\, g_{\mu\nu}\,. \qquad (4)$$

Scale Invariant Vacuum gauge (T=0 and R'=0) Maeder [5]

Definition

In Einstein GR vacuum $T_{\mu\nu} = 0$ and $R'_{\mu\nu} = 0$, thus:

$$\kappa_{\mu;\nu} + \kappa_{\nu;\mu} + 2\kappa_\mu \kappa_\nu - 2g_{\mu\nu}\kappa^\alpha_{;\alpha} + g_{\mu\nu}\kappa^\alpha \kappa_\alpha = \Lambda g_{\mu\nu} \quad (5)$$

Corollary derived in 2017 by Maeder [5]

For homogeneous and isotropic WIG-space $\partial_i \lambda = 0$; therefore, only $\kappa_0 = -\dot{\lambda}/\lambda$ and its time derivative $\dot{\kappa}_0 = -\kappa_0^2$ can be non-zero.

$$3\frac{\dot{\lambda}^2}{\lambda^2} = \Lambda, \quad \text{and} \quad 2\frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \Lambda, \quad (6)$$

$$\text{or} \quad \frac{\ddot{\lambda}}{\lambda} = 2\frac{\dot{\lambda}^2}{\lambda^2}, \quad \text{and} \quad \frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \frac{\Lambda}{3}. \quad (7)$$

Scale Invariant Vacuum gauge (T=0 and R'=0) Maeder [5]

Definition

In Einstein GR vacuum $T_{\mu\nu} = 0$ and $R'_{\mu\nu} = 0$, thus:

$$\kappa_{\mu;\nu} + \kappa_{\nu;\mu} + 2\kappa_\mu \kappa_\nu - 2g_{\mu\nu}\kappa^\alpha_{;\alpha} + g_{\mu\nu}\kappa^\alpha \kappa_\alpha = \Lambda g_{\mu\nu} \quad (5)$$

Corollary derived in 2017 by Maeder [5]

For homogeneous and isotropic WIG-space $\partial_i \lambda = 0$; therefore, only $\kappa_0 = -\dot{\lambda}/\lambda$ and its time derivative $\dot{\kappa}_0 = -\kappa_0^2$ can be non-zero.

$$3\frac{\dot{\lambda}^2}{\lambda^2} = \Lambda, \quad \text{and} \quad 2\frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \Lambda, \quad (6)$$

$$\text{or} \quad \frac{\ddot{\lambda}}{\lambda} = 2\frac{\dot{\lambda}^2}{\lambda^2}, \quad \text{and} \quad \frac{\ddot{\lambda}}{\lambda} - \frac{\dot{\lambda}^2}{\lambda^2} = \frac{\Lambda}{3}. \quad (7)$$

SIV based Cosmology: first in 1977 by Canuto et al. [4] then in 2017 by Maeder [5]

Corollary

The FLRW equations within the Weyl Integrable Geometry:

$$\frac{8\pi G\rho}{3}=\frac{k}{a^2}+\frac{\dot{a}^2}{a^2}+2\frac{\dot{\lambda}\,\dot{a}}{\lambda\,a}+\frac{\dot{\lambda}^2}{\lambda^2}-\frac{\Lambda_{\mathrm{E}}\lambda^2}{3}\,, \tag{8}$$

$$-8\pi Gp=\frac{k}{a^2}+2\frac{\ddot{a}}{a}+2\frac{\ddot{\lambda}}{\lambda}+\frac{\dot{a}^2}{a^2}+4\frac{\dot{a}\,\dot{\lambda}}{a\,\lambda}-\frac{\dot{\lambda}^2}{\lambda^2}-\Lambda_{\mathrm{E}}\,\lambda^2\,. \tag{9}$$

In SIV gauge **the cosmological constant disappears***:*

$$\frac{8\pi G\rho}{3}=\frac{k}{a^2}+\frac{\dot{a}^2}{a^2}+2\,\frac{\dot{a}\dot{\lambda}}{a\lambda}\,, \tag{10}$$

$$-8\pi Gp=\frac{k}{a^2}+2\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}+4\frac{\dot{a}\dot{\lambda}}{a\lambda}\,. \tag{11}$$

Comparing ΛCDM and SIV cosmology models [5, 6].

