Blinded analysis of cosmological data

Asta Heinesen

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CRAL
CENTRE DE RECHERCHE ASTROPHYSIQUE DE LYON
ENS DE LYON
erc

Cosmological data analysis with minimal assumptions

Data analysis for a new epoch of precision measurements

What information can be extracted from data without prior knowledge of geometry or field equations?

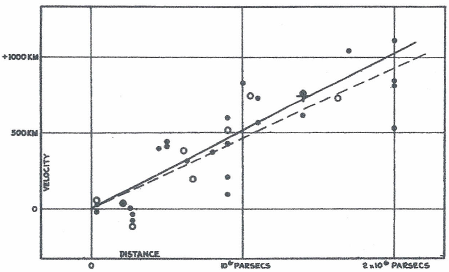
How do anisotropies in our cosmological measurements relate to the geometry in our cosmic neighbourhood?

Standardisable candles/sirens and redshift drift signals

Discussion

First measurements of the expansion of space

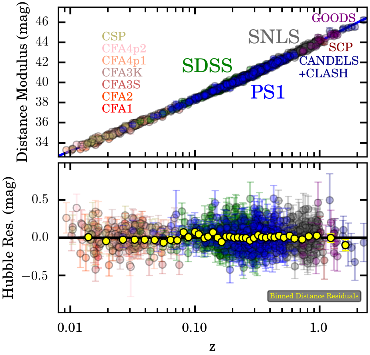
Discovery of the Hubble-Lemaître law $d_L \approx z/H_0$: G. Lemaître (1927), V. M. Slipher (1917), E. P. Hubble (1929).



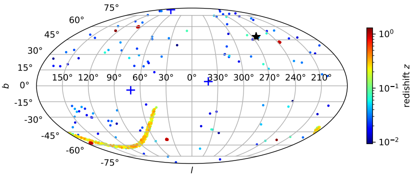
E. P. Hubble (1929): Velocity-Distance Relation among Extra-Galactic Nebulae. Radial velocities, corrected for solar motion, are plotted against distances estimated from involved stars and mean luminosities of nebulae in a cluster. ©US National Academy of Sciences.

Modern supernova Ia datasets

- ⋆ Improved distance calibration
- ⋆ Improved redshift-range and angular coverage



D. M. Scolnic et al. (2018): The Hubble diagram for the Pantheon sample of 1048 SNe Ia. Top: distance modulus for each SN; bottom: residuals to the best fit cosmology.



A. Borderies (2021) Sky distribution of SNe Ia in the Joint Lightcurve Analysis sample in galactic coordinates (l, b). The color scale represents the redshift z. The star correspond to the CMB dipole direction.

Luminosity distance

Distance defined by the ratio of intrinsic luminosity and flux observed from an astronomical object

⋆ Standard candles: Supernovae of type Ia
⋆ Standard sirens: Gravitational waves from black hole and neutron star mergers

Supernova remnant.
credit: NASA, ESA, and J. Banovetz, D. Milisavljevic

Cosmography, **FLRW geometry**

The luminosity distance Hubble law, M. Visser (2004):

$$d_L = \frac{1}{H_0} z + \frac{1-q_0}{2H_0} z^2 + \frac{-1+3q_0^2+q_0-j_0+\Omega_{k0}}{6H_0} z^3 + \mathcal{O}(z^4)$$

$$H \equiv \frac{\dot{a}}{a}\,, \qquad q \equiv -\frac{\ddot{a}}{aH^2}\,, \qquad j \equiv \frac{\dddot{a}}{aH^3}\,, \qquad \Omega_k \equiv \frac{-k}{a^2H^2}$$

$$\dot{} \equiv \frac{d}{dt}\,, \qquad k \in \{-1, 0, 1\}$$

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Purely geometrical result. "FLRW cosmography"

With no exact symmetries: Observables are dependent on the direction on the sky

Luminosity distance cosmography, **general geometry**

A. Heinesen, JCAP05(2021)008 [arXiv:2010.06534]

Isotropic FLRW cosmography $\rightarrow$ cosmography without symmetries

Important papers J. Kristian and R. K. Sachs (1966), S. Seitz, P. Schneider and J. Ehlers (1994), C. Clarkson and O. Umeh (2011), C. Clarkson, G. F. R. Ellis, A. Faltenbacher, R. Maartens, O. Umeh, J. P. Uzan (2012)

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$$H \rightarrow \mathfrak{H}\,, \qquad q \rightarrow \mathfrak{Q}\,, \qquad j \rightarrow \mathfrak{J}\,, \qquad \Omega_k \rightarrow \mathfrak{R}$$

Generalised cosmological parameters $\mathfrak{H}\,,\mathfrak{Q}\,,\mathfrak{J}\,,\mathfrak{R}$ vary with the point of observation and the line of sight.

