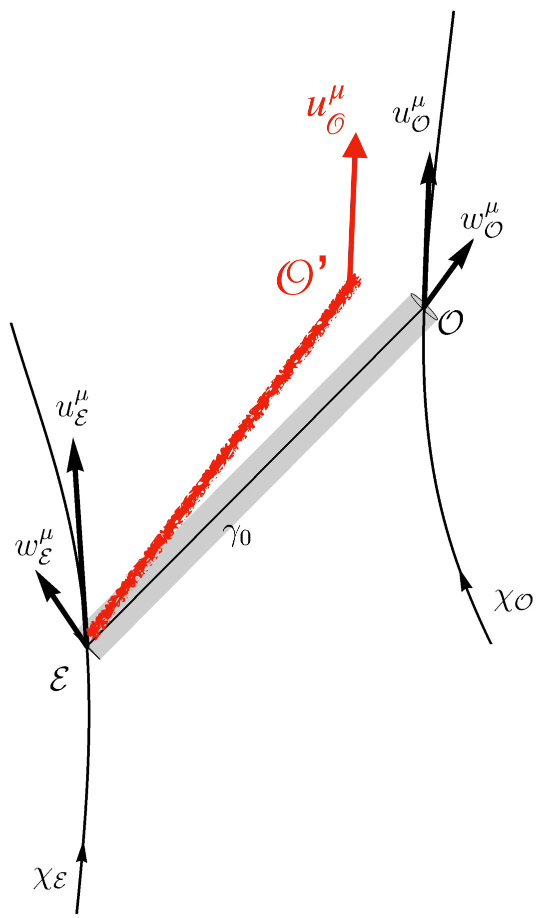
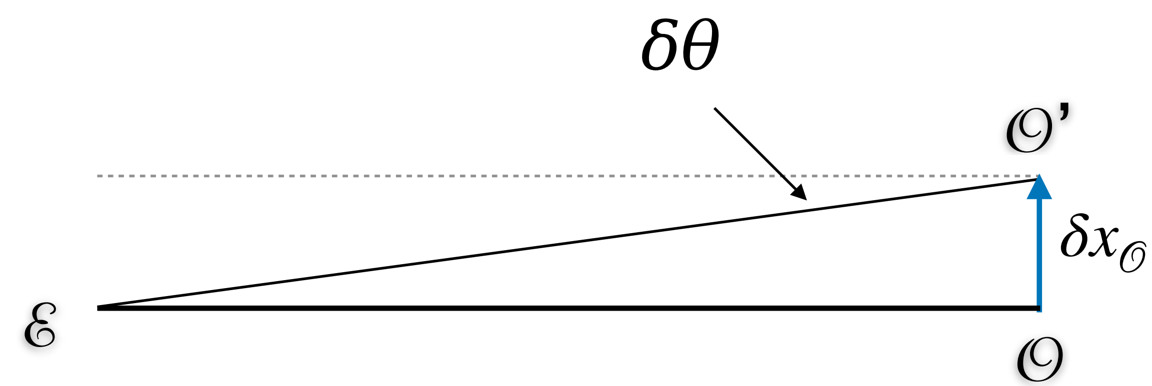
Parallax in general relativity





Mikołaj Korzyński

Center for Theoretical Physics, Polish Academy of Sciences, Warsaw

Hot topics in Modern Cosmology
Spontaneous Workshop XIV
May 8th – 14th, 2022, IESC Cargèse, France

Motivation

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Geometric optics in GR beyond the Sachs formalism, beyond a single emission and observation point

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Based on papers:

M. Grasso, MK, J. Serbenta, *Geometric optics in general relativity using bilocal operators,* Phys. Rev. D **99**, 064038 (2019)

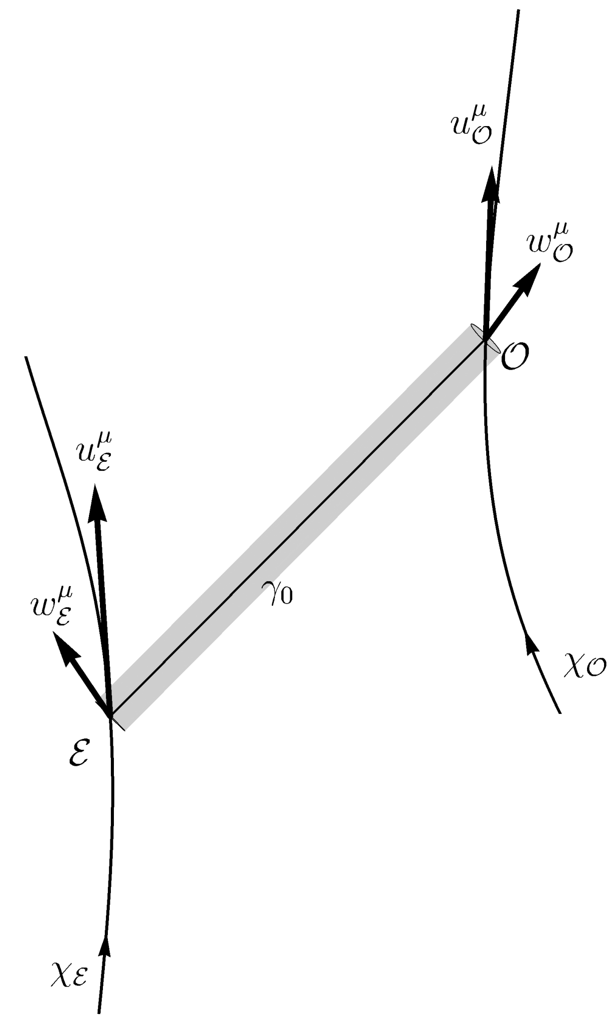
MK, E. Villa, *Geometric optics in relativistic cosmology: New formulation and a new observable,* Phys. Rev. D **101**, 063506 (2020)

MK, J. Serbenta, *Testing the null energy condition with precise distant measurements,* Phys. Rev. D **105**, 084017 (2022)

Distance measures in GR

Distance measure along a null geodesic

$$D \equiv D(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}}, u_{\mathscr{E}})$$



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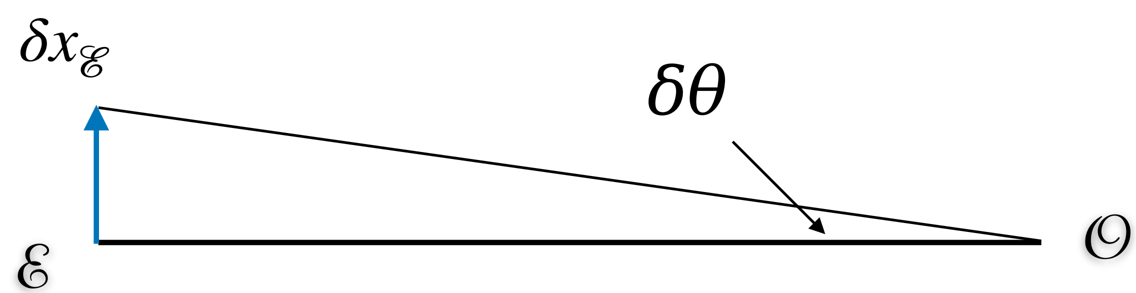
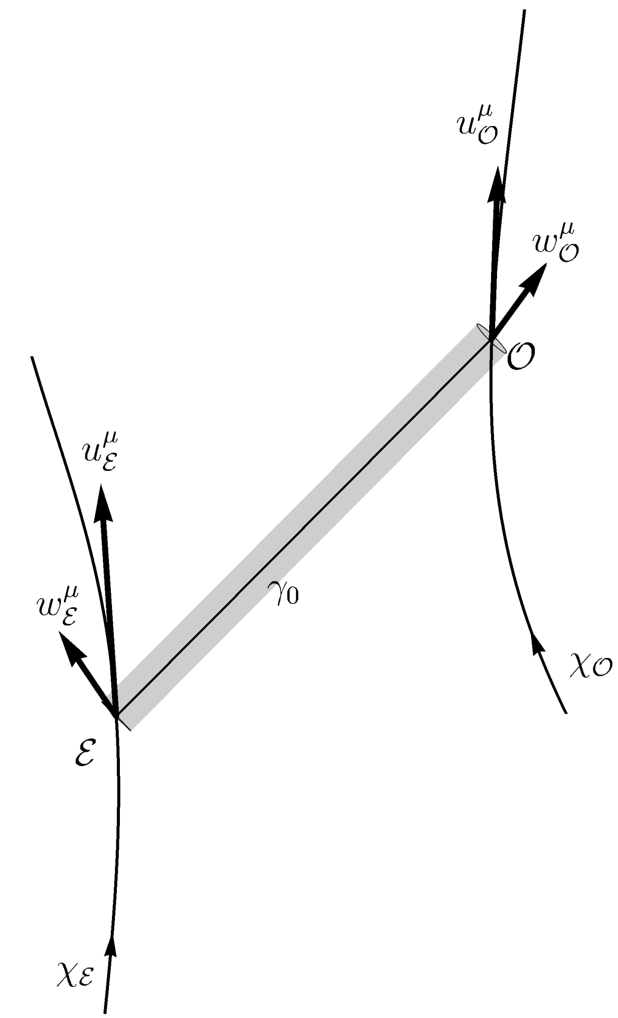
Angular diameter distance

flat spacetime

$$\delta\theta = \frac{\delta x_{\mathscr{E}}}{D}$$





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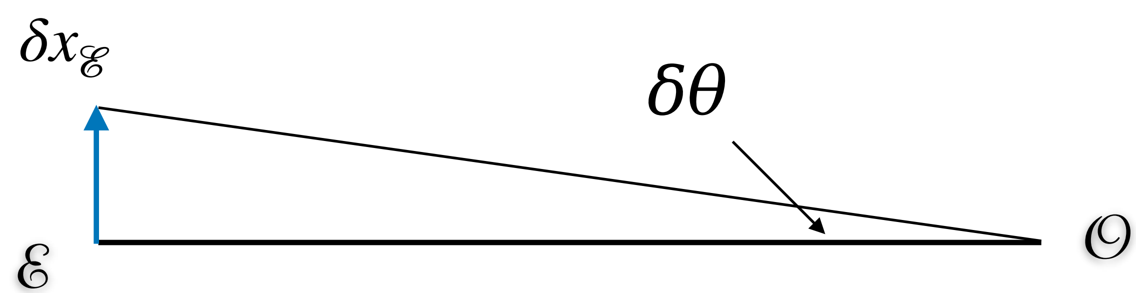
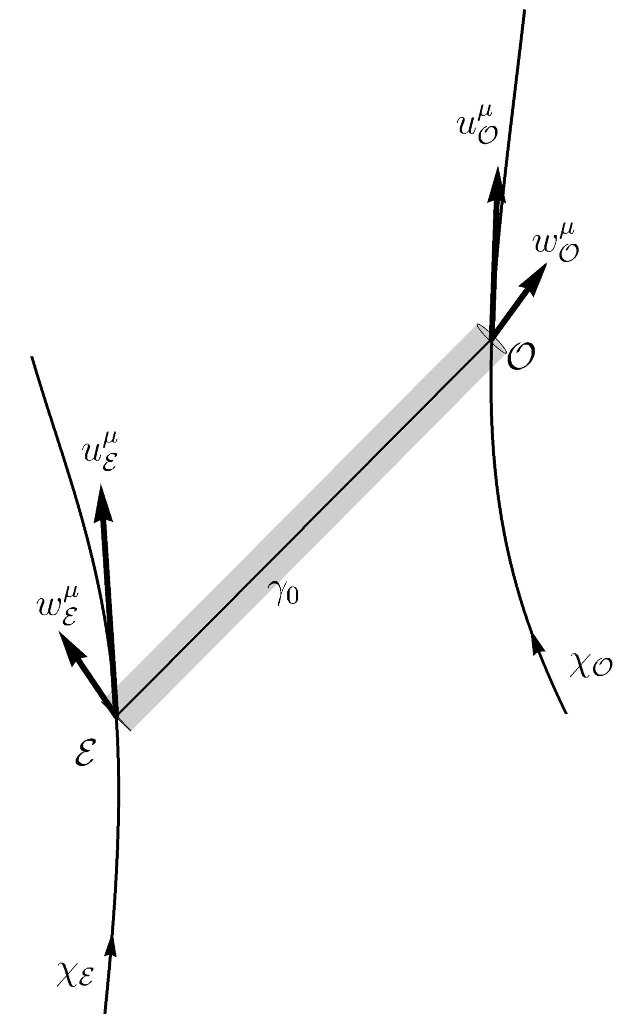




general spacetime

$$\delta\theta^A = M^A{}_B\, \delta x^B_{\mathscr{E}}$$

$$D_{ang} = \left|\det M^A{}_B\right|^{-1/2} = \left|\frac{A_{\mathscr{E}}}{\Omega_{\mathscr{O}}}\right|^{1/2}$$

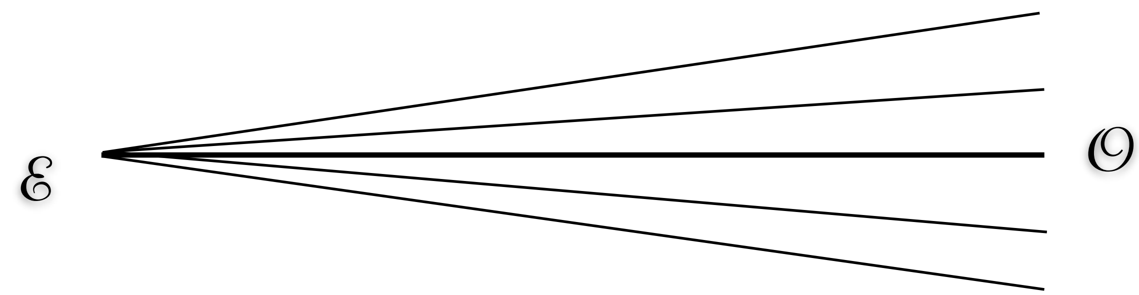
$$D_{ang} \equiv D_{ang}(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}})$$

Distance measures in GR

Luminosity distance

flat spacetime, no relative motion

$$F = \frac{I}{4\pi D^2}$$



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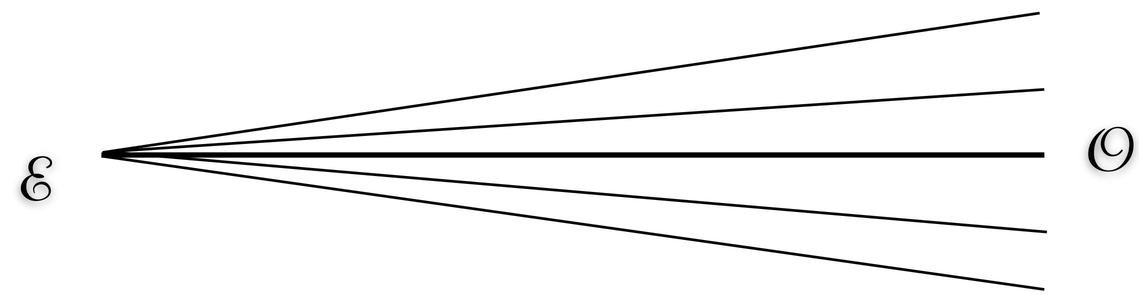

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$$D_{lum} = \sqrt{\frac{I}{4\pi F}} \qquad D_{lum} \equiv D_{lum}(\mathscr{E}, \mathscr{O}, \gamma_0, u_{\mathscr{O}}, u_{\mathscr{E}})$$

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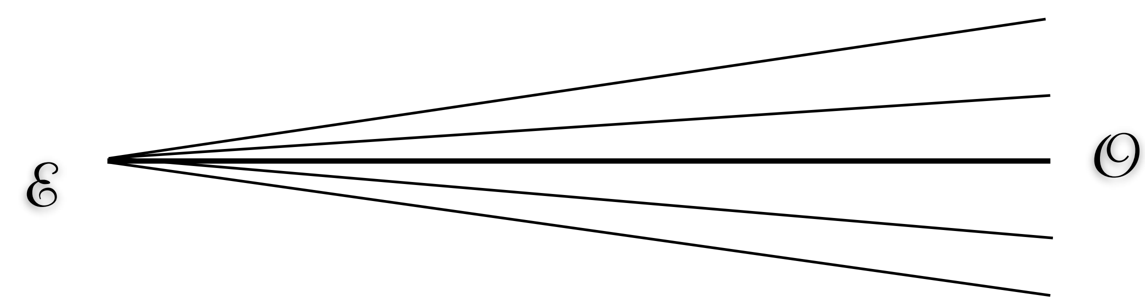

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Related to the angular diameter distance via the Etherington's reciprocity relation

$$D_{lum} = (1+z)^2\, D_{ang}$$

[Etherington 1933, Penrose 1966, … Uzun 2019]

Angular diameter distance

Expressing the distance measures using curvature

Main tool: geodesic deviation equation around a null geodesic

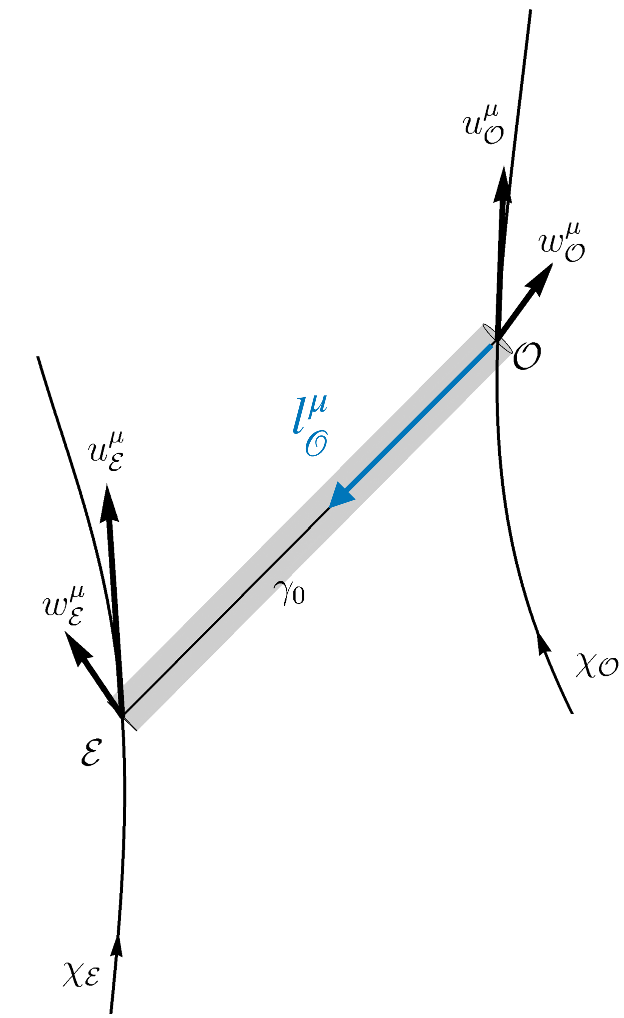
$$M^{A}{}_{B} = (l_{\mathcal{O}\,\mu}\, u^{\mu}_{\mathcal{O}})\, \mathcal{D}^{-1\,A}{}_{B}$$