Differences between the models declines for increasing matter densities.

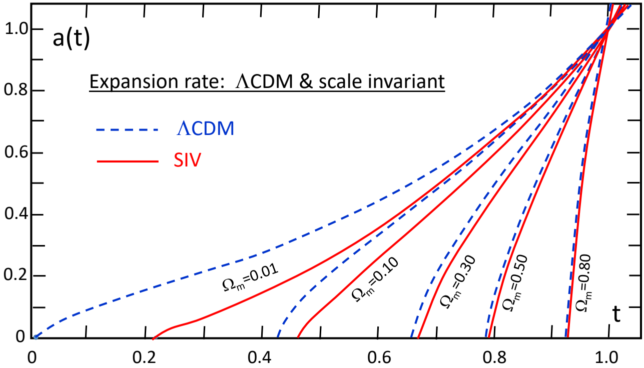

Figure 1: Expansion rates $a(t)$ as a function of time t in the flat $(k = 0)$ ΛCDM and SIV models in the matter dominated era. The curves are labeled by the values of Ω_m.

Comparing MOND and the SIV model to the data [7].

MOND deviates significantly for the data on the Dwarf Spheroidals.

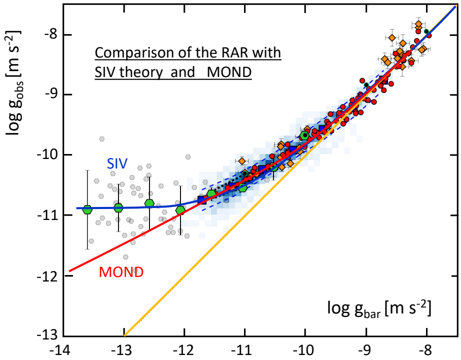

Figure 2: Dwarf Spheroidals, MOND, and SIV (20).

Radial Acceleration Relation (RAR) of g_{obs} and g_{bar} for the galaxies studied by Lelli et al. (2017). The big green hexagons represent the binned data of the dwarf spheroidals. The blue curve shows the relation predicted by the SIV (20). The red curve gives the MOND relation, while the orange curve shows the 1:1-line.

Density fluctuations grow fast enough within the SIV [8].

The overall slope is independent of the choice of recombination epoch.

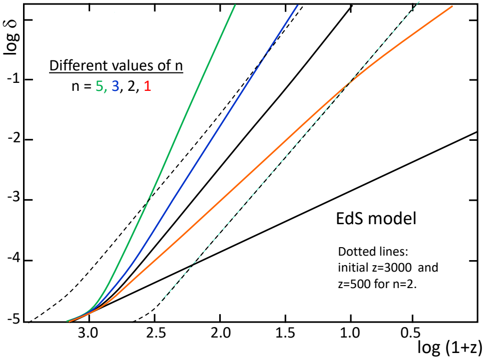

Figure 3: The growth of density fluctuations for different values of parameter n (the gradient of the density distribution in the nascent cluster), for an initial value $\delta = 10^{-5}$ at $z = 1376$ and $\Omega_{\rm m} = 0.10$. The initial slopes are those of the EdS models. The two light broken curves show models with initial $(z+1) = 3000$ and 500, with same $\Omega_{\rm m} = 0.10$ and $n = 2$. These dashed lines are to be compared to the black continuous line of the $n = 2$ model. All the three lines for $n = 2$ are very similar and nearly parallel.

Cosmological constant & the vacuum energy density [9].

Theorem

Using the SIV equations (6) or (7) with $\Lambda = \lambda^2 \Lambda_E$ then one has:

$$\Lambda_E = 3\frac{\dot{\lambda}^2}{\lambda^4}, \quad \text{and} \quad \frac{d\Lambda_E}{dt} = 0.$$

Corollary

In cosmic-time units the solution of the SIV equations is: $\lambda = t_0/t$.

Definition

The vacuum energy density ρ, by using $C = 3/(4\pi G)$, is then:

$$\rho = \frac{\Lambda}{8\pi G} = \lambda^2 \rho' = \lambda^2 \frac{\Lambda_E}{8\pi G} = \frac{3}{8\pi G}\frac{\dot{\lambda}^2}{\lambda^2} = \frac{C}{2}\frac{\dot{l}^2}{l^2} = \frac{C}{2}\dot{\psi}^2$$

Connecting to inflation via $\dot{\psi} = -\dot{\lambda}/\lambda$ or $\psi \propto \ln(t)$ in SIV gauge.