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The luminosity distance Hubble law for a general congruence of observers and emitters in a general space-time:

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$$\mathfrak{H} = -\frac{\frac{\mathrm{d}E}{\mathrm{d}\lambda}}{E^2}\,, \qquad \mathfrak{Q} = -1 - \frac{1}{E}\frac{\frac{\mathrm{d}\mathfrak{H}}{\mathrm{d}\lambda}}{\mathfrak{H}^2}\,,$$

$$\mathfrak{R} = 1 + \mathfrak{Q} - \frac{1}{2E^2}\frac{k^\mu k^\nu R_{\mu\nu}}{\mathfrak{H}^2}\,, \qquad \mathfrak{J} = \frac{1}{E^2}\frac{\frac{\mathrm{d}^2\mathfrak{H}}{\mathrm{d}\lambda^2}}{\mathfrak{H}^3} - 4\mathfrak{Q} - 3$$

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$\frac{\mathrm{d}E}{\mathrm{d}\lambda}$: rate of change of photon energy, E, along null ray.

$\frac{\mathrm{d}}{\mathrm{d}\lambda} \equiv k^\mu \nabla_\mu$: Derivative along photon 4-momentum, k^μ.

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$k^\mu k^\nu R_{\mu\nu}$: Ricci focusing term.

Luminosity distance cosmography, **general geometry**

Simple multipole expansions in direction of incoming light, e^μ, seen by the observer

Physically interpretable multipole coefficients

$$H \to \mathfrak{H} = \frac{1}{3}\theta - e^\mu a_\mu + e^\mu e^\nu \sigma_{\mu\nu} \,,$$

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$$q \to \mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^\mu \overset{1}{\mathfrak{q}}_\mu + e^\mu e^\nu \overset{2}{\mathfrak{q}}_{\mu\nu} + e^\mu e^\nu e^\rho \overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^\mu e^\nu e^\rho e^\kappa \overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})}\,,$$

1 dof $\to$ 16 independent dof

$$j_0 - \Omega_{k0} \to \mathfrak{J} - \mathfrak{R} = 1 + \frac{\overset{0}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{e}\cdot\overset{1}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{ee}\cdot\overset{2}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{eee}\cdot\overset{3}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{eeee}\cdot\overset{4}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{eeeee}\cdot\overset{5}{\boldsymbol{\mathfrak{t}}} + \boldsymbol{eeeeee}\cdot\overset{6}{\boldsymbol{\mathfrak{t}}}}{\mathfrak{H}^3(\boldsymbol{e})}\,,$$

1 dof $\to$ 36 independent dof

Luminosity distance cosmography, **general geometry**

- $\star$ The effective deceleration parameter $\mathfrak{Q}$ does not directly measure deceleration of length scales!

- $\star$ $\mathfrak{Q}$ is generally not constrained in sign by any energy condition.

- $\star$ Negative values of $\mathfrak{Q}$ without violation of the strong energy condition.

- $\star$ The effective curvature parameter $\mathfrak{R} \not\propto {}^{(3)}R$
  ${}^{(3)}R$: spatial Ricci curvature in the frame of observers

Luminosity distance cosmography – some comments

- For a *general* cosmological space-time the luminosity distance series expansion in redshift is given by 9, 25, 61 dof in the $\mathcal{O}(z)$, $\mathcal{O}(z^2)$, $\mathcal{O}(z^3)$ vicinity of the observer.

- This opens the door for model-independent analysis.

- The generalisation $H \to \mathfrak{H}$ might have implications for the Hubble tension, $q \to \mathfrak{Q}$ might have implications for the dark energy problem.

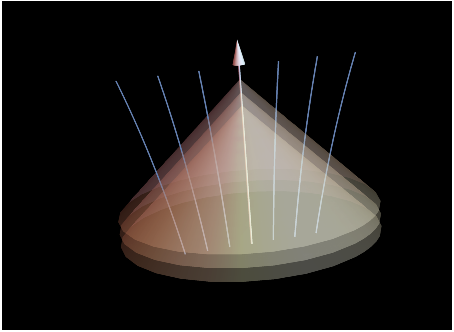
- Investigate luminosity distance of artificial observers within realistic numerical simulations. H. J. Macpherson (University of Cambridge), J. Adamek (University of Zurich), R. Durrer (University of Geneva), M. Kunz (University of Geneva), C. Clarkson (Queen Mary, University of London)

- Motivated search for anisotropies in supernova data. A. Borderies (ENS de Lyon), H. J. Macpherson (University of Cambridge), S. Dhawan (University of Cambridge)