Jacobi matrix

$$\ddot{\mathcal{D}}^{A}{}_{B} - R^{A}{}_{llC}\, \mathcal{D}^{C}{}_{B} = 0$$

$$\mathcal{D}^{A}{}_{B}(\lambda_{\mathcal{O}}) = 0$$

$$\dot{\mathcal{D}}^{A}{}_{B}(\mathcal{O}) = \delta^{A}{}_{B}$$



Angular diameter distance

$$D_{ang} = (l_{\mathcal{O}\,\mu}\, u^{\mu}_{\mathcal{O}})^{-1} \left| \det \mathcal{D}^{A}{}_{B} \right|^{1/2}$$

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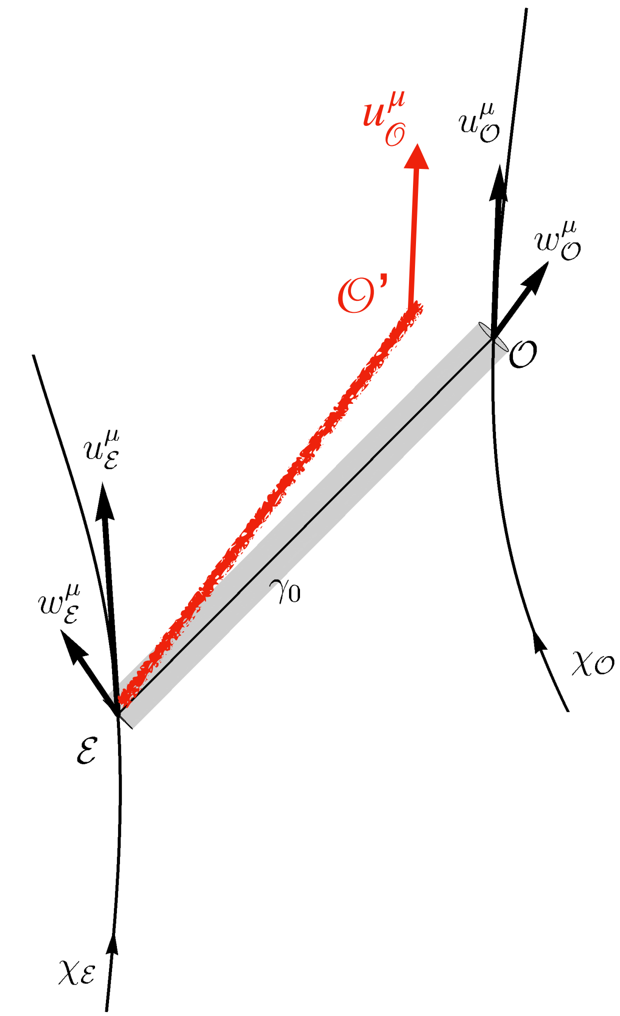
Parallax effect - difference in apparent position of a light source between two nearby observers [Grasso, Korzyński, Serbenta 2019]



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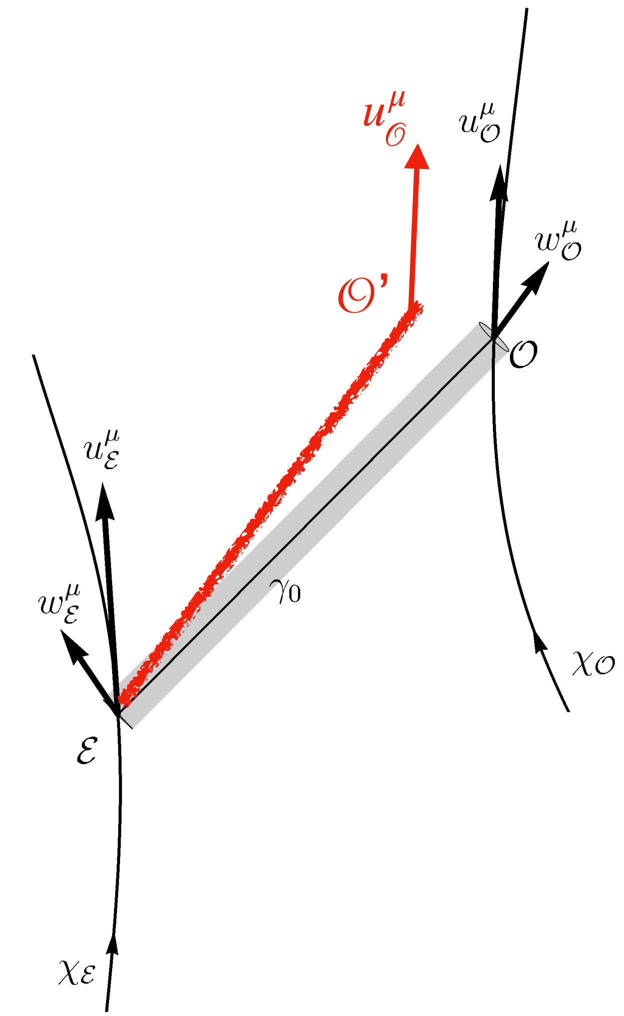

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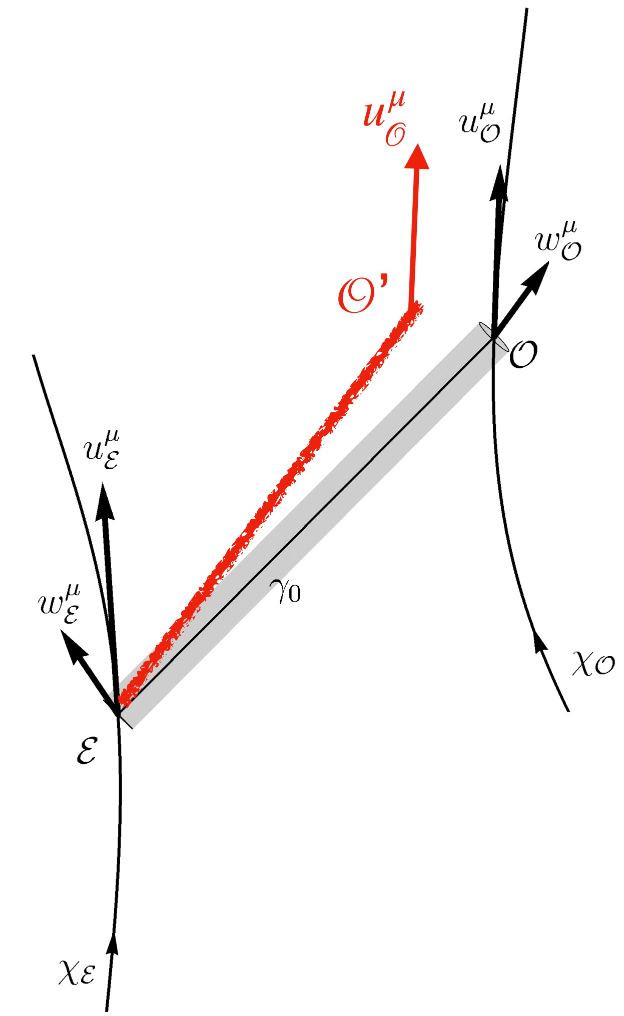

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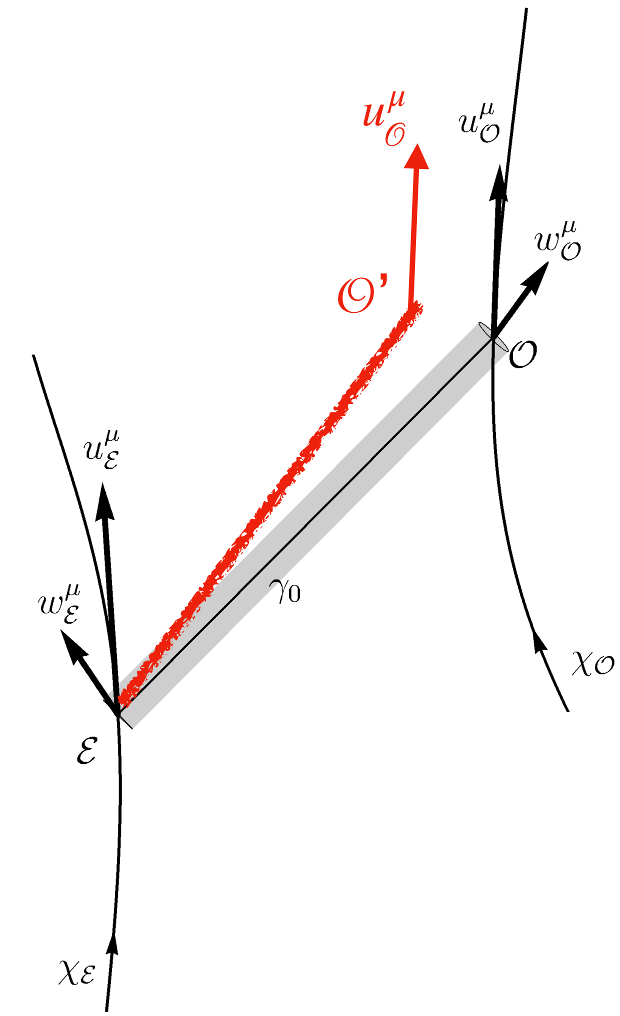

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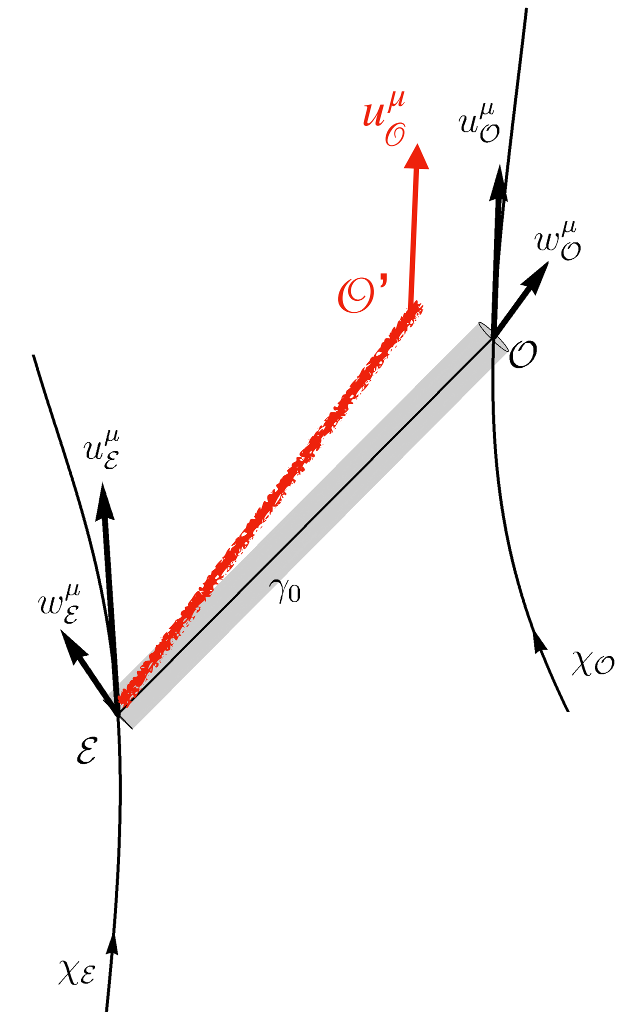

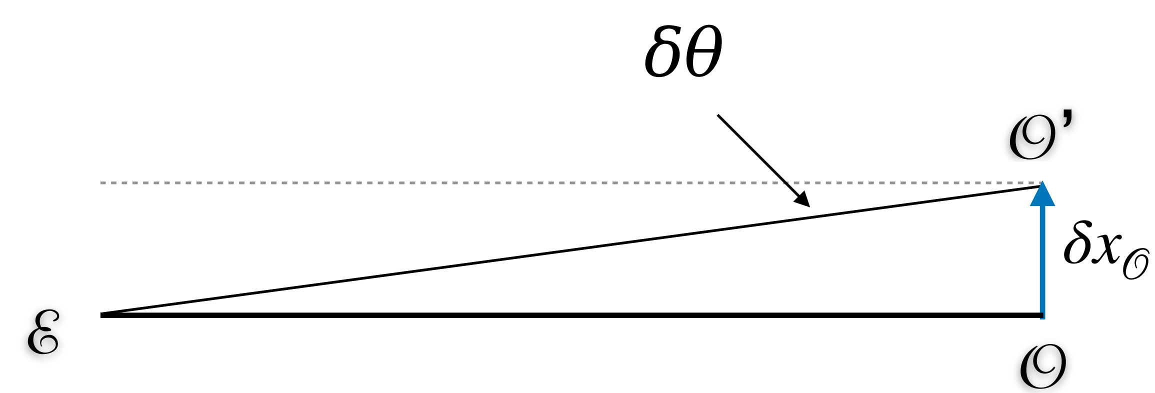

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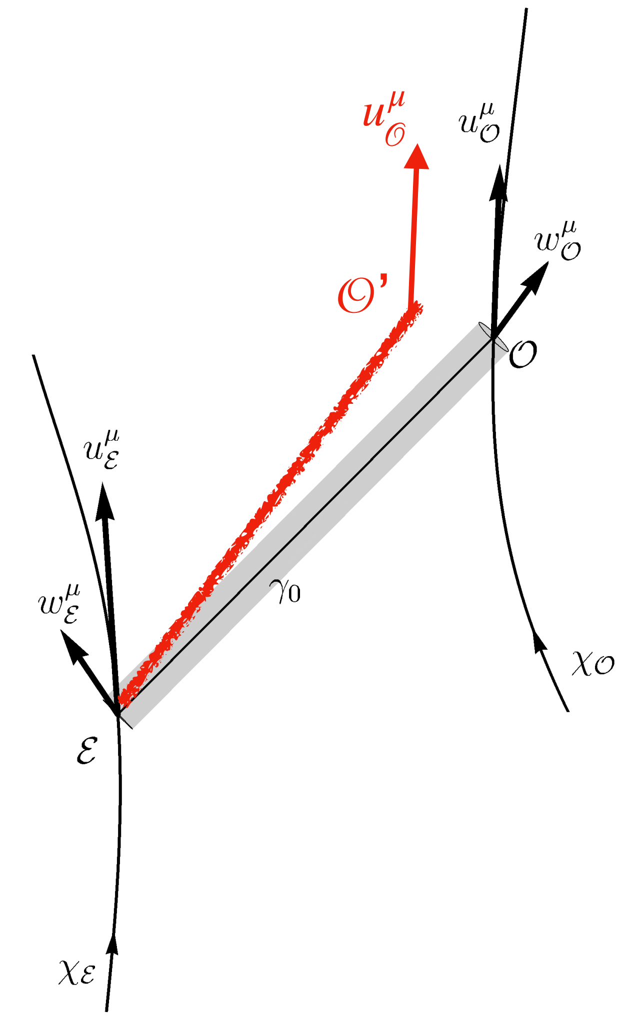

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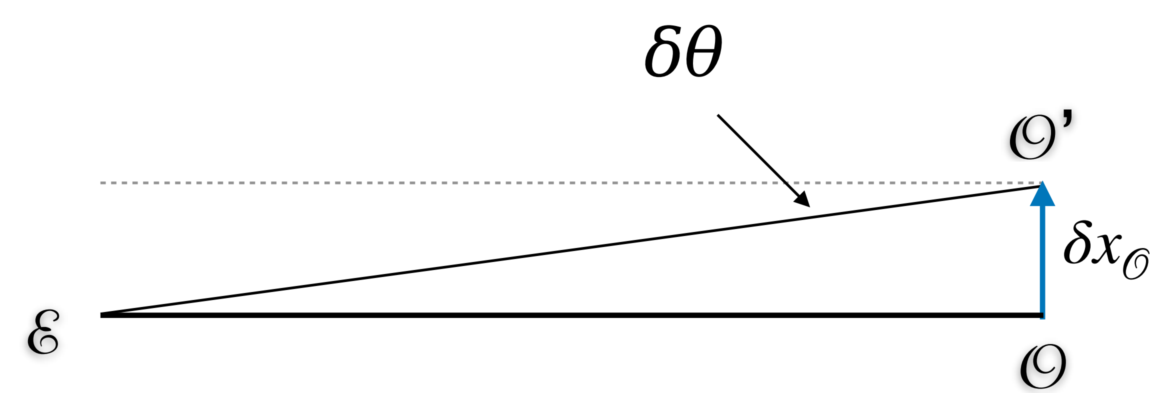
$$\Pi_{AB} = \Pi_{BA}$$



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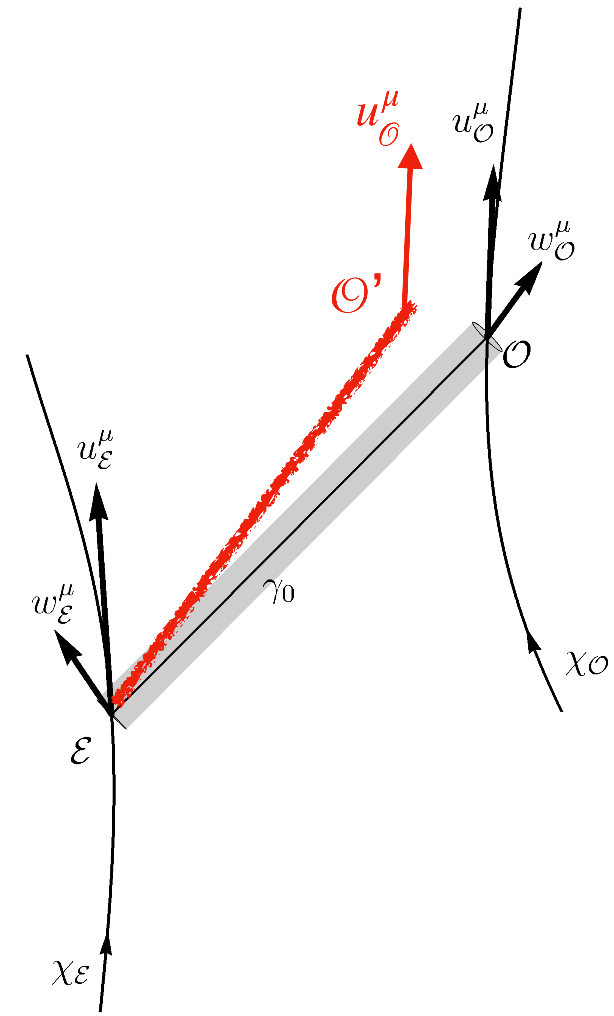