Cosmological constant & the vacuum energy density [9].

Theorem

Using the SIV equations (6) or (7) with $\Lambda = \lambda^2 \Lambda_E$ then one has:

$$\Lambda_E = 3\frac{\dot{\lambda}^2}{\lambda^4}, \quad \text{and} \quad \frac{d\Lambda_E}{dt} = 0.$$

Corollary

In cosmic-time units the solution of the SIV equations is: $\lambda = t_0/t$.

Definition

The vacuum energy density ρ, by using $C = 3/(4\pi G)$, is then:

$$\rho = \frac{\Lambda}{8\pi G} = \lambda^2 \rho' = \lambda^2 \frac{\Lambda_E}{8\pi G} = \frac{3}{8\pi G}\frac{\dot{\lambda}^2}{\lambda^2} = \frac{C}{2}\frac{\dot{l}^2}{l^2} = \frac{C}{2}\dot{\psi}^2$$

Connecting to inflation via $\dot{\psi} = -\dot{\lambda}/\lambda$ or $\psi \propto \ln(t)$ in SIV gauge.

Cosmological constant & the vacuum energy density [9].

Theorem

Using the SIV equations (6) or (7) with $\Lambda = \lambda^2 \Lambda_E$ then one has:

$$\Lambda_E = 3\frac{\dot{\lambda}^2}{\lambda^4}, \quad \text{and} \quad \frac{d\Lambda_E}{dt} = 0.$$

Corollary

In cosmic-time units the solution of the SIV equations is: $\lambda = t_0/t$.

Definition

The vacuum energy density ρ, by using $C = 3/(4\pi G)$, is then:

$$\rho = \frac{\Lambda}{8\pi G} = \lambda^2 \rho' = \lambda^2 \frac{\Lambda_E}{8\pi G} = \frac{3}{8\pi G}\frac{\dot{\lambda}^2}{\lambda^2} = \frac{C}{2}\frac{\dot{l}^2}{l^2} = \frac{C}{2}\dot{\psi}^2$$

Connecting to inflation via $\dot{\psi} = -\dot{\lambda}/\lambda$ or $\psi \propto \ln(t)$ in SIV gauge.

The SIV based Inflation has a graceful exit!

Standard inflation [10, 11, 12, 13]:

$$\left.\begin{matrix}\rho \\ p\end{matrix}\right\} = \frac{1}{2}\dot{\varphi}^2 \pm V(\varphi), \quad (12)$$

$$|\,\dot{H}_{\rm infl}\,| \ll H_{\rm infl}^2 \,. \quad (13)$$

Would the SIV satisfy (13)?:

$$C = 3/(4\pi G)$$

$$\dot{\psi} = -\dot{\lambda}/\lambda, \qquad \varphi \leftrightarrow \sqrt{C}\,\psi,$$
$$V \leftrightarrow CU(\psi), \quad U(\psi) = g\, e^{\mu\,\psi}.$$

The critical ratio (13) for the occurrence of inflation within SIV:

$$\frac{|\,\dot{H}_{\rm infl}\,|}{H_{\rm infl}^2} = \frac{3\,(\mu+1)}{g\,(\mu+2)}\,t^{-\mu-2} \ll 1 \text{ for } \mu < -2, \text{ and } t \ll t_0 = 1. \quad (14)$$

Graceful exit from inflation at $t \approx \sqrt[n]{\frac{n\ g}{3(n+1)}}$ with $n = -\mu - 2 > 0$.

The SIV based Inflation has a graceful exit!

