Magnetic fields in the universe: A new law with applications in cosmology and gravitational collapse

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Motivation

Observational-Theoretical data

- Large-scale magnetic fields form an existing component of the universe's energy content, which potentially contributes to universal dynamics and structure formation (via its effects on density inhomogeneities).
- Although magnetic fields are widely present in the universe (interstellar medium, our galaxy, galaxy clusters, intergalactic space), their origin, evolution and role have not been adequately explained.
- The unique vector nature of magnetic fields allows for their double coupling with spacetime curvature, via not only Einstein's equations but the Ricci identities as well. This property points them out as the sole known energy source with vector beingness.
- In analogy with a spring under pressure, magnetic fieldlines tend to resist their gravitational deformation by developing curvature related (elastic) tension stresses. This tendency presents remarkable implications on the problem of magnetised gravitational collapse.

Motivation

Theoretical models (shortcomings)

- Lack of an exact model of magnetic field's evolution. Conventionally, cosmic magnetic fields are treated as linear perturbations on a FRW background and their evolution is therefore derived to be $B \propto a^{-2}$ (flux conservation).

- Most studies of magnetised stellar/protogalactic collapse are Newtonian and as such, they do not take into account the special coupling of magnetic fields with spacetime curvature.

Method & Aim

- Employ General Relativity and make use of a covariant approach.
- Adopt the MHD approximation (magnetic field frozen into the fluid).
- Consider two basic fields, a timelike 4-velocity u^a (associated with a fundamental observer) and a spacelike vector n^a along the magnetic forcelines.
- Split the kinematic/dynamic quantities (vectors and tensors) into their components with respect to u^a and n^a.

Method & Aim

- Decompose and solve Faraday's equation to derive the exact evolution formula for the magnetic field (allowing for anisotropy).
- Study the magnetised gravitational collapse of an ideal fluid and establish a non-collapse criterion.
- Present the linearly perturbed Bianchi I geometry as a simple example model of magnetised gravitational collapse.

Kinematic variables and (1+3) spacetime splitting

4-D Gradient of the 4-velocity field

$$\nabla_b u_a = \frac{1}{3}\Theta h_{ab} + \sigma_{ab} + \omega_{ab} - \dot{u}_a u_b\,,$$

where
$\Theta \equiv \mathrm{D}^a u_a$: is the volume scalar ($\Theta > 0$ means expansion whilst $\Theta < 0$ contraction),
$\sigma_{ab} \equiv \mathrm{D}_{\langle b} u_{a\rangle}$: is the shear (shape distortions), $\omega_{ab} \equiv \mathrm{D}_{[b} u_{a]}$: is the vorticity (rotation) and $\dot{u}_a \equiv u^b \nabla_b u_a$: is the 4-acceleration (non-gravitational forces).

Derivative and projection operators

Temporal and spatial derivative:

$$\dot{} = u^a \nabla_a \qquad \textit{and} \qquad \mathrm{D}_a = h_a{}^b \nabla_b \qquad \text{respectively}\,.$$

Projection operators:

$$h_{ab} = g_{ab} + u_a u_b \qquad \textit{and} \qquad \tilde{h}_{ab} = h_{ab} - n_a n_b\,,$$

projecting in the observer's local 3-D space ($h_{ab} u^b = 0$) and on the 2-D surface normal to the magnetic forcelines ($\tilde{h}_{ab} n^b = 0$) respectively.

Kinematic variables and (1+2) space splitting

Decomposition of vectors and 2nd rank tensors

Our 3-D vectors split parallel and orthogonal to $n^a \parallel B^a$ as:

$$\dot{u}^a = \mathcal{A}n^a + \mathcal{A}^a\,, \qquad B^a = \mathcal{B}n^a \qquad \textit{and} \qquad \omega^a = \Omega n^a + \Omega^a\,.$$

Similarly, 2nd rank symmetric and trace-free tensors split according to:

$$\sigma_{ab} = \Sigma(n_a n_b - \frac{1}{2}\tilde{h}_{ab}) + 2\Sigma_{(a}n_{b)} + \Sigma_{ab}\,,$$

where

$$\Sigma \equiv \sigma_{ab}n^a n^b = -\tilde{h}^{ab}\sigma_{ab}\,, \qquad \Sigma_a \equiv \tilde{h}_a{}^b n^c \sigma_{bc}\,, \qquad \Sigma_{ab} \equiv (\tilde{h}_{(a}{}^c \tilde{h}_{b)}{}^d - (1/2)\tilde{h}_{ab}\tilde{h}^{cd})\sigma_{cd}\,.$$