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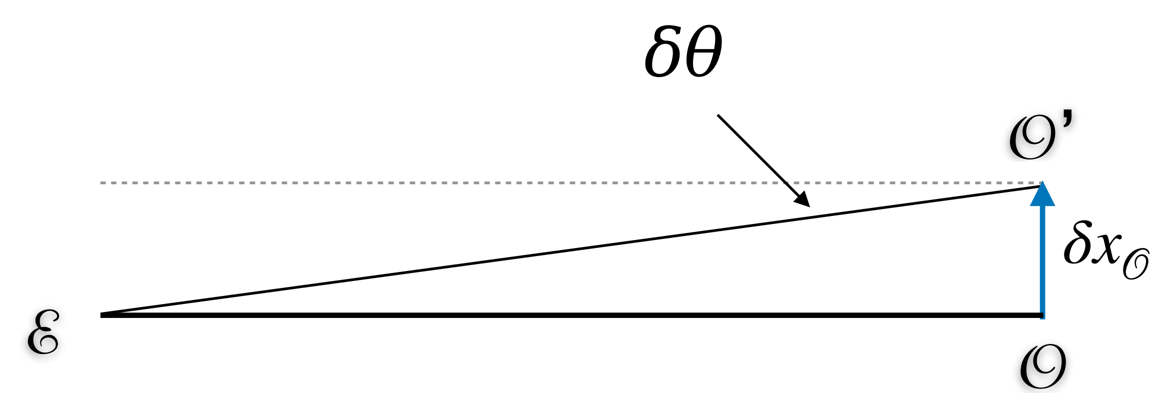

$$D_{par} = \left|\det \Pi^A{}_B\right|^{-1/2} \qquad D_{par} \equiv D_{par}(\mathcal{E}, \mathcal{O}, \gamma_0, u_{\mathcal{O}})$$

Parallax distance

Expressing the distance measures using curvature

$$\Pi^A{}_B = (l_{\mathcal{O}\,\mu}\, u^\mu_{\mathcal{O}})\, \mathcal{D}^{-1\,A}{}_C \left(\delta^C{}_B + m^C{}_B\right)$$

$$\dot{m}^A{}_B - R^A{}_{llC}\, m^C{}_B = R^A{}_{llB}$$

$$m^A{}_B(\mathcal{O}) = 0$$

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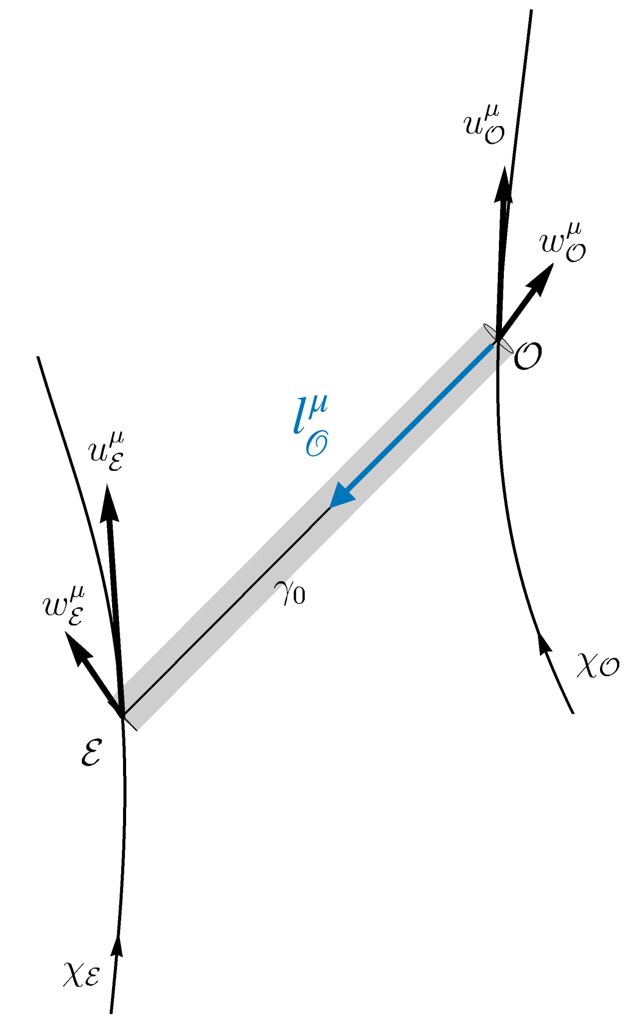

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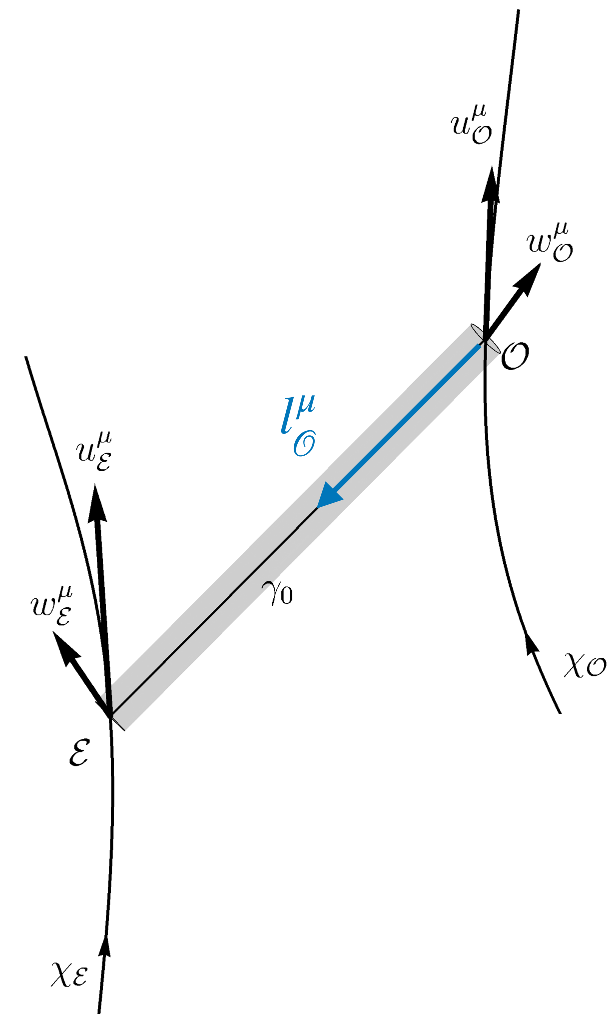

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curvature correction

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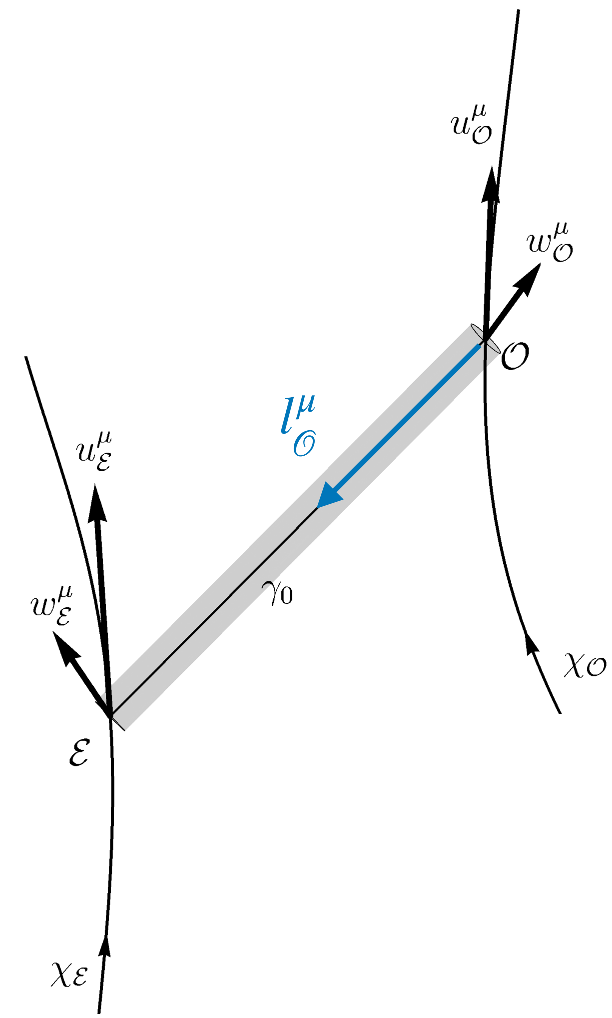

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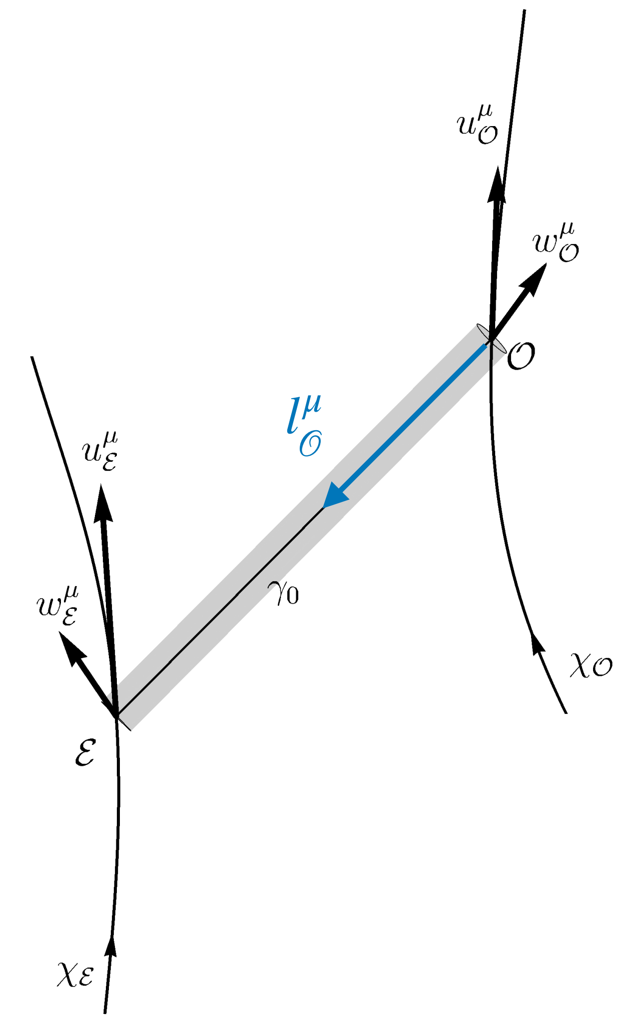

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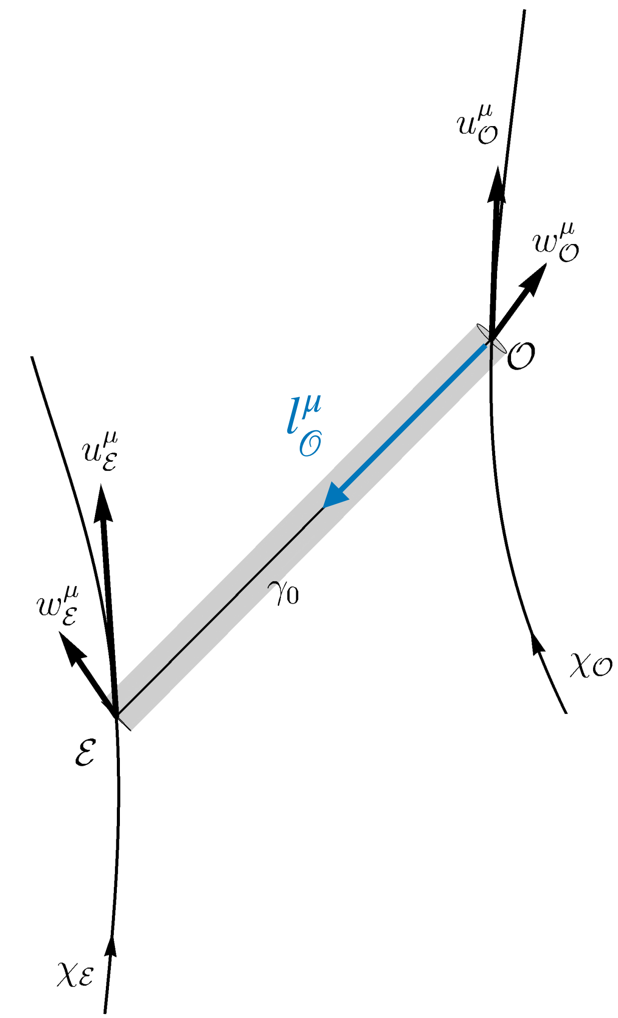
Distance slip

Define a scalar, dimensionless quantity $$\mu = 1 - \frac{\det \Pi^A{}_B}{\det M^A{}_B}$$



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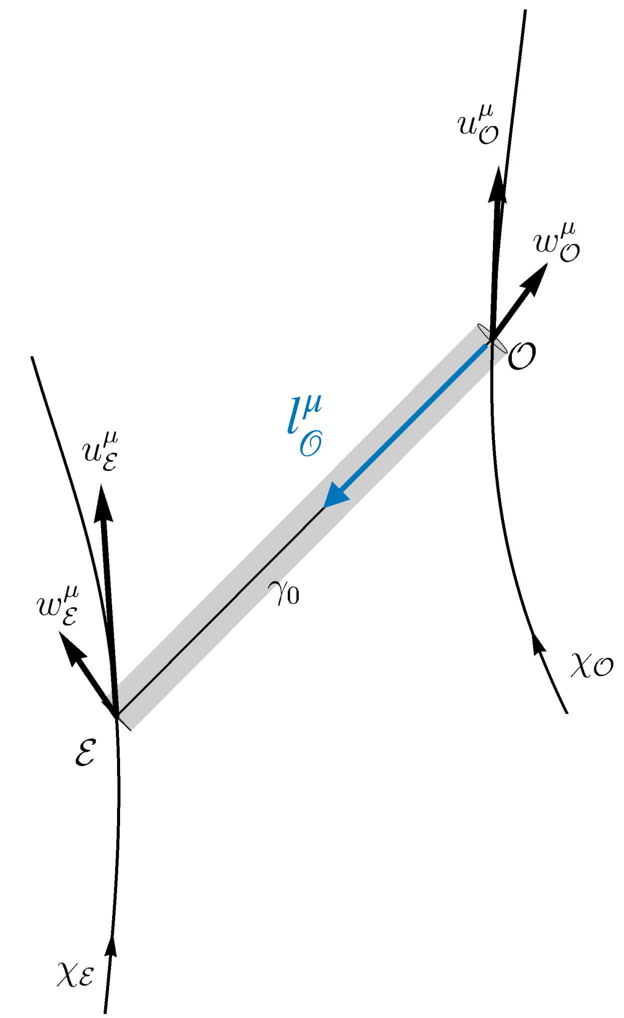
±1, but usually 1



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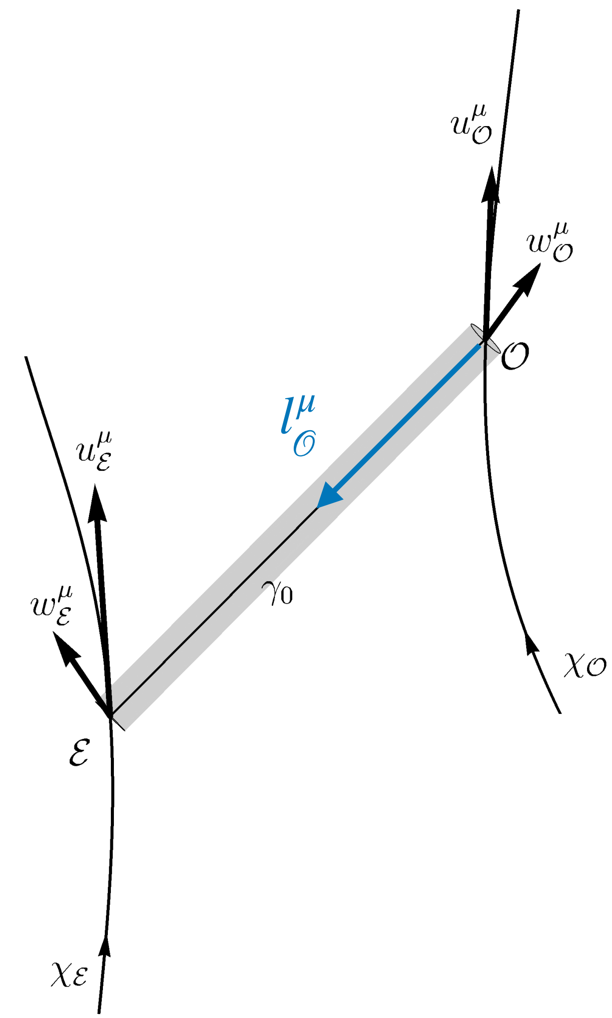

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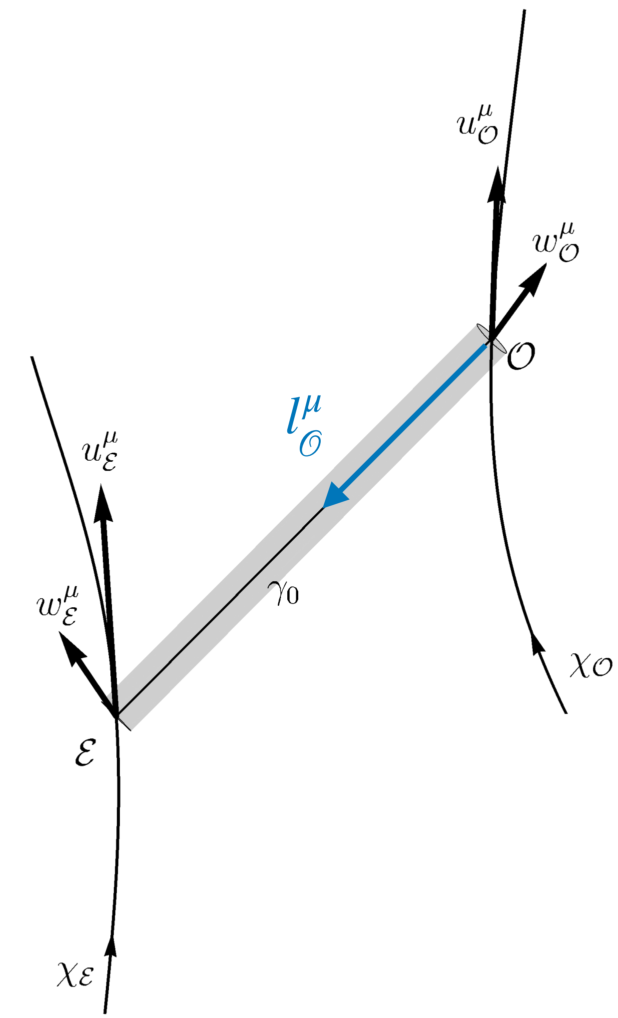