Standard inflation [10, 11, 12, 13]:

$$\left.\begin{matrix}\rho \\ p\end{matrix}\right\} = \frac{1}{2}\dot{\varphi}^2 \pm V(\varphi), \quad (12)$$

$$|\,\dot{H}_{\rm infl}\,| \ll H^2_{\rm infl}\,. \quad (13)$$

Would the SIV satisfy (13)?:

$$C = 3/(4\pi G)$$

$$\dot{\psi} = -\dot{\lambda}/\lambda, \qquad \varphi \leftrightarrow \sqrt{C}\,\psi,$$
$$V \leftrightarrow CU(\psi), \quad U(\psi) = g\,e^{\mu\,\psi}\,.$$

The critical ratio (13) for the occurrence of inflation within SIV:

$$\frac{|\,\dot{H}_{\rm infl}\,|}{H^2_{\rm infl}} = \frac{3\,(\mu+1)}{g\,(\mu+2)}\,t^{-\mu-2} \ll 1 \; for \; \mu < -2, \; and \; t \ll t_0 = 1. \quad (14)$$

Graceful exit from inflation at $t \approx \sqrt[n]{\frac{n\ g}{3(n+1)}}$ with $n = -\mu - 2 > 0$.

The SIV based Inflation has a graceful exit!

Standard inflation [10, 11, 12, 13]:

$$\left.\begin{array}{c}\rho\\ p\end{array}\right\} = \frac{1}{2}\dot{\varphi}^2 \pm V(\varphi), \quad (12)$$

$$|\,\dot{H}_{\text{infl}}\,| \ll H^2_{\text{infl}}\,. \quad (13)$$

Would the SIV satisfy (13)?:

$$C = 3/(4\pi G)$$

$$\dot{\psi} = -\dot{\lambda}/\lambda, \qquad \varphi \leftrightarrow \sqrt{C}\,\psi,$$
$$V \leftrightarrow CU(\psi), \quad U(\psi) = g\,e^{\mu\,\psi}\,.$$

The critical ratio (13) for the occurrence of inflation within SIV:

$$\frac{|\,\dot{H}_{\text{infl}}\,|}{H^2_{\text{infl}}} = \frac{3\,(\mu+1)}{g\,(\mu+2)}\,t^{-\mu-2} \ll 1 \; for \;\; \mu < -2, \;\; and \;\; t \ll t_0 = 1. \quad (14)$$

Graceful exit from inflation at $t \approx \sqrt[n]{\frac{n\;g}{3(n+1)}}$ with $n = -\mu - 2 > 0$.

Summary

- The SIV cosmology **is a viable alternative to ΛCDM.**
  - In SIV gauge (10) the cosmological constant **disappears**.
  - Diminishing differences at higher densities (Fig. 1) [5, 6].
  - SIV gives the correct RAR for dwarf spheroidals (Fig.2) [7].
  - SIV has fast growth of the density fluctuations (Fig.3) [8].
- **Early inflation is natural** within the SIV cosmology [9]!
  - It exhibits **a graceful exit from inflation** (14).
- Other research directions
  - Primordial Nucleosynthesis - on hold at the moment!
  - Testing SIV cosmology against ΛCDM successes - CMB etc.

Summary

- The SIV cosmology **is a viable alternative to ΛCDM.**
  - In SIV gauge (10) the cosmological constant **disappears**.
  - Diminishing differences at higher densities (Fig. 1) [5, 6].
  - SIV gives the correct RAR for dwarf spheroidals (Fig.2) [7].
  - SIV has fast growth of the density fluctuations (Fig.3) [8].
- **Early inflation is natural** within the SIV cosmology [9]!
  - It exhibits **a graceful exit from inflation** (14).
- Other research directions
  - Primordial Nucleosynthesis - on hold at the moment!
  - Testing SIV cosmology against ΛCDM successes - CMB etc.

Summary

- The SIV cosmology **is a viable alternative to ΛCDM.**
  - In SIV gauge (10) the cosmological constant **disappears**.
  - Diminishing differences at higher densities (Fig. 1) [5, 6].
  - SIV gives the correct RAR for dwarf spheroidals (Fig.2) [7].
  - SIV has fast growth of the density fluctuations (Fig.3) [8].
- **Early inflation is natural** within the SIV cosmology [9]!
  - It exhibits **a graceful exit from inflation** (14).
- Other research directions
  - Primordial Nucleosynthesis - on hold at the moment!
  - Testing SIV cosmology against ΛCDM successes - CMB etc.

References I

[1] Gueorguiev, V. G., Maeder, A., *The Scale Invariant Vacuum Paradigm: Main Results and Current Progress.* Universe 2022, **8** (4) 213; DOI:10.3390/universe8040213, e-Print:2202.08412 [gr-qc].

