# Relativistic structure formation: extreme objects

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*Hot topics in Modern Cosmology*

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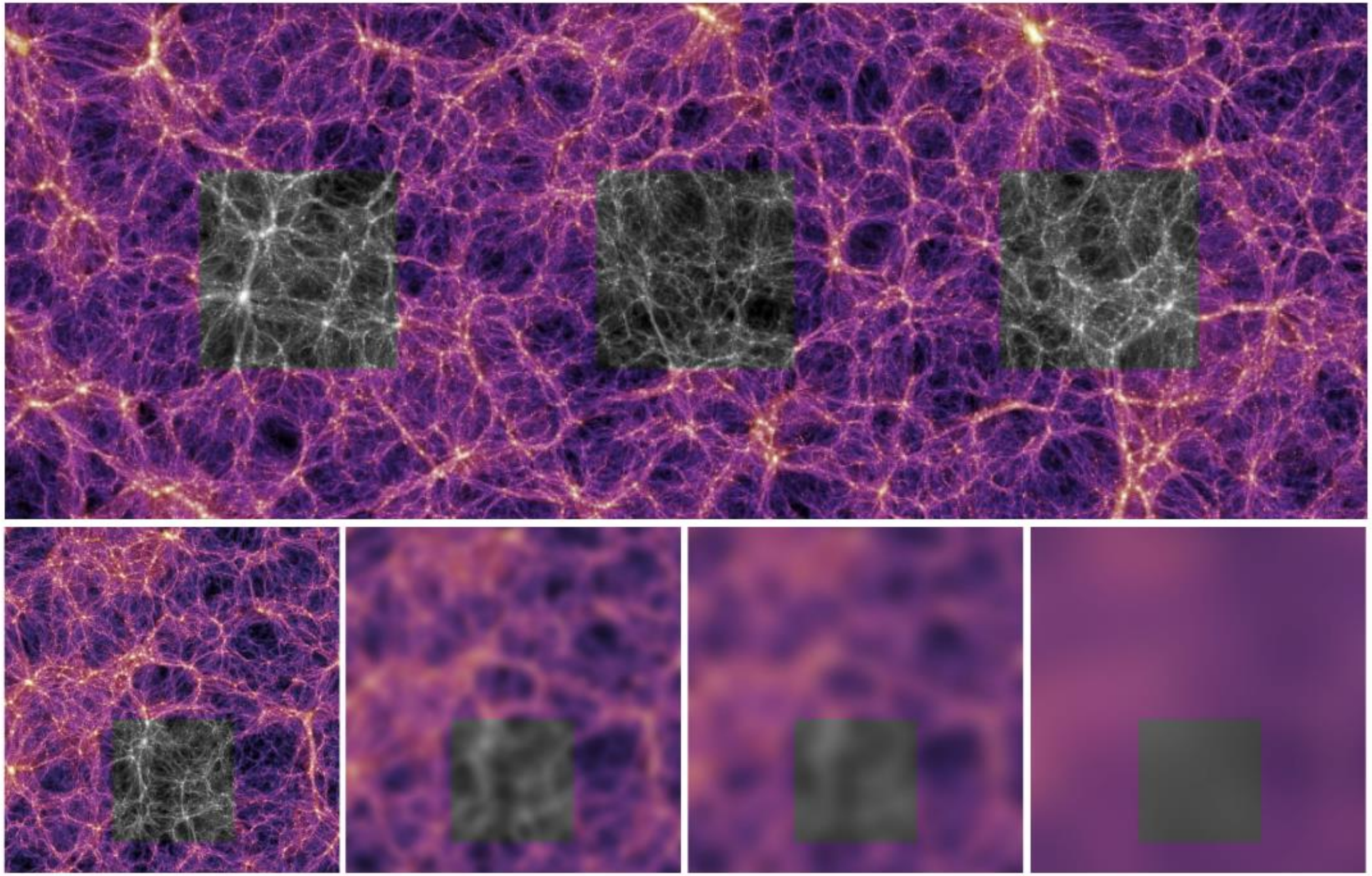


*Clarkson, Rept. Prog. Phys. 74 (2011)*

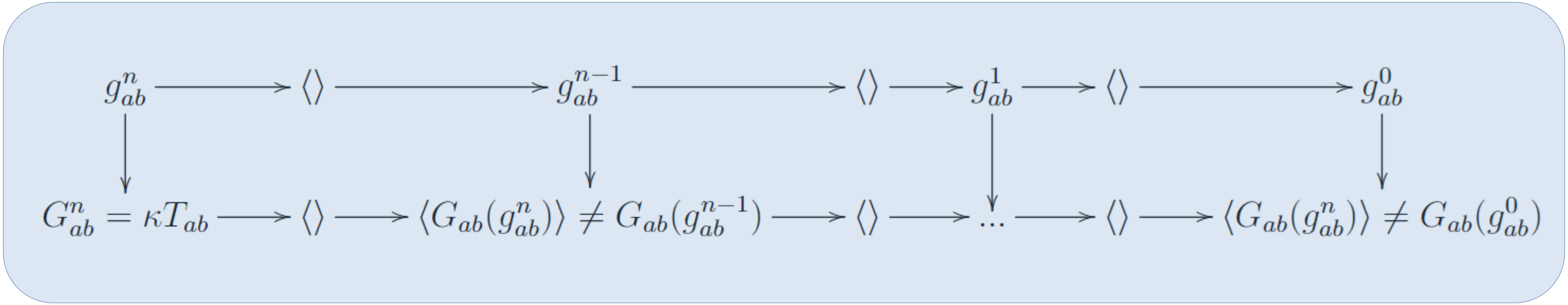
**Averaging** Coarse-graining of structure, such that small-scale effects are hidden to reveal large scale geometry and dynamics.