Vanishes in a flat spacetime

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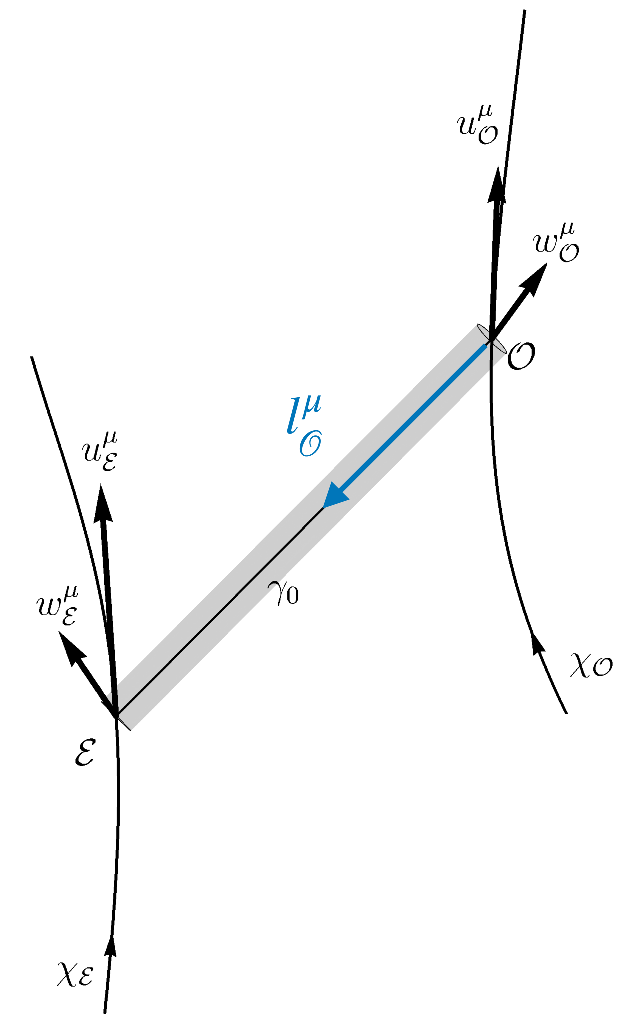
Frames-independent $\mu \equiv \mu(\mathscr{E}, \mathscr{O}, \gamma_0)$



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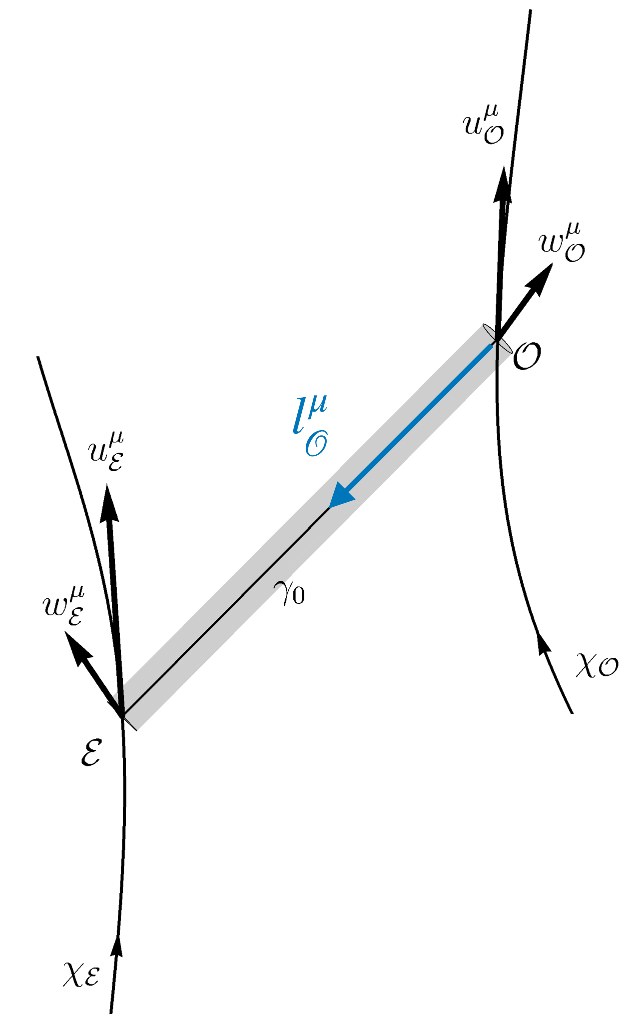

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no $C^\mu{}_{\nu\alpha\beta}$ or Λ

Short distance approximation:
$$\mu = \frac{8\pi G}{c^4} \int_{\mathcal{O}}^{\mathscr{E}} T_{ll}(\lambda)\ \left(\lambda_{\mathscr{E}} - \lambda\right)\ d\lambda + O(\mathrm{R}^2)$$

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Magnitude of the effect locally:

negligible pressure (dust) $T^{\mu\nu} = \rho\, U^\mu U^\nu$

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Galactic scales

mass density of the thin disc of the Milky Way $\rho \approx 1\, M_\odot\, \mathrm{pc}^{-3}$

most distant trigonometric parallax measured $r \approx 20\, \mathrm{kpc}$

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$\mu \approx 2 \cdot 10^{-4}$

μ in cosmology

MK, E. Villa, *Geometric optics in relativistic cosmology: New formulation and a new observable,* Phys. Rev. **D 101**, 063506 (2020)

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Need sources for which two methods of distance determination are possible (+ big sample)

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SN1a + host galaxy identification D_{lum}, z, D_{par}

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quasars as standard candles [Panda *et al* 2018; Risalti & Lusso 2018]

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MK, E. Villa, *Geometric optics in relativistic cosmology: New formulation and a new observable,* Phys. Rev. **D 101**, 063506 (2020)

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Assume this measurement is possible. Signal? What can we learn?

μ in FLRW spacetime

μ in FLRW spacetime

$$ds^2 = -\,dt^2 + a(t)^2\left(d\chi^2 + S_k(\chi)^2\, d\Omega^2\right)$$

$$S_k(\chi) = \begin{cases} \frac{1}{\sqrt{k}}\sin(\sqrt{k}\chi) & \text{if } k > 0 \\ \chi & \text{if } k = 0 \\ \frac{1}{\sqrt{|k|}}\sinh(\sqrt{|k|}\chi) & \text{if } k < 0, \end{cases}$$

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$$ds^2 = -\,dt^2 + a(t)^2\left(d\chi^2 + S_k(\chi)^2\,d\Omega^2\right)$$

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$$C_k(\chi) \equiv \frac{\mathrm{d}S_k}{\mathrm{d}\chi} = \begin{cases} \cos(\sqrt{k}\chi) & \text{if } k > 0 \\ 1 & \text{if } k = 0 \\ \cosh(\sqrt{|k|}\chi) & \text{if } k < 0. \end{cases}$$

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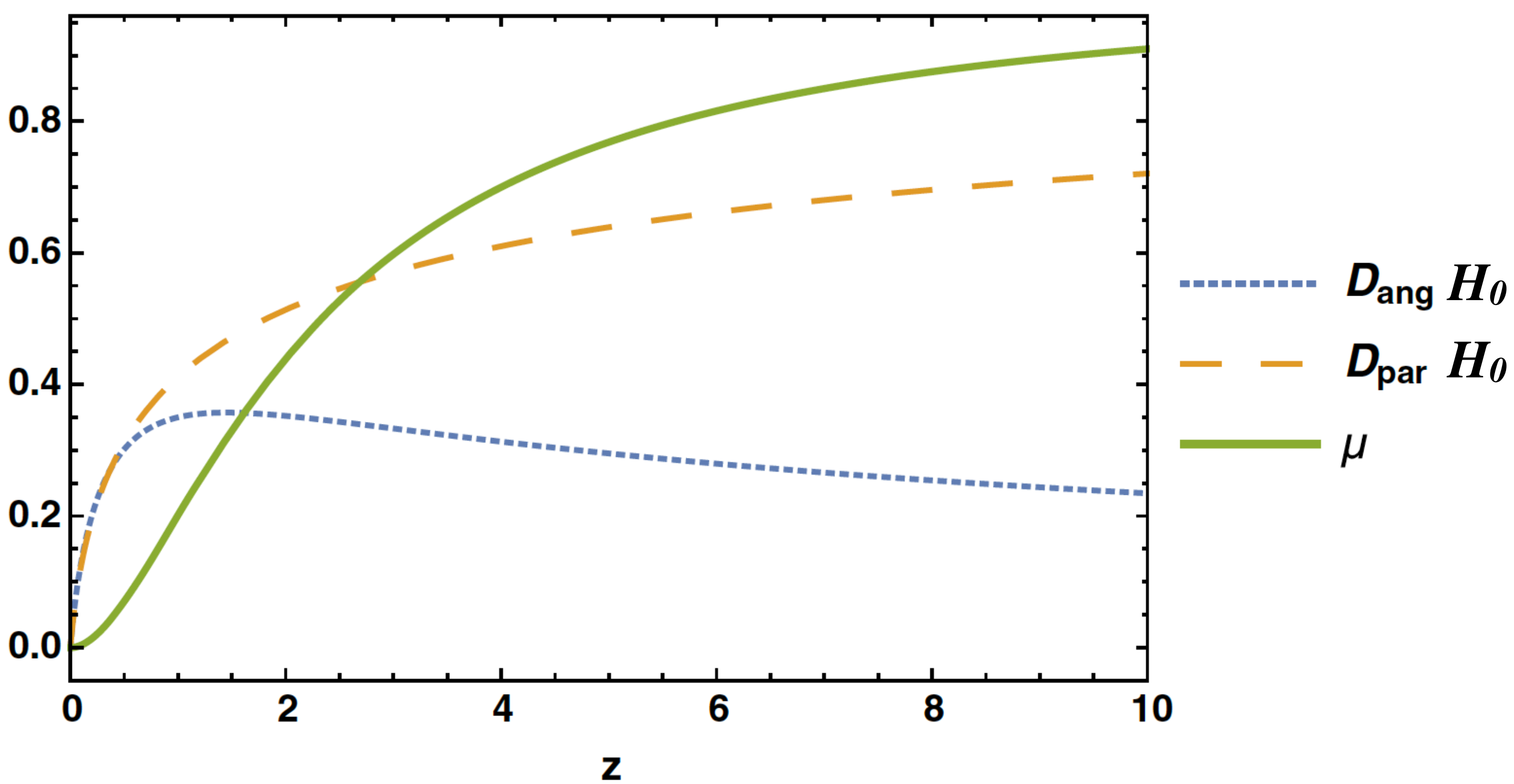

ΛCDM, Planck values

$H_0 = 67.4$

$\Omega_{m0} = 0.266018$

$\Omega_{k0} = 0$

$\Omega_{\Lambda 0} = 0.732982$

μ in cosmology

Low redshift expansion

$$\mu(z) = \frac{3}{2}\Omega_{m0}\, z^2 + \left(-\frac{1}{2}\Omega_{m0} - \frac{3}{2}\Omega_{m0}\,\Omega_{k0} - \frac{9}{4}\Omega_{m0}^2\right) z^3 + O(z^4)$$

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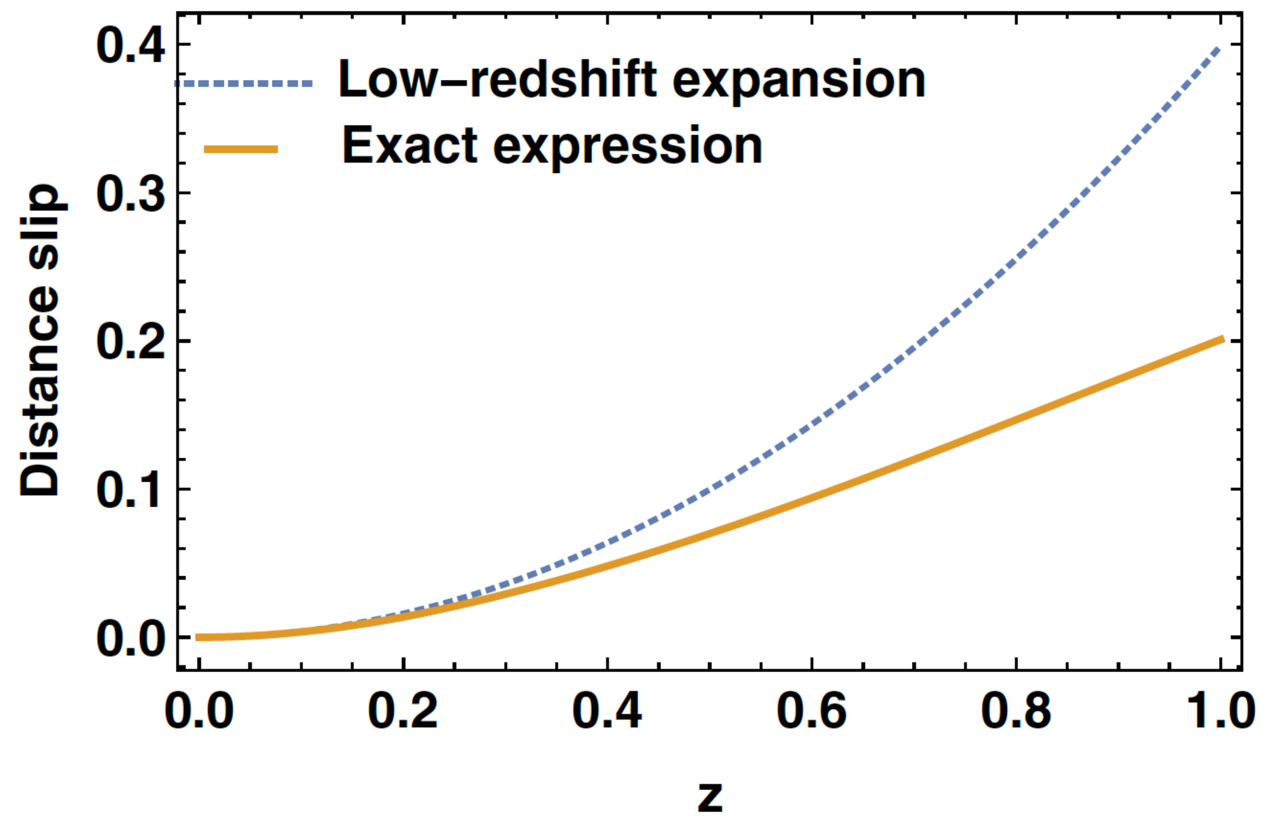

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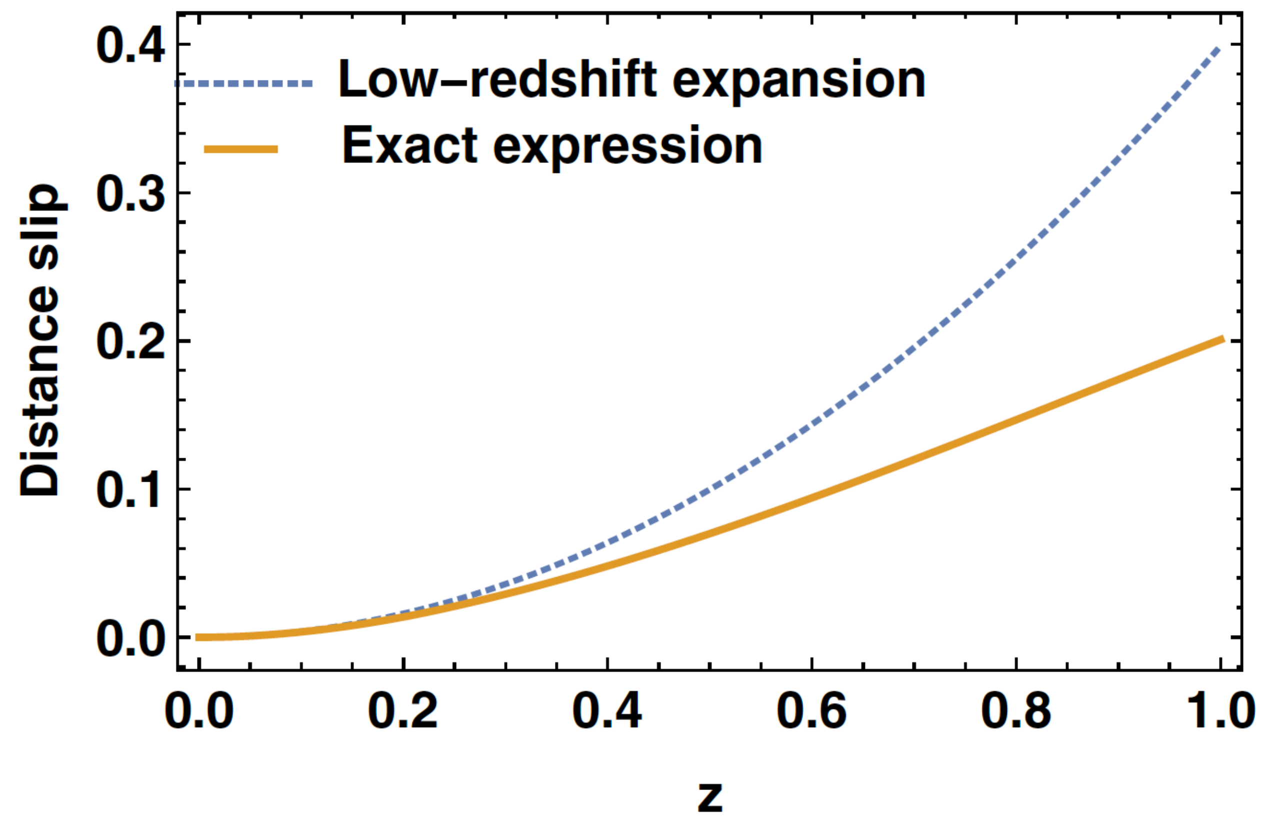
dimensionless μ vs dimensionless $z \Longrightarrow$ no H_0