Redshift drift

Change in redshift of an astronomical object in time of the observer

- ⋆ "Lyman-α forest" from quasars
- ⋆ Neutral hydrogen 21-cm emission lines



The drift in time of an observer's light cone.
credit: M. Korzyński

Redshift drift in a general cosmology

A. Heinesen, Phys. Rev. D **103** (2021), 023537 [arXiv:2011.10048]

Redshift drift in a general space-time (the *exact* signal):

$$\dot{z} = (1+z)\mathfrak{H}_o - \mathfrak{H}_e + \mathcal{S}_{e\to o}$$

(FLRW: $\dot{z} = (1+z)H_o - H_e$)

$\mathcal{S}_{e\to o}$: zero in FLRW universe models and represents anisotropies and inhomogeneities along the null congruence

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Redshift drift as a probe of the strong energy condition

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$$\dot{z} = (1+z)\mathfrak{H}_o - \mathfrak{H}_e + \mathcal{S}_{e\to o} = E_e \int_{\lambda_e}^{\lambda_o} d\lambda\, \Pi$$

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Obtain bounds on Π from imposing the strong energy condition:

Strong energy condition satisfied $\rightarrow \Pi \leq 0 \rightarrow \dot{z} \leq 0$

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Redshift drift cosmography

A. Heinesen, Phys. Rev. D **104** (2021), 123527 [arXiv:2107.08674]

Redshift drift series expansion to first order in redshift:

$$\dot{z} = -\mathfrak{d}_0 \mathfrak{H}_0 z + \mathcal{O}(z^2)$$

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$$H \to \mathfrak{H}$$

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$H \to \mathfrak{H}$ 1 dof $\to$ 9 dof

$q \to \mathfrak{d}$ 1 dof $\to$ 12 independent dof

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The generalised deceleration parameter $\mathfrak{d} \neq \mathfrak{Q}$

Redshift drift – some comments

- In general the redshift drift signal acquires non-trivial contributions from structure along the null ray.

- Anisotropies tend to systematically contribute negatively to the redshift drift signal → Redshift drift as a model independent test of the strong energy condition.
  Consistent with numerical results:
  S. M. Koksbang, JCAP 10 (2019) 036,
  S. M. Koksbang, MNRAS 498 (2020) L135

- Understanding local motion effects vs. global cosmological effects in drift measurements. M. Korzyński (University of Warsaw)

- Reduction of number of degrees of freedom in cosmography expressions for realistic space-time approximations.
  A. Heinesen and H. J. Macpherson, JCAP 03 (2022) 057

- Connection to cosmological backreaction? T. Buchert (2000, 2001)
  T. Buchert, H. v. Elst, A. Heinesen (2021)

Discussion

Model independent cosmological analysis of distance–redshift data and redshift drift data is possible!

Requires sufficient data, sky coverage, and control over systematic errors in data.

$\sim 10^5$ SN1a within this decade (LSST, WFIRST, ...)

New cosmological measurements: Redshift drift and position drift. Measurements of redshift drift in the $2 < z < 5$ range available in ~ 1 decade of observation time (ELT). Low redshift detections available in a few decades (SKA).

Until then: Constrained analyses with available distance–redshift data • Realistic model space-times for assessing light propagation and measurements of typical observers • Develop the theory side • Forecasts for upcoming surveys

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e^μ: Direction of incoming light as seen by the observer

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$u^\mu u^\nu R_{\mu\nu}$: Ricci focusing term;
only non-vanishing term in FLRW limit

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$u^\mu u^\nu R_{\mu\nu}$: Ricci focusing term

Now assume that $a^\mu = 0$, then

$\Pi^o \leq 0$ if $u^\mu u^\nu R_{\mu\nu} \geq 0$ (Strong Energy Condition)

$\Rightarrow \dot{z} \leq 0$ if $u^\mu u^\nu R_{\mu\nu} \geq 0$ if monopole, Π^o, is dominating

Positivity of redshift drift = detection of SEC violation (Dark energy)

Luminosity distance cosmography, **general geometry**

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^{\mu}\overset{1}{\mathfrak{q}}_{\mu} + e^{\mu}e^{\nu}\overset{2}{\mathfrak{q}}_{\mu\nu} + e^{\mu}e^{\nu}e^{\rho}\overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^{\mu}e^{\nu}e^{\rho}e^{\kappa}\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})} \,,$$