[2] Weyl, H. 1923, Raum, Zeit, Materie. Vorlesungen über allgemeine Relativitätstheorie. Re-edited by Springer Verlag, Berlin (1970).

[3] Dirac, P. A. M. 1973, Proc. R. Soc. Lond. A, **333**, 403 (1973).

[4] Canuto, V., Adams, P. J., Hsieh, S.-H., & Tsiang, E., 1977, Phys. Rev. D **16**, 1643 (1977).

[5] Maeder, A. 2017, *An Alternative to the LambdaCDM Model: the case of scale invariance,* Astrophys. J. , **834**, 194 (2017). e-Print: 1701.03964 [astro-ph.CO].

[6] Maeder, A.; Gueorguiev, V.G. 2020, *The Scale-Invariant Vacuum (SIV) Theory: A Possible Origin of Dark Matter and Dark Energy,* Universe **6**, 46 (2020). DOI: 10.3390/universe6030046.

[7] Maeder, A.; Gueorguiev, V.G. *Scale-invariant dynamics of galaxies, MOND, dark matter, and the dwarf spheroidals,* MNRAS**492**, 2698 (2019). e-Print: 2001.04978 [gr-qc]

[8] Maeder, A., Gueorguiev, V., G., *The growth of the density fluctuations in the scale-invariant vacuum theory,* Phys. Dark Univ. **25**, (2019) 100315. e-Print: 1811.03495 [astro-ph.CO]

[9] Maeder, A., Gueorguiev, V. G., *Scale invariance, horizons, and inflation.* MNRAS **504**, 4005 (2021). e-Print: 2104.09314 [gr-qc].

[10] Guth, A. 1981, Phys. Rev. D23, 347.

[11] Linde, A. 1995, Lectures on Inflationary Cosmology, Particle Physics and Cosmology, Ninth lake Louise, 44 p. e-Print: 9410082 [hep-th].

[12] Linde, A. 2005, Particle Physics and Inflationary Universe, Contemp. Physics, 5, 1-362.

[13] Weinberg, S. 2008, Cosmology, Oxford Univ. Press, 593 p.

[14] Gueorguiev, V. G., Maeder, A., *Geometric Justification of the Fundamental Interaction Fields for the Classical Long-Range Forces.* Symmetry **13**, 379 (2021). e-Print: 1907.05248 [math-ph].

The Weak Field Approximation (WFA) within SIV

The equation of the geodesics Dirac, 1973 [3] ($u^\mu = dx^\mu/ds$)

$$u^\mu_{*\nu} = 0 \Rightarrow \frac{du^\mu}{ds} + {}^*\Gamma^\mu_{\nu\rho} u^\nu u^\rho + \kappa_\nu u^\nu u^\mu = 0 \,. \tag{15}$$

Derivable from an action by Bouvier & Maeder in 1978:

$$\delta\mathscr{A} = \int_{P_0}^{P_1} \delta\,(d\widetilde{s}) = \int \delta\,(\beta\, ds) = \int \delta\left(\beta \frac{ds}{d\tau}\right) d\tau = 0. \tag{16}$$

WFA: where $i \in 1,2,3$, while the potential $\Phi = GM/r$ is scale invariant.

$$\begin{aligned} g_{ii} = -1,\ g_{00} &= 1 + 2\Phi/c^2 \Rightarrow \Gamma^i_{00} = \frac{1}{2}\frac{\partial g_{00}}{\partial x^i} = \frac{1}{c^2}\frac{\partial \Phi}{\partial x^i}, \\ \frac{d^2\overrightarrow{r}}{dt^2} &= -\frac{GM}{r^2}\frac{\overrightarrow{r}}{r} + \kappa_0(t)\frac{d\overrightarrow{r}}{dt}. \end{aligned} \tag{17}$$

The Weak Field Approximation (WFA) within SIV

The equation of the geodesics Dirac, 1973 [3] ($u^\mu = dx^\mu/ds$)

$$u^\mu_{*\nu} = 0 \Rightarrow \frac{du^\mu}{ds} + {}^*\Gamma^\mu_{\nu\rho} u^\nu u^\rho + \kappa_\nu u^\nu u^\mu = 0\,. \tag{15}$$