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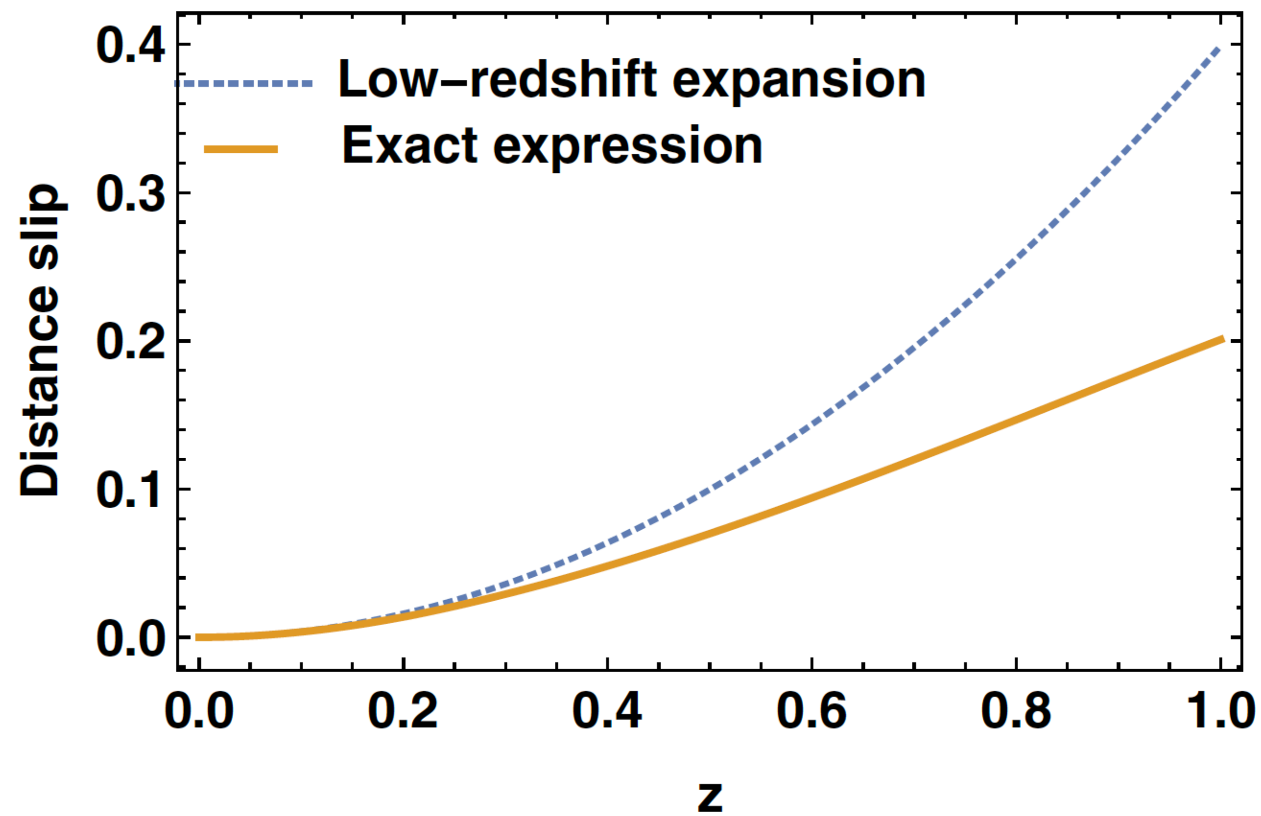
leading order term gives a measurement of Ω_{m0}



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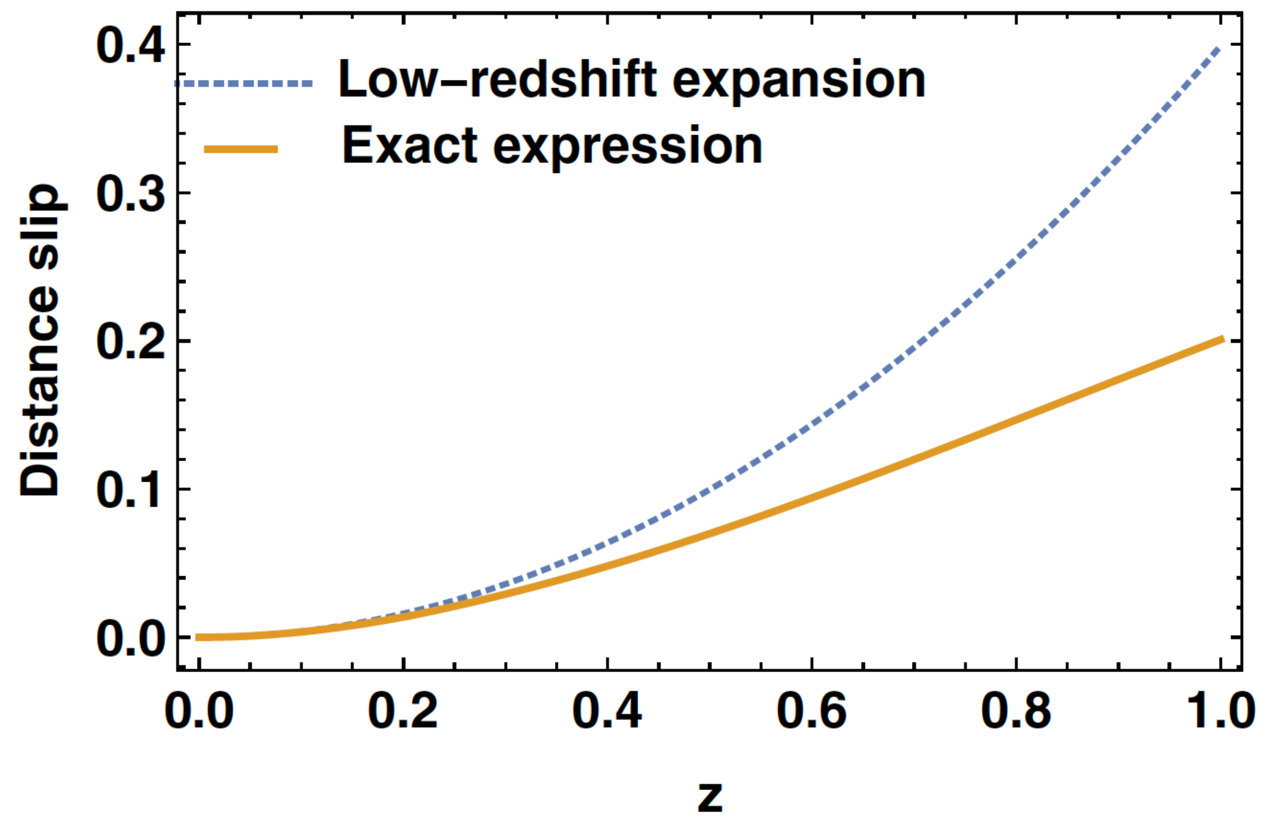
independent from any other measurements



μ in cosmology

$\boldsymbol{\mu}$ vs D_{ang} diagram

$$\mu(D_{ang}) = \frac{3}{2}\Omega_{m0}\, H_0^2\, D_{ang}^2 + \frac{5}{2}\Omega_{m0}\, H_0^3\, D_{ang}^3 + O(D_{ang}^4)$$

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bypassing z as observable

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potentially very robust measurement, independent from others

Distance inequality

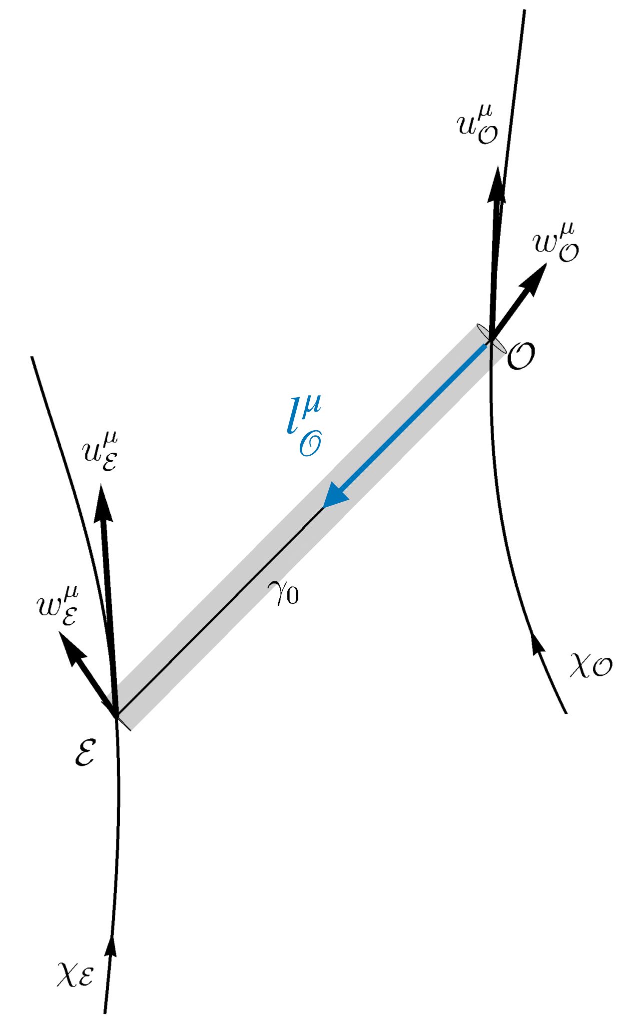
MK, J. Serbenta 2022:

Theorem:

- Null energy condition (NEC) holds, i.e. $R_{\mu\nu}\, l^\mu\, l^\nu \geq 0$
- No optical „singular points" between $\mathcal{O}$ and $\mathcal{E}$ (such as focal points)

then

- $D_{par} \geq D_{ang} \quad (\, \mu \geq 0\,)$
- moreover, $D_{par} = D_{ang}$ $(\, \mu = 0\,)$ if and only if ${R^A}_{llB} = 0$ along γ_0



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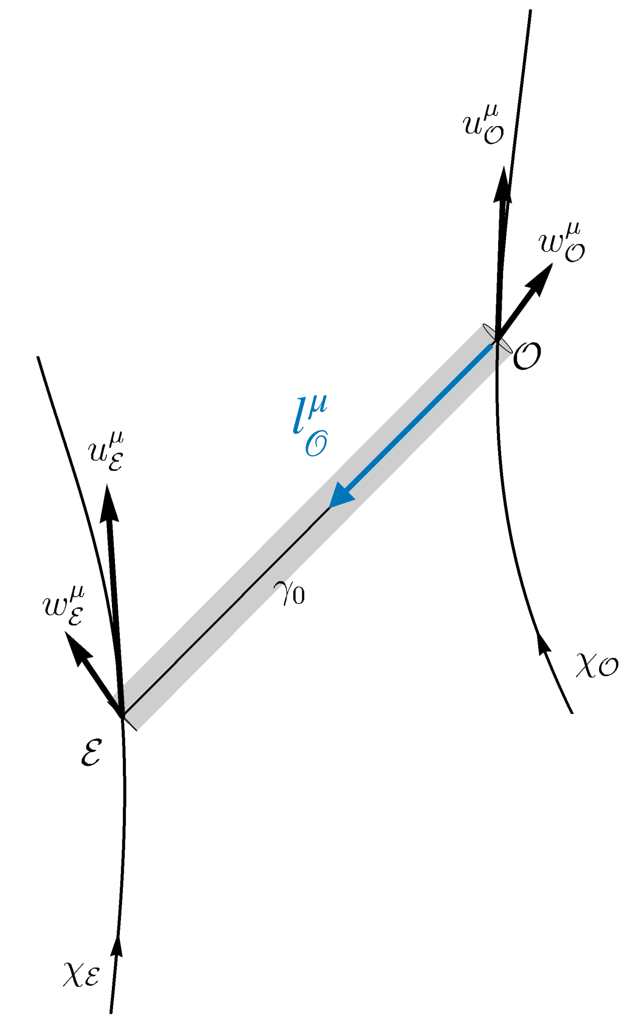


Rephrasing:

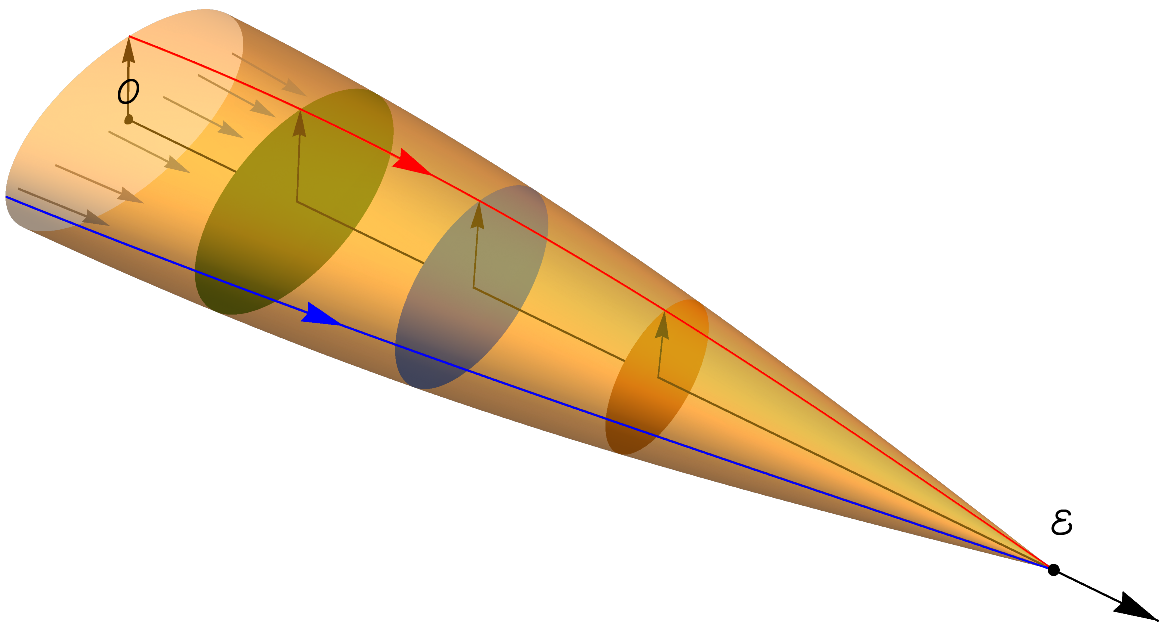
If the NEC holds then

both focusing of light by matter and tidal distortion of light rays makes D_{par} larger than D_{ang} at least up to the first focal point

Distance inequality

Sketch of the proof

Geometry of the null congruence parallel at $\mathscr{O}$



$$\mu(\lambda) = 1 - \frac{\mathscr{A}(\lambda)}{\mathscr{A}(\mathscr{O})}$$

$$\mathscr{A}(\lambda) = \mathscr{A}(\mathscr{O})\,\exp\left(\int_{\mathscr{O}}^{\lambda} \theta(\lambda')\,d\lambda'\right)$$

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\,\theta^2 - \sigma^2 - R_{\mu\nu}\,l^\mu\,l^\nu$$

$$\theta(\mathscr{O}) = 0$$

Distance inequality

Possible applications

- S. Räsänen 2014 - consistency test of FLRW metric using D_{ang}/D_{lum}, z and D_{par}

- Distance inequality $\Longrightarrow$ sign of difference between D_{ang} and D_{par} carries information about the NEC. Observational test of NEC (+ GR + light propagation)

Distance inequality

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If we observe $D_{par} < D_{ang}$ then either the NEC does not hold, or modified GR or light propagation

Violation of NEC: $R_{\mu\nu}\, l^\mu\, l^\nu < 0 \Longleftrightarrow T_{\mu\nu}\, l^\mu\, l^\nu < 0$

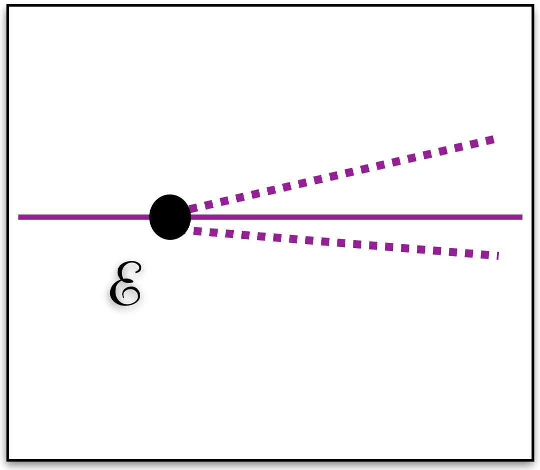
equation of state $p = w\,\rho \qquad w < -1$

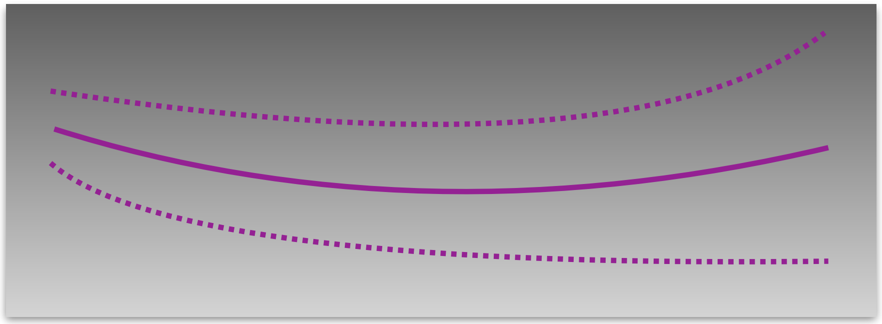
Summary and take-home message

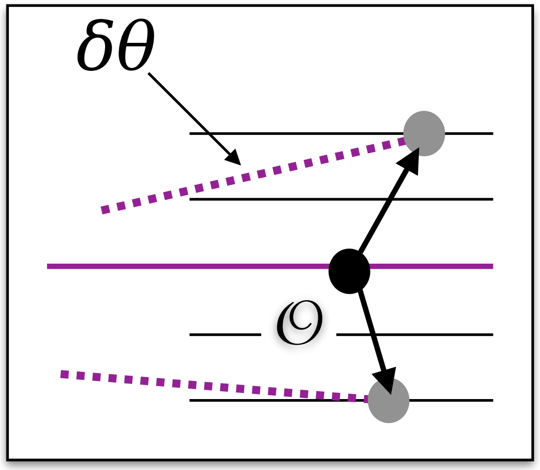
- By comparing D_{ang} and D_{par} measured to a single source we get the distance slip μ - new (potential) observable
- Interesting properties: frame-invariance, measures curvature and matter along the line of sight
- μ very small on short distances, too difficult to measure nowadays, but…
- It can provide independent matter density measurements
- $\mu \geq 0$ if the null energy condition holds

Thank you!