$$\begin{aligned}
\overset{0}{\mathfrak{q}} &\equiv \tfrac{1}{3}\tfrac{\mathrm{d}\theta}{\mathrm{d}\tau} + \tfrac{1}{3}D_{\mu}a^{\mu} - \tfrac{2}{3}a^{\mu}a_{\mu} - \tfrac{2}{5}\sigma_{\mu\nu}\sigma^{\mu\nu} \\
\overset{1}{\mathfrak{q}}_{\mu} &\equiv -h^{\nu}_{\mu}\tfrac{\mathrm{d}a_{\nu}}{\mathrm{d}\tau} - \tfrac{1}{3}D_{\mu}\theta + a^{\nu}\omega_{\mu\nu} + \tfrac{9}{5}a^{\nu}\sigma_{\mu\nu} - \tfrac{2}{5}D_{\nu}\sigma^{\nu}_{\ \mu} \\
\overset{2}{\mathfrak{q}}_{\mu\nu} &\equiv h^{\alpha}_{\mu}h^{\beta}_{\nu}\tfrac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}\tau} + D_{\langle\mu}a_{\nu\rangle} + a_{\langle\mu}a_{\nu\rangle} - 2\sigma_{\alpha(\mu}\omega^{\alpha}_{\ \nu)} - \tfrac{6}{7}\sigma_{\alpha\langle\mu}\sigma^{\alpha}_{\ \nu\rangle} \\
\overset{3}{\mathfrak{q}}_{\mu\nu\rho} &\equiv -D_{\langle\mu}\sigma_{\nu\rho\rangle} - 3a_{\langle\mu}\sigma_{\nu\rho\rangle} \\
\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa} &\equiv 2\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}
\end{aligned}$$

D_{μ}: spatial derivative , $\langle\rangle$: trace-free part of spatial tensor

O. Umeh, PhD thesis (2013)

Luminosity distance cosmography, **general geometry**

$$\mathfrak{Q} = -1 - \frac{\overset{0}{\mathfrak{q}} + e^{\mu}\overset{1}{\mathfrak{q}}_{\mu} + e^{\mu}e^{\nu}\overset{2}{\mathfrak{q}}_{\mu\nu} + e^{\mu}e^{\nu}e^{\rho}\overset{3}{\mathfrak{q}}_{\mu\nu\rho} + e^{\mu}e^{\nu}e^{\rho}e^{\kappa}\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa}}{\mathfrak{H}^2(\boldsymbol{e})} \, ,$$

$$\overset{0}{\mathfrak{q}} \equiv \frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau} + \frac{1}{3}D_{\mu}a^{\mu} - \frac{2}{3}a^{\mu}a_{\mu} - \frac{2}{5}\sigma_{\mu\nu}\sigma^{\mu\nu}$$
$$\overset{1}{\mathfrak{q}}_{\mu} \equiv -h^{\nu}_{\mu}\frac{\mathrm{d}a_{\nu}}{\mathrm{d}\tau} - \frac{1}{3}D_{\mu}\theta + a^{\nu}\omega_{\mu\nu} + \frac{9}{5}a^{\nu}\sigma_{\mu\nu} - \frac{2}{5}D_{\nu}\sigma^{\nu}_{\ \mu}$$
$$\overset{2}{\mathfrak{q}}_{\mu\nu} \equiv h^{\alpha}_{\mu}h^{\beta}_{\nu}\frac{\mathrm{d}\sigma_{\alpha\beta}}{\mathrm{d}\tau} + D_{\langle\mu}a_{\nu\rangle} + a_{\langle\mu}a_{\nu\rangle} - 2\sigma_{\alpha(\mu}\omega^{\alpha}_{\ \nu)} - \frac{6}{7}\sigma_{\alpha\langle\mu}\sigma^{\alpha}_{\ \nu\rangle}$$
$$\overset{3}{\mathfrak{q}}_{\mu\nu\rho} \equiv -D_{\langle\mu}\sigma_{\nu\rho\rangle} - 3a_{\langle\mu}\sigma_{\nu\rho\rangle}$$
$$\overset{4}{\mathfrak{q}}_{\mu\nu\rho\kappa} \equiv 2\sigma_{\langle\mu\nu}\sigma_{\rho\kappa\rangle}$$

$\frac{1}{3}\frac{\mathrm{d}\theta}{\mathrm{d}\tau}$: local acceleration of length scales;
only non-vanishing term in the comoving FLRW limit.

The effective deceleration parameter $\mathfrak{Q}$ does not directly measure deceleration of length scales!

Luminosity distance, **A numerical relativity study**

H. J. Macpherson and A. Heinesen, Phys.Rev.D 104 (2021) 023525 [arXiv:2103.11918]

General relativistic simulation (Einstein Toolkit) with realistic cosmological initial conditions.

Cosmological constant $= 0$ $\rightarrow$ Model universe well described on the largest scales by an Einstein de Sitter model.

Sky map of effective cosmological parameters of typical observer (coarsegraining scale of ∼200Mpc/h scale corresponding to $\sim$ 5% density contrasts):