Derivable from an action by Bouvier & Maeder in 1978:

$$\delta\mathscr{A} = \int_{P_0}^{P_1} \delta\left(d\widetilde{s}\right) = \int \delta\left(\beta\, ds\right) = \int \delta\left(\beta \frac{ds}{d\tau}\right) d\tau = 0. \tag{16}$$

WFA: where $i \in 1,2,3$, while the potential $\Phi = GM/r$ is scale invariant.

$$\begin{aligned} g_{ii} = -1,\ g_{00} &= 1 + 2\Phi/c^2 \Rightarrow \Gamma^i_{00} = \frac{1}{2}\frac{\partial g_{00}}{\partial x^i} = \frac{1}{c^2}\frac{\partial \Phi}{\partial x^i}, \\ \frac{d^2\overrightarrow{r}}{dt^2} &= -\frac{GM}{r^2}\frac{\overrightarrow{r}}{r} + \kappa_0(t)\frac{d\overrightarrow{r}}{dt}. \end{aligned} \tag{17}$$

Towards the Radial Acceleration Relation (RAR) [7]

Ration of the correction term $\kappa_0(t)\,\upsilon$ to the usual Newtonian term:

$$x = \frac{\kappa_0 \upsilon r^2}{GM} = \frac{H_0}{\xi}\frac{\upsilon\, r^2}{GM} = \frac{H_0}{\xi}\frac{(r\, g_{\rm obs})^{1/2}}{g_{\rm bar}} \sim \frac{g_{\rm obs}-g_{\rm bar}}{g_{\rm bar}} \tag{18}$$

Explicit scale invariance by cancelling the proportionality factor:

$$\left(\frac{g_{\rm obs}-g_{\rm bar}}{g_{\rm bar}}\right)_2 \div \left(\frac{g_{\rm obs}-g_{\rm bar}}{g_{\rm bar}}\right)_1 = \left(\frac{g_{\rm obs,2}}{g_{\rm obs,1}}\right)^{1/2}\left(\frac{g_{\rm bar,1}}{g_{\rm bar,2}}\right) \tag{19}$$

RAR: where $g = g_{\rm obs,2}$, $g_N = g_{\rm bar,2}$, and $k = k_{(1)}$ are the system-1 terms.

$$\frac{g}{g_{\rm N}} - 1 = k_{(1)}\frac{g^{1/2}}{g_{\rm N}} \Rightarrow g = g_{\rm N} + \frac{k^2}{2} \pm \frac{1}{2}\sqrt{4 g_{\rm N} k^2 + k^4}\,, \tag{20}$$

$$\text{when } g_{\rm N} \gg k^2 : g \to g_{\rm N}\,, \text{but } g_{\rm N} \to 0\,, \quad \text{then } g \to k^2\,. \tag{21}$$

Towards the Radial Acceleration Relation (RAR) [7]

Ration of the correction term $\kappa_0(t)\,\upsilon$ to the usual Newtonian term:

$$x = \frac{\kappa_0 \upsilon r^2}{GM} = \frac{H_0}{\xi}\frac{\upsilon\, r^2}{GM} = \frac{H_0}{\xi}\frac{(r\, g_{\rm obs})^{1/2}}{g_{\rm bar}} \sim \frac{g_{\rm obs} - g_{\rm bar}}{g_{\rm bar}} \tag{18}$$

Explicit scale invariance by cancelling the proportionality factor:

$$\left(\frac{g_{\rm obs} - g_{\rm bar}}{g_{\rm bar}}\right)_2 \div \left(\frac{g_{\rm obs} - g_{\rm bar}}{g_{\rm bar}}\right)_1 = \left(\frac{g_{\rm obs,2}}{g_{\rm obs,1}}\right)^{1/2} \left(\frac{g_{\rm bar,1}}{g_{\rm bar,2}}\right) \tag{19}$$

RAR: where $g = g_{\rm obs,2}$, $g_N = g_{\rm bar,2}$, and $k = k_{(1)}$ are the system-1 terms.

$$\frac{g}{g_{\rm N}} - 1 = k_{(1)}\frac{g^{1/2}}{g_{\rm N}} \Rightarrow g = g_{\rm N} + \frac{k^2}{2} \pm \frac{1}{2}\sqrt{4 g_{\rm N} k^2 + k^4}\,, \tag{20}$$

$$\text{when } g_{\rm N} \gg k^2 : g \to g_{\rm N}\,, \text{but } g_{\rm N} \to 0\,, \quad \text{then } g \to k^2\,. \tag{21}$$