Auxiliary relations

Observing that $(n^b \mathrm{D_b n_a})\mathrm{u^a} = 0 = (\mathrm{D_a u_b})\mathrm{n^b}$ one arrives at the crucial relations:

$$\Sigma = -\frac{1}{3}\Theta \qquad \textit{and} \qquad \Sigma_a = -\epsilon_{ab}\Omega^b\,.$$

Faraday's equation and its solution

MHD limit and decomposition

At the MHD limit (practically zero electric component) Faraday's equation reads:

$$\dot{B}_{\langle a\rangle} = h_a{}^b\dot{B}_b = (-\frac{2}{3}\Theta h_{ab} + \sigma_{ab} + \epsilon_{abc}\omega^c)B^b\,.$$

Projecting the above along $n^a \parallel B^a$ ($B^a = \mathcal{B}n^a$), leads to:

$$\dot{\mathcal{B}} = -\Theta\mathcal{B}\,.$$

Solution-The magnetic field's law of variation

The above equation accepts the general solution:

$$\mathcal{B} \propto a^{-3}\,,$$

where a is the average scale factor ($\Theta \equiv 3\dot{a}/a$). It is worth noting that our solution:

- Is exact (not approximate), in contrast to the conventional $\mathcal{B} \propto a^{-2}$, and it takes into account spatial anisotropy (i.e. $\sigma_{ab} \neq 0$).
- Provides us with the keystone for studying magnetic fields in cosmological and astrophysical problems.

Magnetised gravitational collapse

The Raychaudhuri equation I

Monitors the average volume expansion/contraction of a self-gravitating fluid

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - R_{ab}u^a u^b - 2\left(\sigma^2 - \omega^2\right) + \mathrm{D}_a\dot{u}^a + \dot{u}_a\dot{u}^a\,,$$

where $R_{ab}u^a u^b = (1/2)(\rho + 3P + \mathcal{B}^2) > 0$. Negative terms favour contraction-collapse ($\Theta < 0$) whilst positive ones favour expansion ($\Theta > 0$). We put aside $\dot{u}_a\dot{u}^a > 0$, which always resists the collapse.

Using Euler's equation to calculate $\mathrm{D}_a\dot{u}^a$

The equation of the fluid's motion reads:

$$(\rho + P + B^2)\dot{u}_a = -\mathrm{D}_a P - \frac{1}{2}\mathrm{D}_a B^2 + B^b\mathrm{D}_b B_a + \dot{u}^b B_b B_a\,,$$

where the Lorentz-force splits into its pressure (blue) and tension (red) component. Assuming a nearly homogeneous fluid, i.e. $\mathrm{D}_a\rho \simeq 0 \simeq \mathrm{D}_a\mathcal{B}^2$ but $\mathrm{D}_a\mathrm{B}_b \neq 0$, and taking the divergence of Euler's equation we get:

$$\mathrm{D}^a\dot{u}_a = c_{\mathcal{A}}^2\mathcal{R}_{ab}n^a n^b + 2(\sigma_B^2 - \omega_B^2)\,,$$

where $c_{\mathcal{A}}^2\mathcal{R}_{ab}n^a n^b$ comes from the magneto-geometric coupling via the Ricci identities: $2\mathrm{D}_{[a}\mathrm{D}_{b]}B_c = -2\omega_{ab}\dot{B}_{\langle c\rangle} + \mathcal{R}_{dcba}B^d$.

Magnetised gravitational collapse

The Raychaudhuri equation II: Revealing the elastic magnetic (tension) stresses

$$\dot{\Theta} + \frac{1}{3}\Theta^2 = -(R_{ab}u^a u^b - c_{\mathcal{A}}^2 \mathcal{R}_{ab} n^a n^b) - 2(\sigma^2 - \sigma_B^2) + 2(\omega^2 - \omega_B^2)\,,$$

where

- $\sigma_B^2 = \mathrm{D}_{\langle b}B_{a\rangle}\mathrm{D}^{\langle b}B^{a\rangle}/2(\rho + P + B^2)$ and $\omega_B^2 = \mathrm{D}_{[b}B_{a]}\mathrm{D}^{[b}B^{a]}/2(\rho + P + B^2)$: magnetic (tension) stresses resisting to shear and rotational deformations of the fluid (note opposite signs). They are due to kinematic (Newtonian) effects.
- $c_{\mathcal{A}}^2 \mathcal{R}_{ab} n^a n^b$ (with $c_{\mathcal{A}}^2 = \mathcal{B}^2/(\rho + P + B^2)$ being the Alfvén speed): magneto-curvature (tension) stress resisting to 3-D gravitational distortions $\mathcal{R}_{ab} n^a n^b$ of the fluid along the magnetic forcelines. It has a purely relativistic nature-origin and it is triggered by gravity $R_{ab}u^a u^b > 0$.