Parallax







$$\mathscr{D}^A{}_B\,\Delta l^B = -\,\delta\hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B\,\delta x^B_{\mathcal{O}}$$

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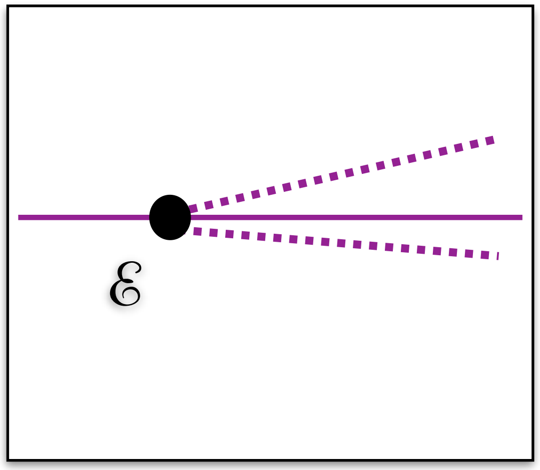

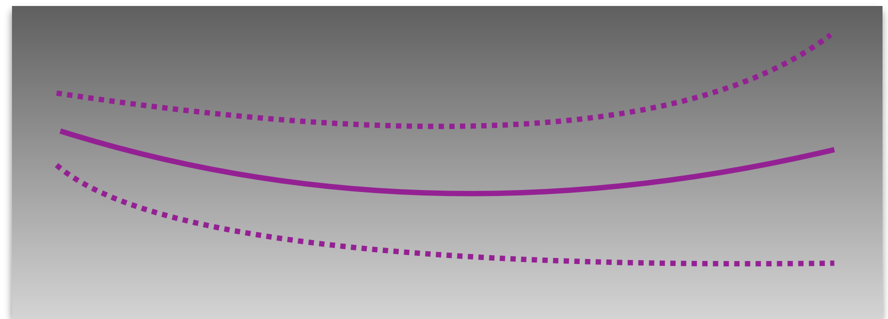

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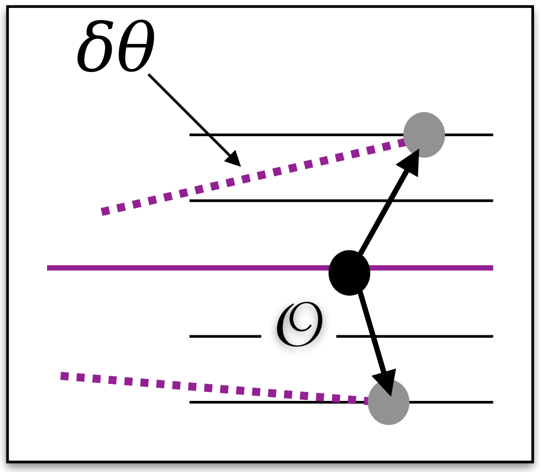

$$\delta\theta^A \approx \delta r^A = -\,\frac{1}{u^\sigma_{\mathscr{O}}\, l_{\mathscr{O}\,\sigma}}\,\mathscr{D}^{-1A}{}_C \left(\delta^C{}_B + m_\perp{}^C{}_B\right)\delta x^B_{\mathscr{O}}$$

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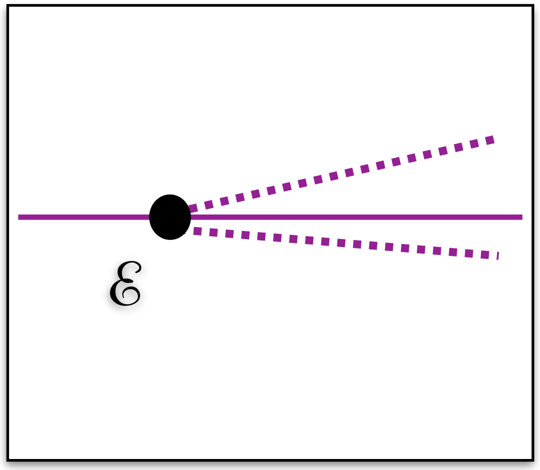

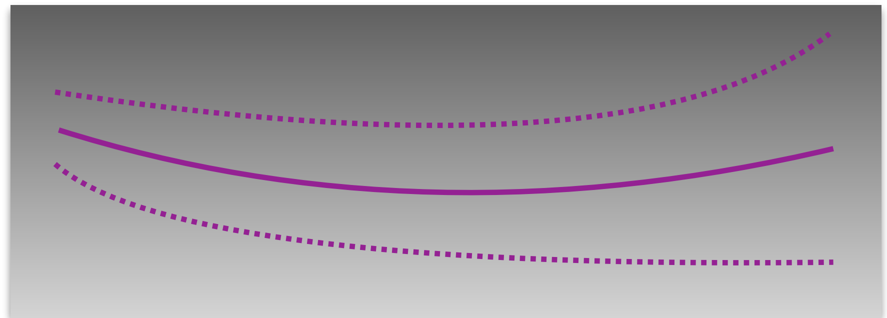

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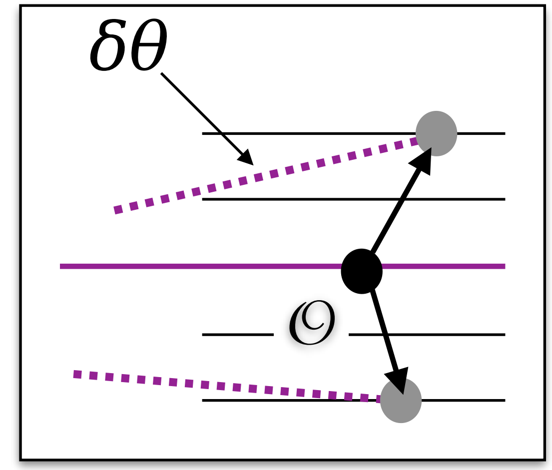

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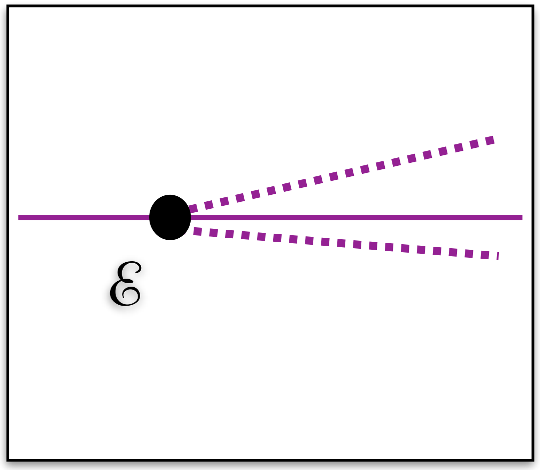
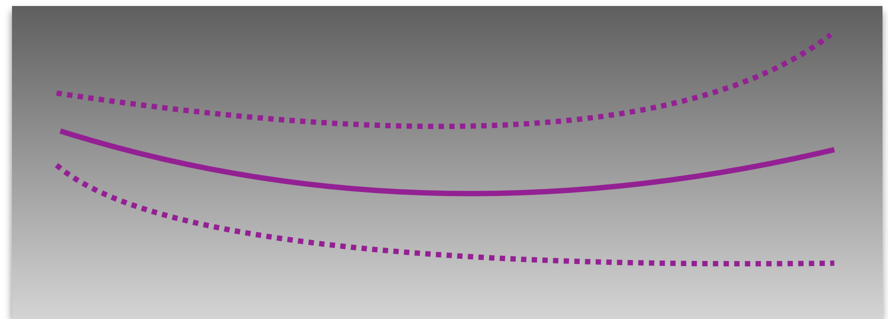
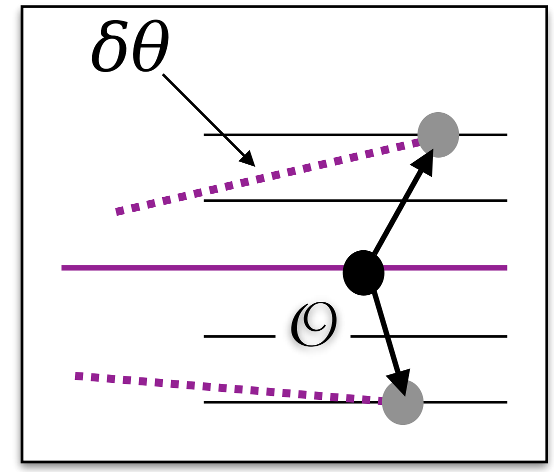
parallax matrix $\Pi^A{}_B$

Parallax







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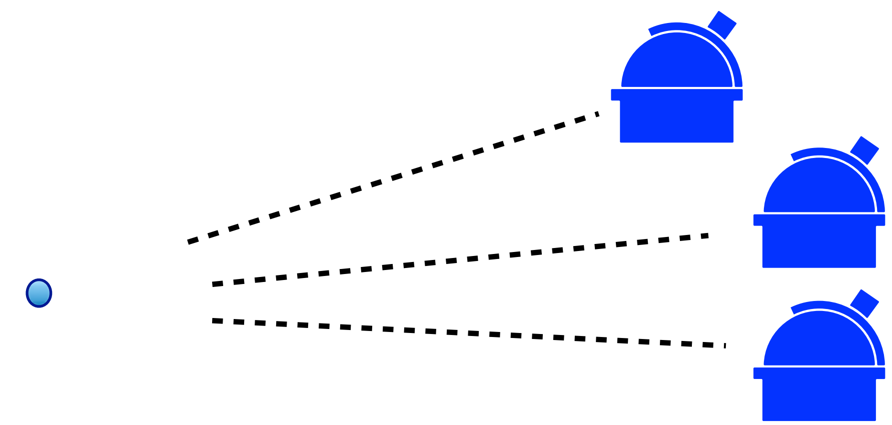

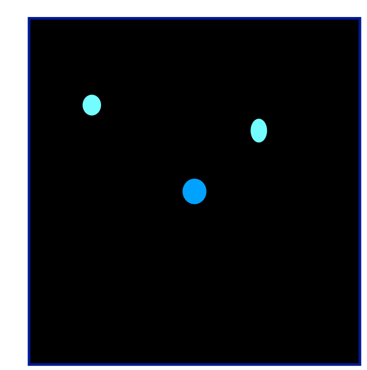

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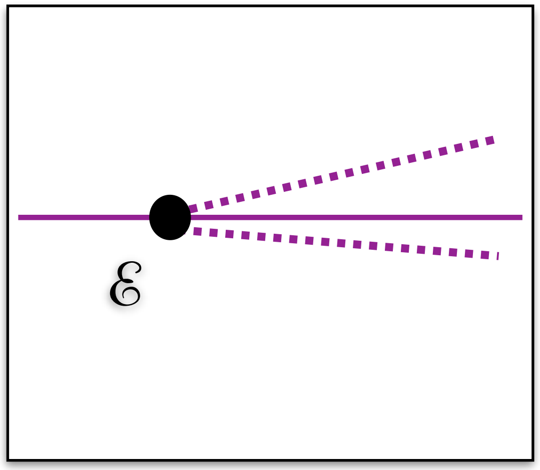

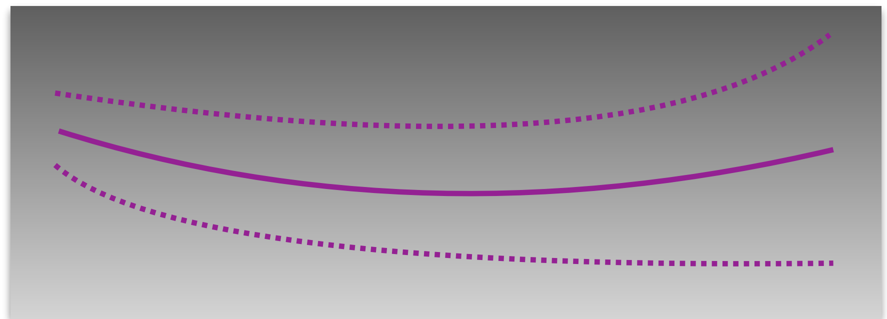

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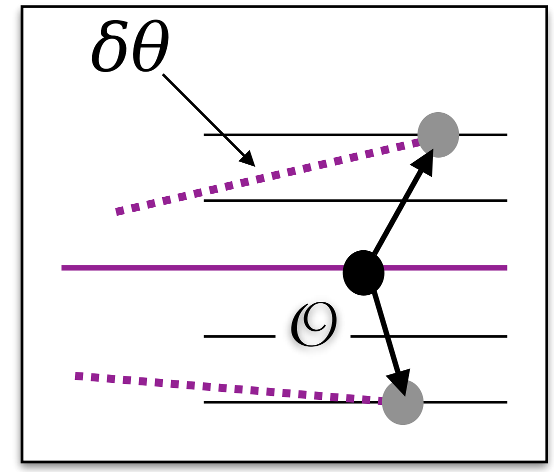

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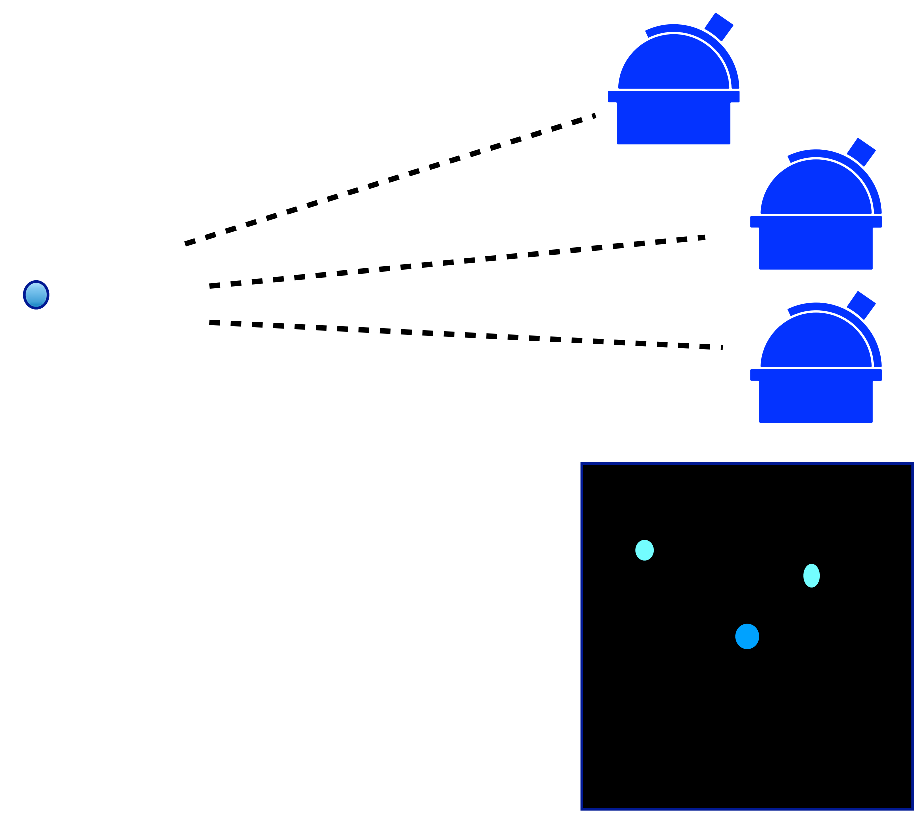

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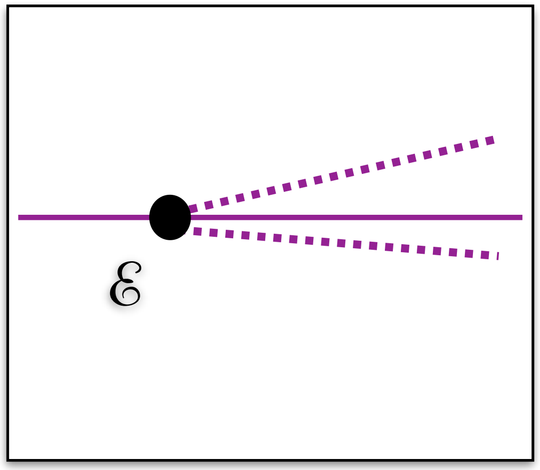

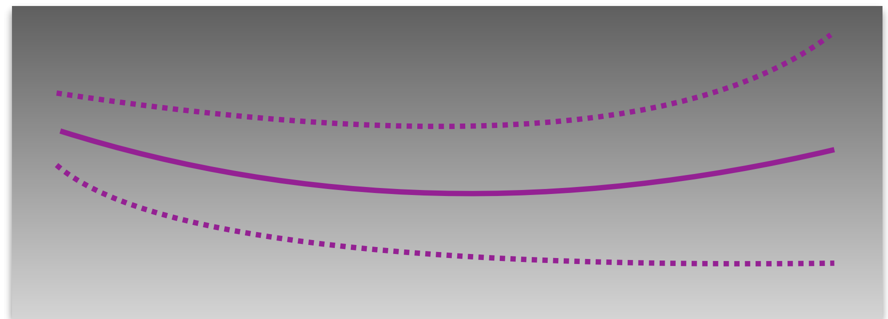

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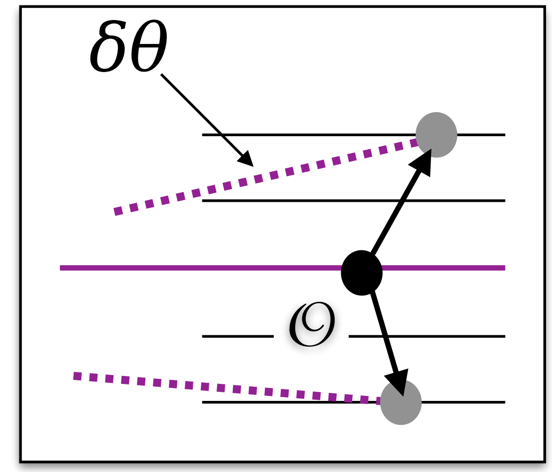