Continuity, Poisson, and Euler Equations within the SIV [8]

Continuity, Poisson, & Euler Equations ($\kappa = \kappa_0 = -\dot{\lambda}/\lambda = 1/t$):

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = \kappa \left[\rho + \vec{r} \cdot \vec{\nabla} \rho \right], \quad \vec{\nabla}^2 \Phi = \triangle \Phi = 4\pi G \rho, \tag{22}$$

$$\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \left(\vec{v} \cdot \vec{\nabla} \right) \vec{v} = -\vec{\nabla} \Phi - \frac{1}{\rho} \vec{\nabla} p + \kappa \vec{v}. \tag{23}$$

Application to a density perturbations $\rho(\vec{x}, t) = \rho_b(t)(1 + \delta(\vec{x}, t))$.

$$\dot{\delta} + \vec{\nabla} \cdot \dot{\vec{x}} = \kappa \vec{x} \cdot \vec{\nabla} \delta = n\kappa(t)\delta \quad , \quad \vec{\nabla}^2 \Psi = 4\pi G a^2 \rho_b \delta, \tag{24}$$

$$\ddot{\vec{x}} + 2H\dot{\vec{x}} + (\dot{\vec{x}} \cdot \vec{\nabla})\dot{\vec{x}} = -\frac{\vec{\nabla}\Psi}{a^2} + \kappa(t)\dot{\vec{x}}. \tag{25}$$

$$\Rightarrow \ddot{\delta} + (2H - (1+n)\kappa)\dot{\delta} = 4\pi G \rho_b \delta + 2n\kappa(H - \kappa)\delta. \tag{26}$$

Continuity, Poisson, and Euler Equations within the SIV [8]

Continuity, Poisson, & Euler Equations ($\kappa = \kappa_0 = -\dot{\lambda}/\lambda = 1/t$):

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = \kappa\left[\rho + \vec{r}\cdot\vec{\nabla}\rho\right], \; \vec{\nabla}^2\Phi = \triangle\Phi = 4\pi G\rho, \tag{22}$$

$$\frac{d\vec{v}}{dt} = \frac{\partial\vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v} = -\vec{\nabla}\Phi - \frac{1}{\rho}\vec{\nabla}p + \kappa\vec{v}. \tag{23}$$

Application to a density perturbations $\rho(\vec{x},t) = \rho_b(t)(1+\delta(\vec{x},t))$.

$$\dot{\delta} + \vec{\nabla}\cdot\dot{\vec{x}} = \kappa\vec{x}\cdot\vec{\nabla}\delta = n\kappa(t)\delta \quad , \quad \vec{\nabla}^2\Psi = 4\pi G a^2\rho_b\delta, \tag{24}$$

$$\ddot{\vec{x}} + 2H\dot{\vec{x}} + (\dot{\vec{x}}\cdot\vec{\nabla})\dot{\vec{x}} = -\frac{\vec{\nabla}\Psi}{a^2} + \kappa(t)\dot{\vec{x}}. \tag{25}$$

$$\Rightarrow \ddot{\delta} + (2H - (1+n)\kappa)\dot{\delta} = 4\pi G\rho_b\delta + 2n\kappa(H-\kappa)\delta. \tag{26}$$

The equation of energy conservation within SIV [5, 9]

Energy conservation within SIV:

$$\frac{d(\rho a^3)}{da} + 3\,pa^2 + (\rho + 3\,p)\frac{a^3}{\lambda}\frac{d\lambda}{da} = 0\,. \tag{27}$$

Equivalent form of the expression (27):

$$\dot{\rho} + 3\,\frac{\dot{a}}{a}(\rho + p) + \frac{\dot{\lambda}}{\lambda}(\rho + 3p) = 0\,. \tag{28}$$

Substituting the ρ and p (12) within the SIV expression (28):

$$\ddot{\psi} + U' + 3H_{\text{infl}}\,\dot{\psi} - 2\,(\dot{\psi}^2 - U) = 0\,. \tag{29}$$

The condition for inflation within SIV [9]!