A non-collapse criterion I

- i) Advanced stage of implosion$\rightarrow$ ii) strong-gravity environment$\rightarrow$ iii) counterbalance of the two relativistic terms$\rightarrow$ iv) implosion's outcome
- If at some time,

$$c_{\mathcal{A}}^2 \mathcal{R}_{ab} n^a n^b > R_{ab}u^a u^b\,,$$

the collapse will be prevented from reaching a singularity.

Magnetised gravitational collapse

A non-collapse criterion II

Projecting the Gauss-Codacci formula:

$$\mathcal{R}_{ab} = \frac{2}{3}\left(\kappa\rho - \frac{1}{3}\Theta^2 + \sigma^2 - \omega^2\right)h_{ab} - E_{ab} + \frac{1}{2}\kappa\pi_{ab} - \frac{1}{3}\Theta(\sigma_{ab} + \omega_{ab})$$
$$+\sigma_{c\langle a}\sigma^{c}{}_{b\rangle} - \omega_{c\langle a}\omega^{c}{}_{b\rangle} + 2\sigma_{c[a}\omega^{c}{}_{b]}$$

twice along n^a, we find out that:

$$\mathcal{R}_{ab}n^a n^b = (2/3)\rho + \mathcal{E}\,,$$

where $\mathcal{R}_{ab}n^a n^b$ and $\mathcal{E} \equiv E_{ab}n^a n^b$ are the 3-D spatial deformation and the tidal tensor (electric Weyl) along the magnetic forcelines.

Taking into account that $\mathcal{B}^2 \propto a^{-6}$ and $\rho \propto a^{-3(1+w)}$, our criterion becomes:

$$\mathcal{E} > \frac{1}{2}\mathcal{B}^2\,,$$

namely the collapse will be impeded if the tidal stress tensor along the fieldlines prevails over the magnetic energy density.

When does this happen? It seems to depend on the geometric background in hand.

Magnetised gravitational collapse

The linearly perturbed Bianchi I: A simple model of magnetised gravitational implosion

We consider a linearly perturbed (collapsing) Bianchi I model under the requirements of: 1) natural host of magnetic fields, 2) almost homogeneity and 3) closed (perturbed) spatial sections $\mathcal{R}_{ab}n^a n^b > 0$.

The evolution of $\mathcal{E}$ (I)

$$\dot{E}_{\langle ab\rangle} = -\Theta E_{ab} - \frac{1}{2}(\rho + P)\sigma_{ab} - \frac{1}{2}\dot{\pi}_{ab} - \frac{1}{6}\Theta\pi_{ab} + 3\sigma_{\langle a}{}^{c}\left(E_{b\rangle c} - \frac{1}{6}\pi_{b\rangle c}\right) .$$

Projecting twice along $n^a \parallel B^a$ we get:

$$\dot{\mathcal{E}} + \frac{5}{2}\Theta\mathcal{E} - \frac{1}{6}(1+w)\Theta\rho + \frac{1}{2}\Theta\mathcal{B}^2 = 0 ,$$

which is solved (adopting a comoving frame where the connection vanishes) giving ($\mathfrak{D}_1, \mathfrak{D}_2, \mathfrak{D}_3$ are constants):

$$\mathcal{E} = \mathfrak{D}_1 a^{-7.5} + \mathfrak{D}_2 a^{-6} + \left(\frac{1+w}{9-6w}\right)\mathfrak{D}_3 a^{-3(1+w)} .$$

Magnetised gravitational collapse

The evolution of $\mathcal{E}$ (II)

Hence, at an advanced stage of the collapse, the dominant mode is:

$$\mathcal{E} \propto a^{-7.5} \qquad \text{(also, recall that} \qquad \mathcal{B}^2 \propto a^{-6}\text{)}\,.$$

The above result generally satisfies our non-collapse criterion:

$$\mathcal{E} > \frac{1}{2}\mathcal{B}^2\,,$$

which means that, given enough time, magnetic fields always prevent the collapse from reaching a singularity in our example model.