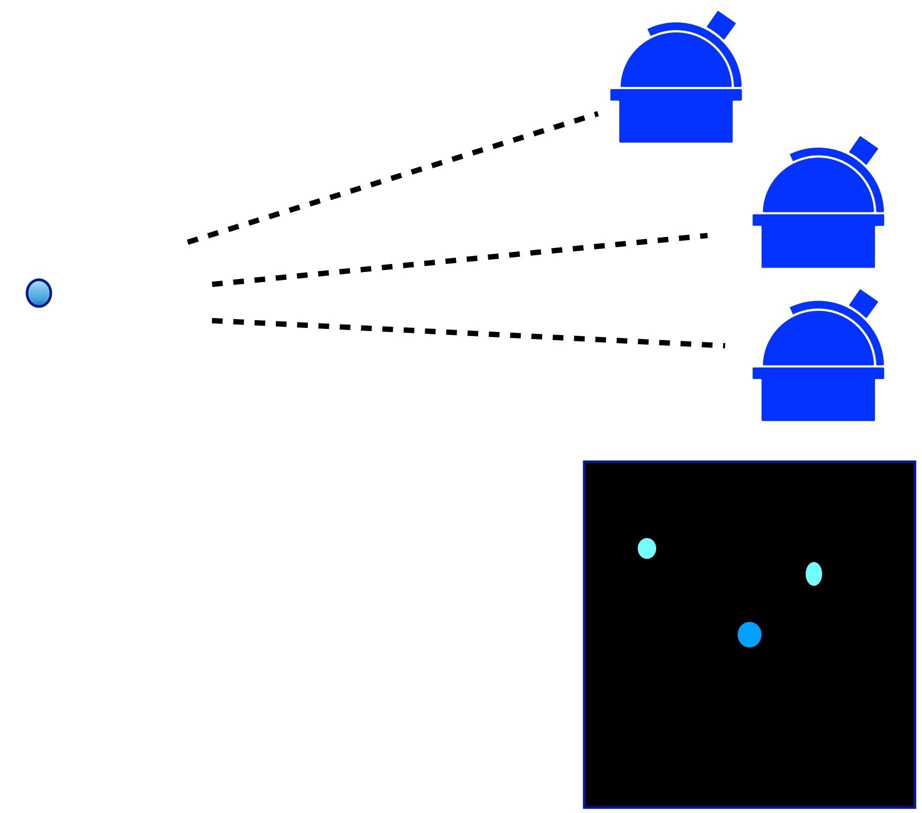

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parallax distance

$$D_{par} = u^\sigma_{\mathscr{O}}\,l_{\mathscr{O}\,\sigma}\,\left|\det \mathscr{D}^A{}_B\right|^{1/2}\,\left|\det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

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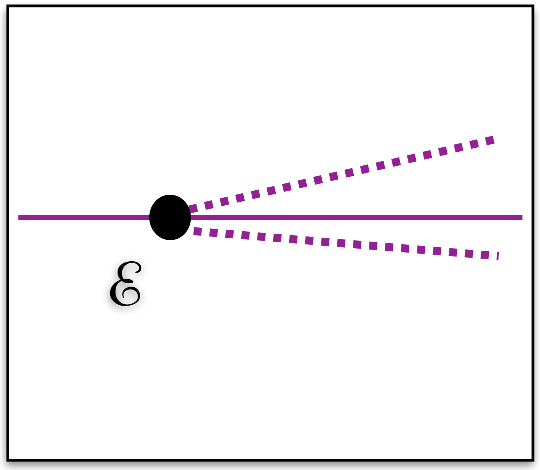
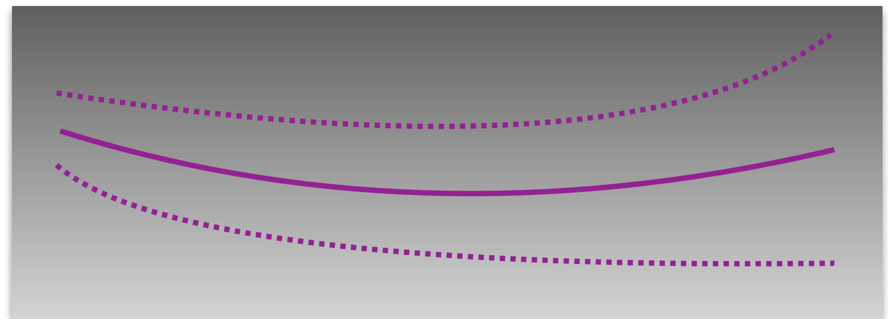
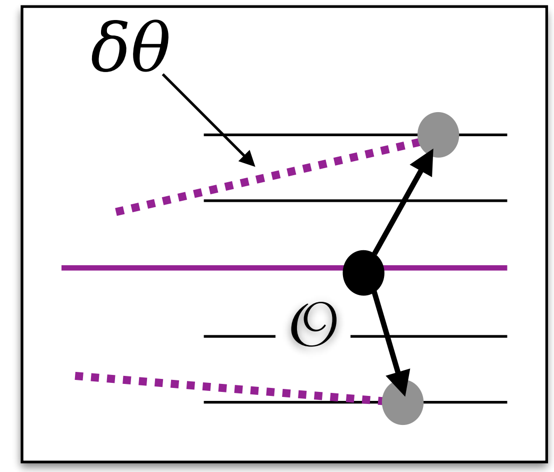
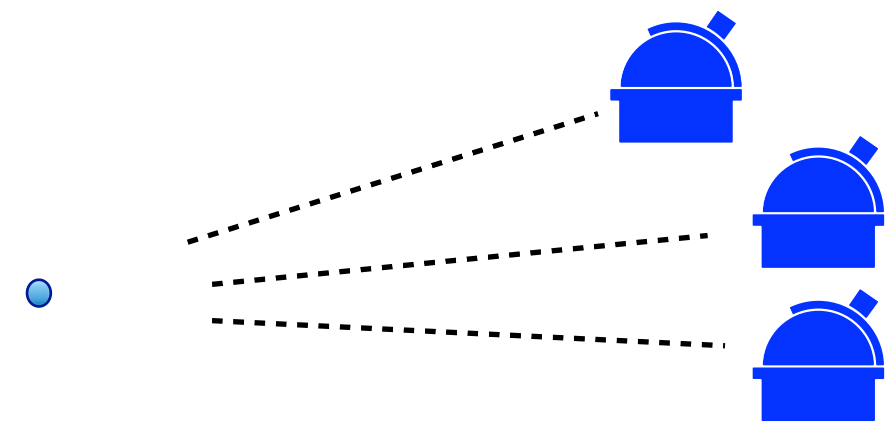






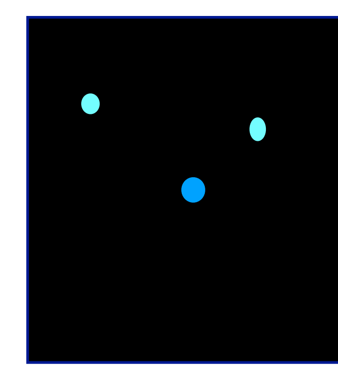
$$\mathscr{D}^A{}_B\,\Delta l^B = -\,\delta\hat{x}^A_{\mathcal{O}} - m_\perp{}^A{}_B\,\delta x^B_{\mathcal{O}}$$

$$\delta\theta^A \approx \delta r^A = -\frac{1}{u^\sigma_{\mathcal{O}}\,l_{\mathcal{O}\,\sigma}}\,\mathscr{D}^{-1\,A}{}_C\left(\delta^C{}_B + m_\perp{}^C{}_B\right)\delta x^B_{\mathcal{O}}$$



parallax matrix $\Pi^A{}_B$

$$\Pi_{AB} = \Pi_{BA}$$



parallax distance

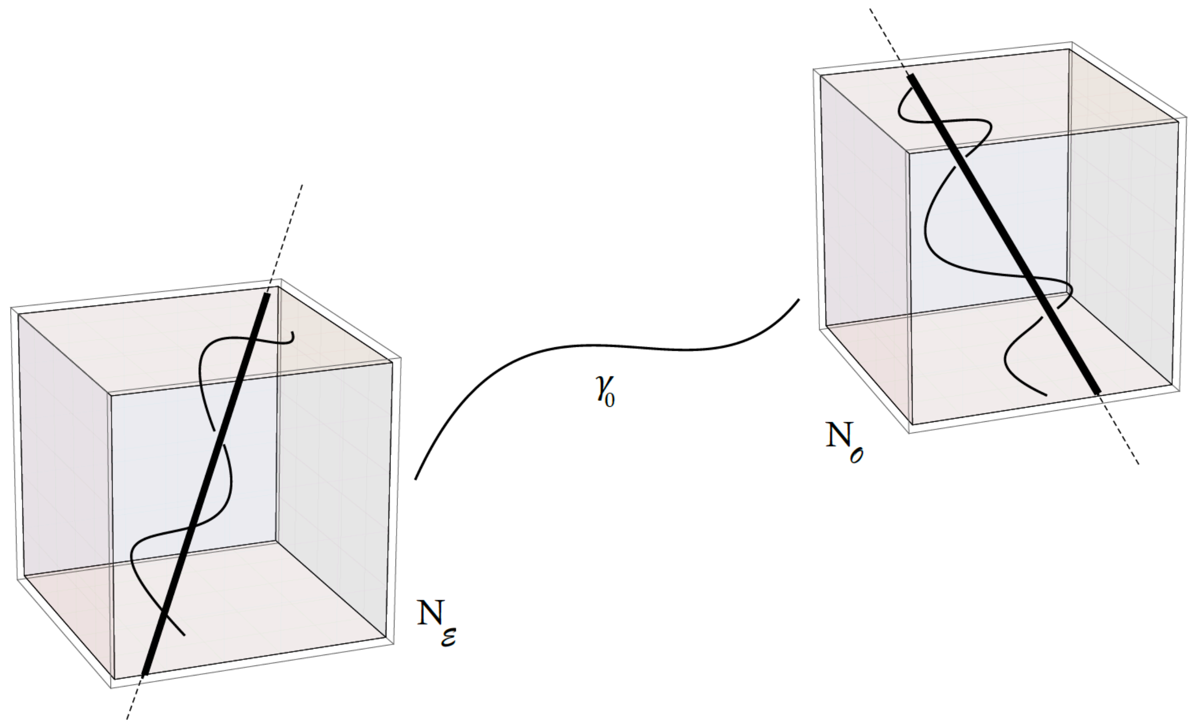
$$D_{par} = u^\sigma_{\mathcal{O}}\,l_{\mathcal{O}\,\sigma}\,\left|\det \mathscr{D}^A{}_B\right|^{1/2}\,\left|\det\left(\delta^A{}_B + m_\perp{}^A{}_B\right)\right|^{-1/2}$$

$$\Pi^A{}_B \equiv \Pi^A{}_B\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

$$D_{par} \equiv D_{par}\left(R^\mu{}_{\nu\alpha\beta}\Big|_{\gamma_0}, u^\mu_{\mathcal{O}}\right)$$

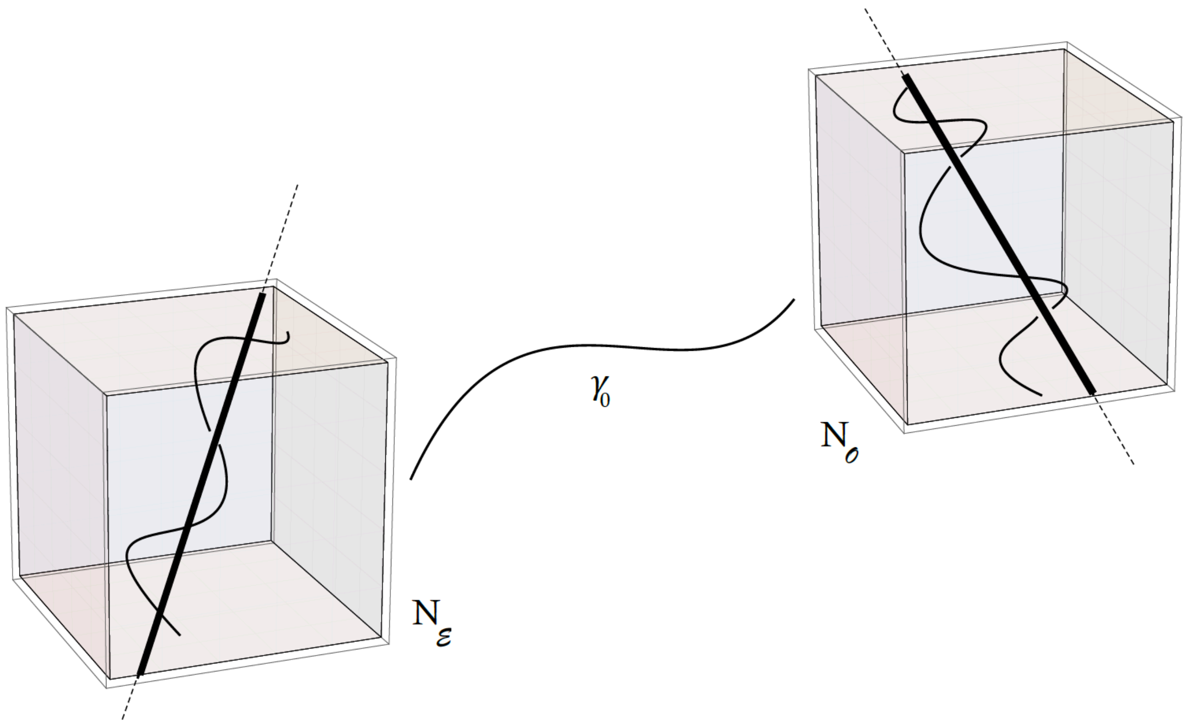
Parallax in a general situation

Parallax in a general situation



Both observer and emitter in bound systems

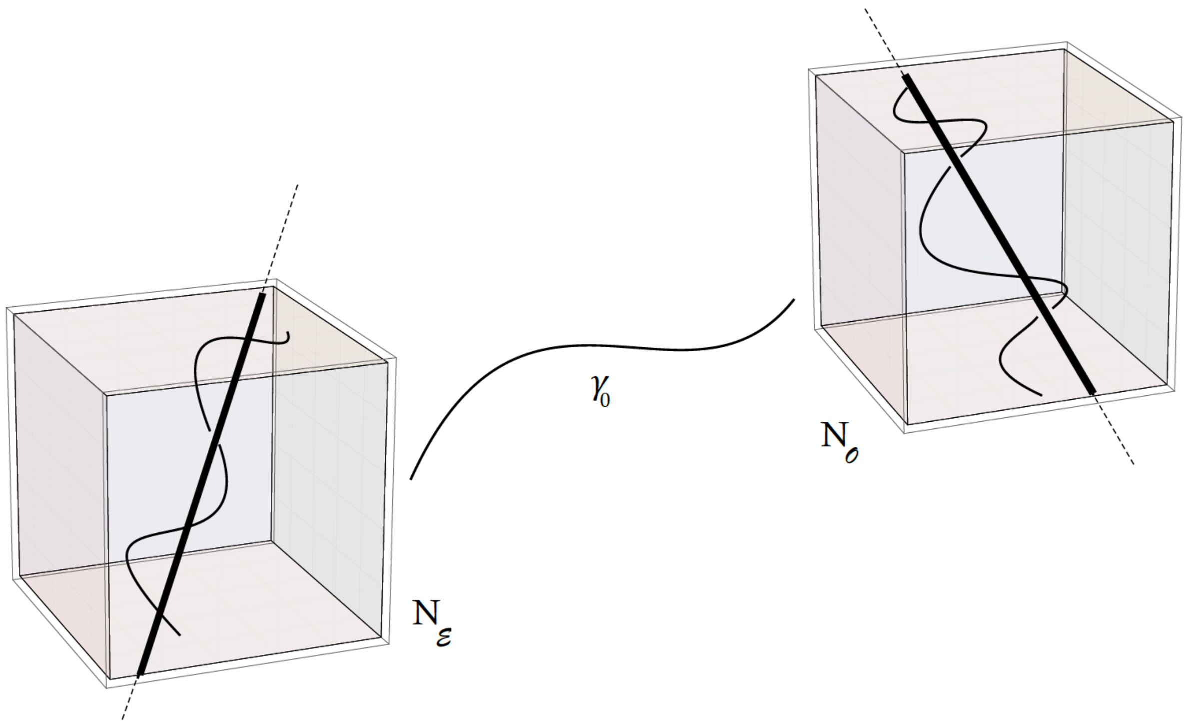
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Parallax in a general situation

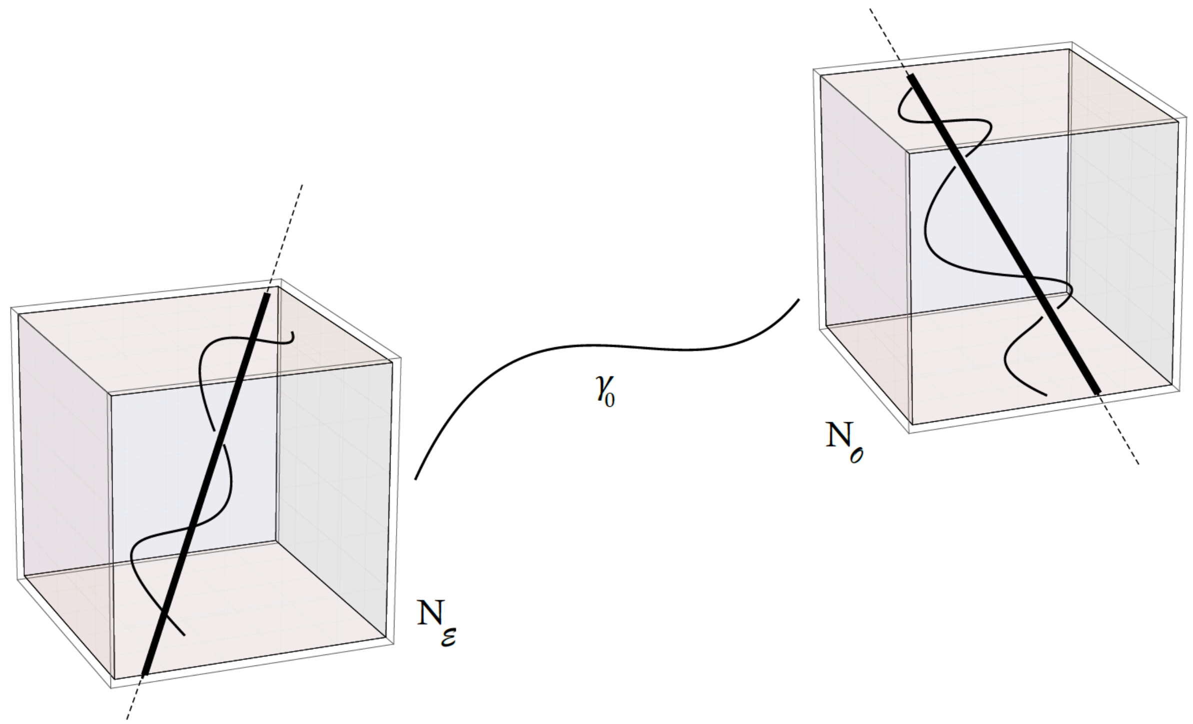


Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the local aberration and light bending effects

Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

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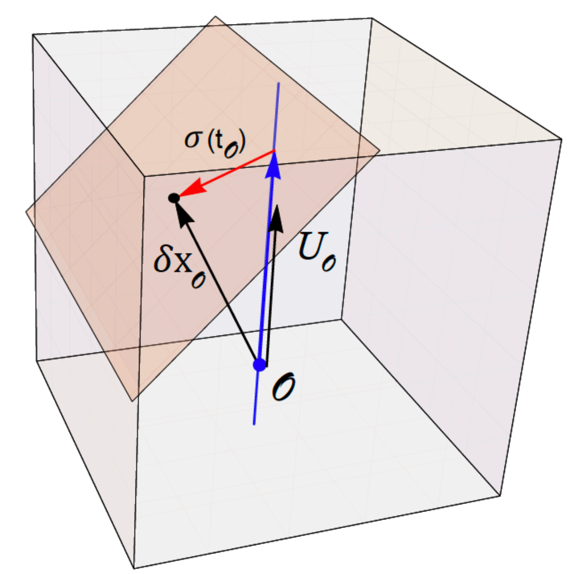
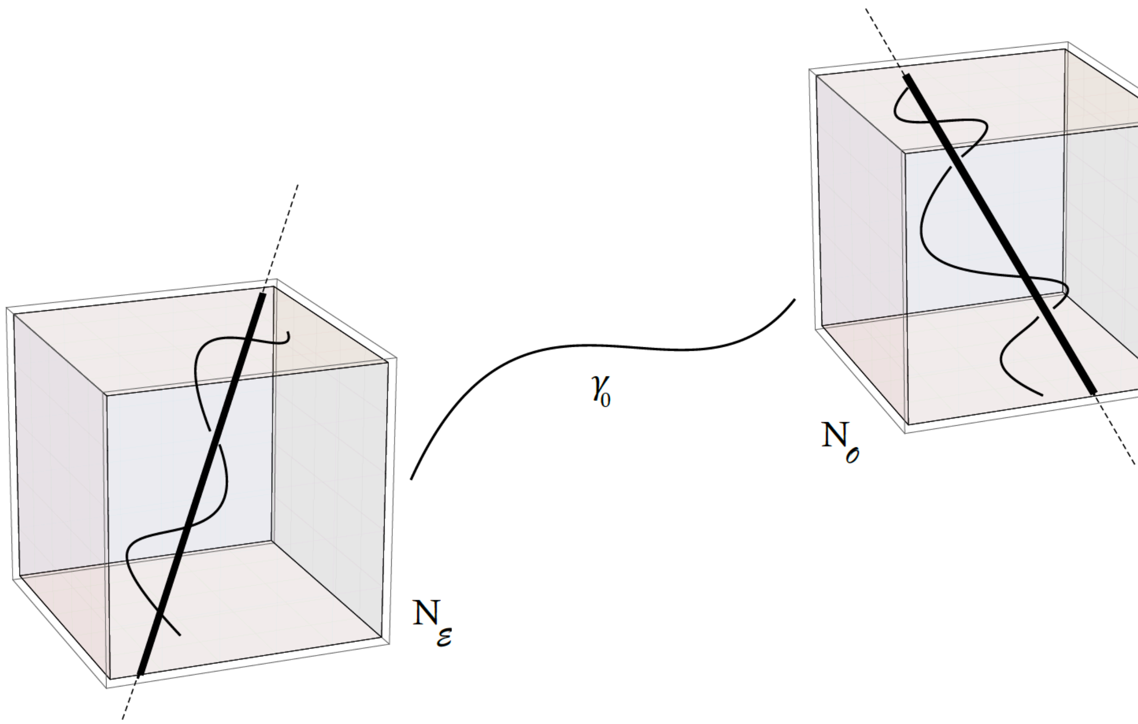


$$\delta x^\mu_{\mathcal{O}} = U^\mu_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^\mu(t_{\mathcal{O}}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

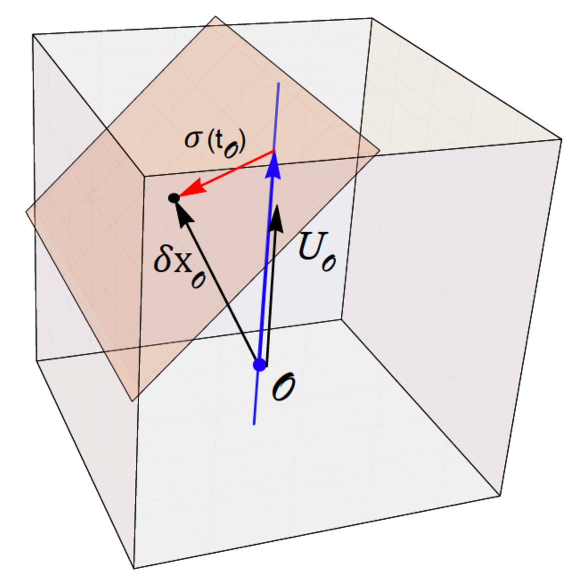
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

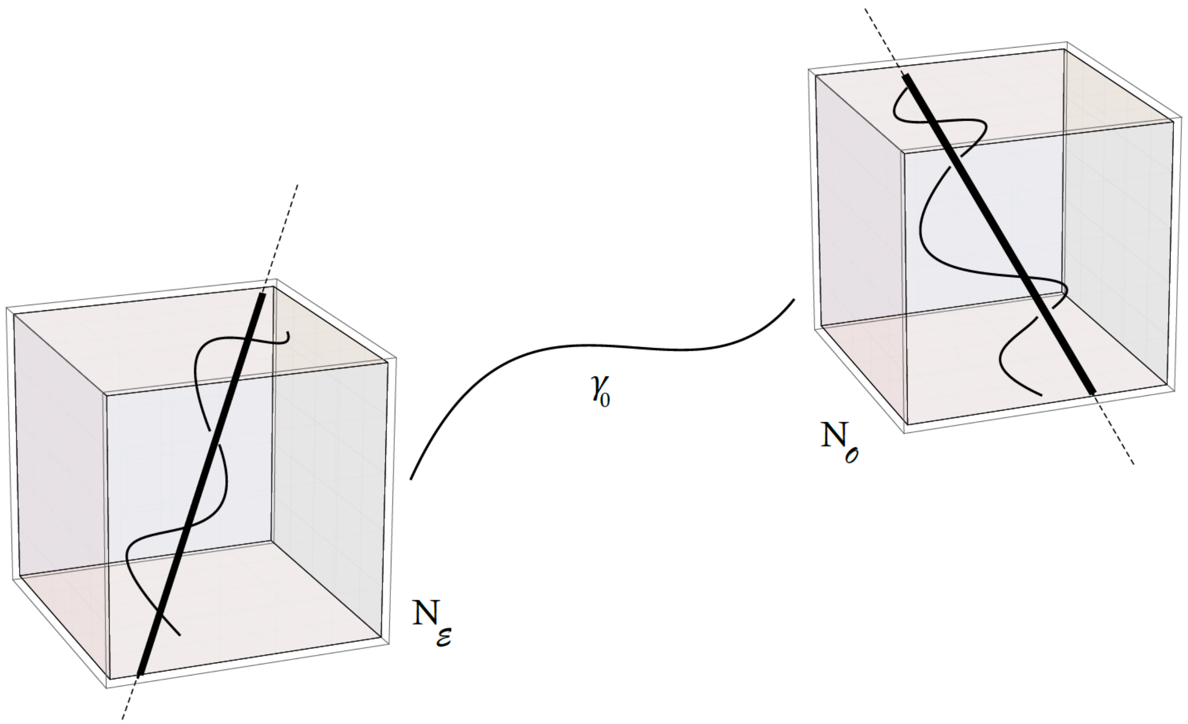
Question: parallax without the local aberration and light bending effects



$$\delta x^{\mu}_{\mathcal{O}} = U^{\mu}_{\mathcal{O}}\, t_{\mathcal{O}} + \sigma^{\mu}(t_{\mathcal{O}}) \qquad \sigma^{\mu}\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^{\mu}_{\mathcal{E}} = U^{\mu}_{\mathcal{E}}\, t_{\mathcal{E}} + \rho^{\mu}(t_{\mathcal{E}}) \qquad \rho^{\mu}\, l_{\mathcal{E}\,\mu} = 0$$

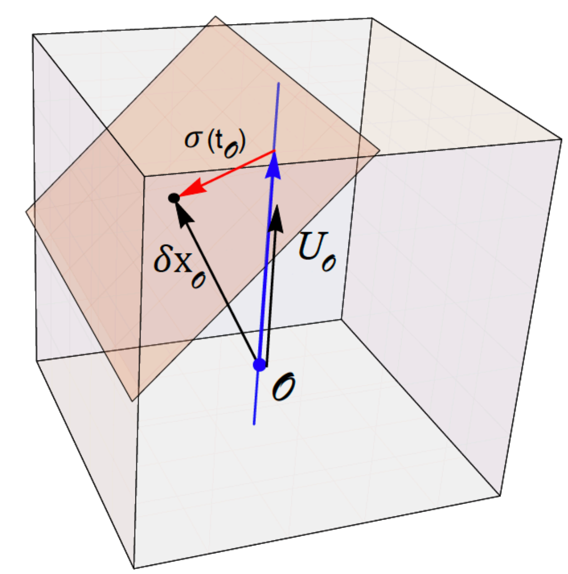
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the local aberration and light bending effects



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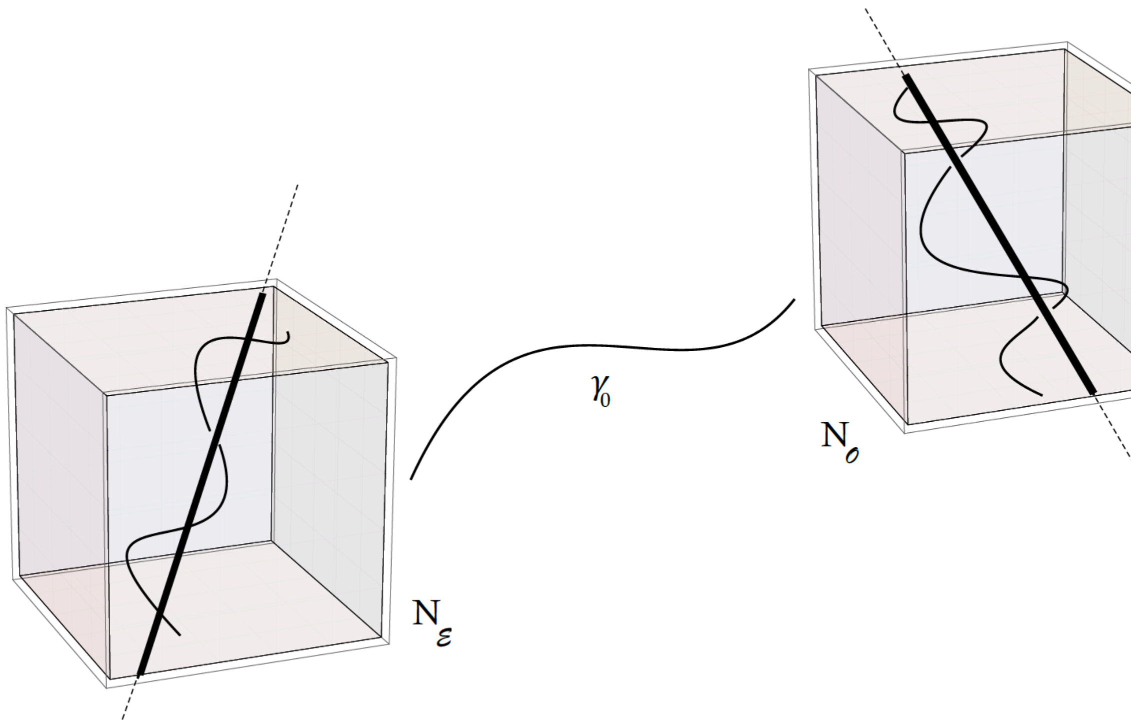
$$\delta x^\mu_\mathcal{E} = U^\mu_\mathcal{E}\, t_\mathcal{E} + \rho^\mu(t_\mathcal{E}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_\mathcal{O} r^A\, t_\mathcal{O} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_\mathcal{O}\right) - \Pi^A{}_B\, \sigma^B(t_\mathcal{O})$$

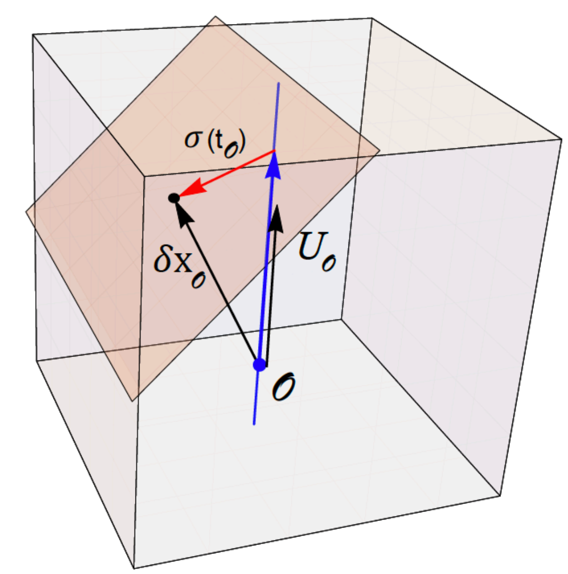
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the local aberration and light bending effects



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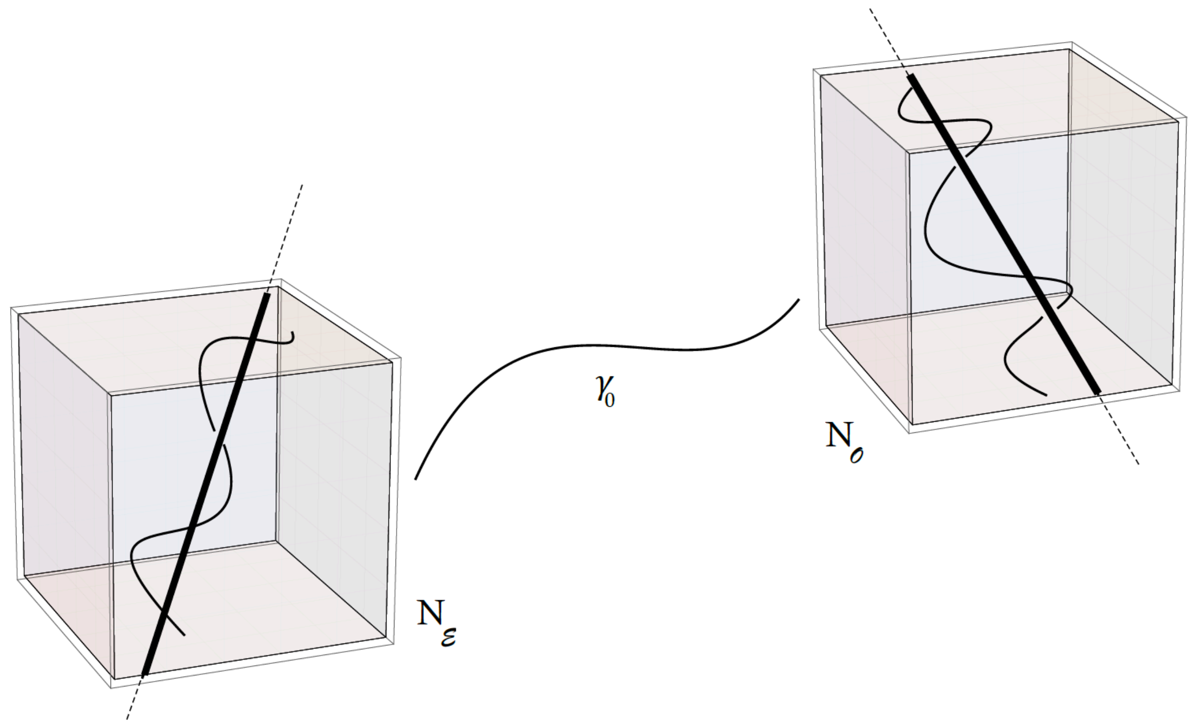
$$\delta\theta^{A} = \delta_{\mathcal{O}} r^{A}\, t_{\mathcal{O}} + M^{A}{}_{B}\, \rho^{B}\left((1+z)^{-1}\, t_{\mathcal{O}}\right) - \Pi^{A}{}_{B}\, \sigma^{B}(t_{\mathcal{O}})$$

barycenter drift (linear)

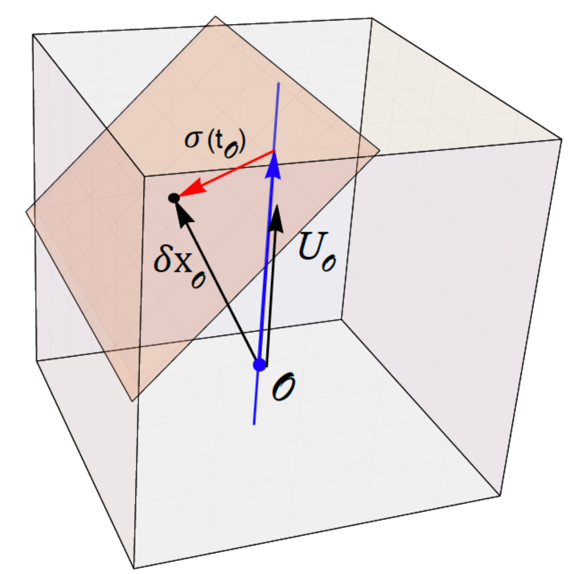
Parallax in a general situation



Both observer and emitter in bound systems

Barycenters in free fall

Question: parallax without the local aberration and light bending effects



$$\delta x^\mu_\mathcal{O} = U^\mu_\mathcal{O}\, t_\mathcal{O} + \sigma^\mu(t_\mathcal{O}) \qquad \sigma^\mu\, l_{\mathcal{O}\,\mu} = 0$$

$$\delta x^\mu_\mathcal{E} = U^\mu_\mathcal{E}\, t_\mathcal{E} + \rho^\mu(t_\mathcal{E}) \qquad \rho^\mu\, l_{\mathcal{E}\,\mu} = 0$$

$$\delta\theta^A = \delta_\mathcal{O} r^A\, t_\mathcal{O} + M^A{}_B\, \rho^B\left((1+z)^{-1}\, t_\mathcal{O}\right) - \Pi^A{}_B\, \sigma^B(t_\mathcal{O})$$

barycenter drift (linear)

parallax (periodic)