The Hubble parameter and its time derivative:

By using: $\lambda = t_0/t$, $\dot{\psi} = -\dot{\lambda}/\lambda$, $\ddot{\psi} = -\dot{\psi}^2$, $U(\psi) = g\, e^{\mu\,\psi}$ one has:

$$H_{\text{infl}} = \dot{\psi} - \frac{2\,U}{3\,\dot{\psi}} - \frac{U'}{3\,\dot{\psi}} \qquad = \frac{1}{t} - \frac{(2+\mu)\,g}{3}\,t^{\mu+1}\,, \tag{30}$$

$$\dot{H}_{\text{infl}} = -\dot{\psi}^2 - \frac{2U}{3} - U' - \frac{U''}{3} \quad = -\frac{1}{t^2} - \frac{(\mu+2)(\mu+1)\,g}{3}\,t^{\mu}\,, \tag{31}$$

The critical ratio (13) for the occurrence of inflation:

$$\frac{|\,\dot{H}_{\text{infl}}\,|}{H^2_{\text{infl}}} = \frac{3\,(\mu+1)}{g\,(\mu+2)}\,t^{-\mu-2}\,. \tag{32}$$

Equivalent Lagrangians & Reparametrization Invariance [14]

Equivalent Lagrangians: same Euler-Lagrange equations of motion!

$$L_1(x,v)=\sqrt{g_{\mu\nu}v^{\mu}v^{\nu}},\ L_n=L_1^n,\ \frac{d}{d\tau}\left(\frac{\partial L}{\partial v^{\alpha}}\right)=\frac{\partial L}{\partial x^{\alpha}}. \tag{33}$$

For action $A_f=\int f(L(x,v))dt$ there is equivalence only if L is conserved.

$$\frac{dp_{\mu}}{dt}=\partial_{\mu}L-\frac{f''}{f'}\dot{L}p_{\mu},\ p_{\mu}=\frac{\partial L}{\partial v^{\mu}}\xrightarrow{f(L)=\sqrt{L}}\frac{dp_{\mu}}{dt}=\partial_{\mu}L+\frac{\dot{L}}{2L}p_{\mu}. \tag{34}$$

Fictitious forces/accelerations in un-proper time parametrization:

The (un-) proper time parametrization $d\tau^2=g_{\alpha\beta}(x,v)dx^{\alpha}dx^{\beta}$ and connection to the SIV paradigm ($L=\lambda^{-2}L_{\mathrm{GR}}$ with $\dot{L}_{\mathrm{GR}}=0$) i.e. (17) :

$$\frac{dp_{\mu}}{dt}=\partial_{\mu}L+\kappa_0 p_{\mu},\quad \text{where } \kappa_0=-\dot{\lambda}/\lambda. \tag{35}$$

Equivalent Lagrangians & Reparametrization Invariance [14]

Equivalent Lagrangians: same Euler-Lagrange equations of motion!

$$L_1(x,v)=\sqrt{g_{\mu\nu}v^{\mu}v^{\nu}},\ L_n=L_1^n,\ \frac{d}{d\tau}\left(\frac{\partial L}{\partial v^{\alpha}}\right)=\frac{\partial L}{\partial x^{\alpha}}. \quad (33)$$

For action $A_f=\int f(L(x,v))dt$ there is equivalence only if L is conserved.

$$\frac{dp_{\mu}}{dt}=\partial_{\mu}L-\frac{f''}{f'}\dot{L}p_{\mu},\ p_{\mu}=\frac{\partial L}{\partial v^{\mu}}\xrightarrow{f(L)=\sqrt{L}}\frac{dp_{\mu}}{dt}=\partial_{\mu}L+\frac{\dot{L}}{2L}p_{\mu}. \quad (34)$$

Fictitious forces/accelerations in un-proper time parametrization:

The (un-) proper time parametrization $d\tau^2=g_{\alpha\beta}(x,v)dx^{\alpha}dx^{\beta}$ and connection to the SIV paradigm ($L=\lambda^{-2}L_{\rm GR}$ with $\dot{L}_{\rm GR}=0$) i.e. (17) :

$$\frac{dp_{\mu}}{dt}=\partial_{\mu}L+\kappa_0 p_{\mu},\quad \text{where } \kappa_0=-\dot{\lambda}/\lambda. \quad (35)$$