**Backreaction** Gravity gravitates, so local gravitational inhomogeneities may affect the cosmological dynamics. How this is calculated depends on the degree of coarse graining.

**Fitting** How do we appropriately fit an idealized model to observations made from one location in a lumpy universe, given that this 'background' does not in fact exist?

*C. Clarkson, G. Ellis, J. Larena, and O. Umeh, Rept. Prog. Phys. 74 (2011):*

# Averaging/smoothing does not commute with evaluating the inverse metric, connections and curvature



- There is no overall agreement on how to construct the cosmological background

- We can test different backgrounds with the large scale structure formation

- We can try to say something about the large scale structures without or with a minimal reference to the background

# The silent universe approach

- Initial conditions compatible with the concordance model

- Non-linear evolution of density in 1+3 formalism; silent universe equations : no rotation, no gravitational waves, no heat flux

- Numerical evaluation of the mass function of cosmological structures

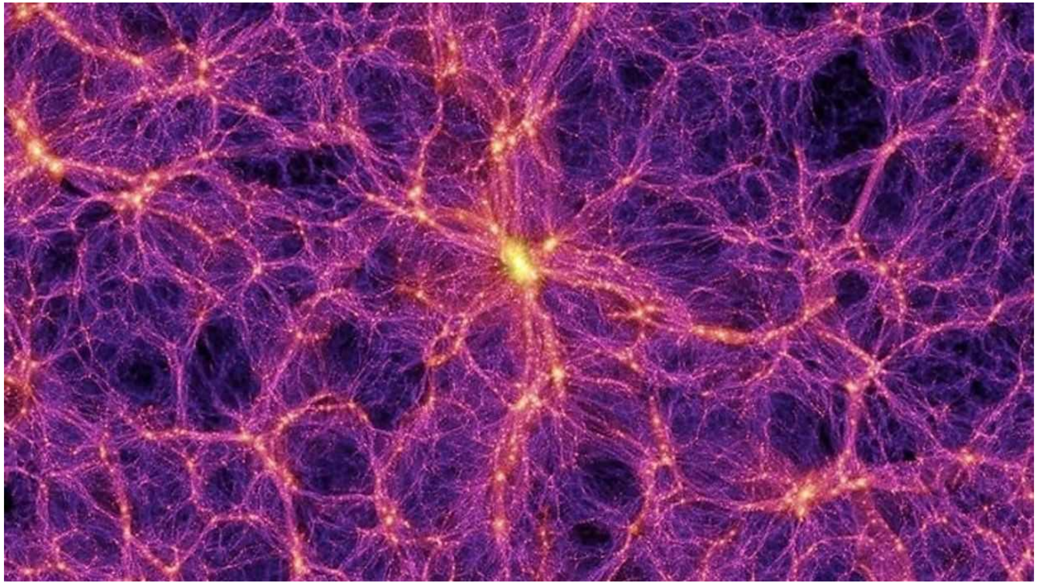
- Poisson statistics of the most massive objects in the Universe

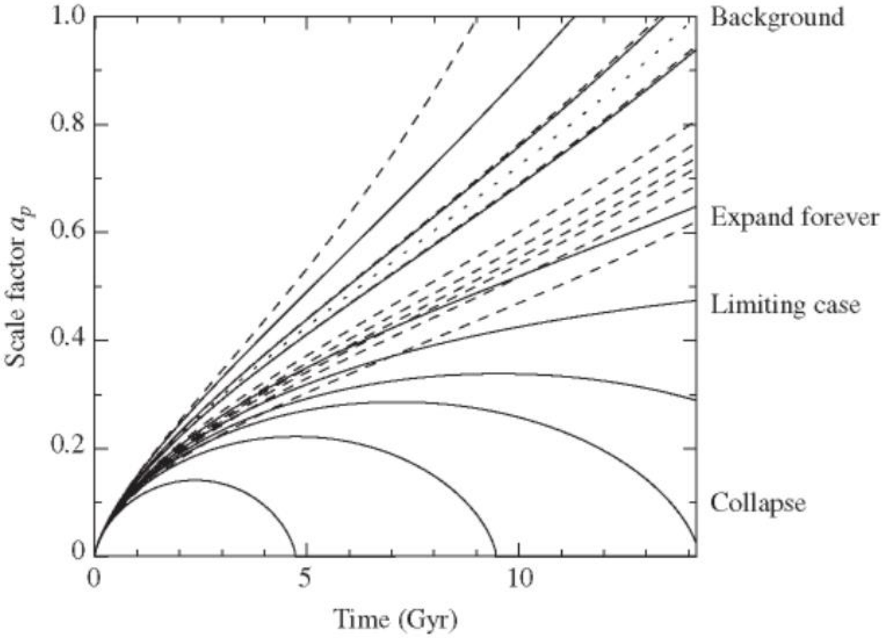
Silent universe equations:

$$\begin{aligned}
\dot{\rho} &= -\rho\,\Theta \\
\dot{\Theta} &= -\frac{1}{3}\Theta^2 - \frac{1}{2}\,\kappa\rho - 6\,\sigma^2 + \Lambda \\
\dot{\sigma} &= -\frac{2}{3}\Theta\,\sigma + \sigma^2 - \mathcal{W} \\
\dot{\mathcal{W}} &= -\Theta\,\mathcal{W} - \frac{1}{2}\,\kappa\rho\,\sigma - 3\sigma\,\mathcal{W}
\end{aligned}$$

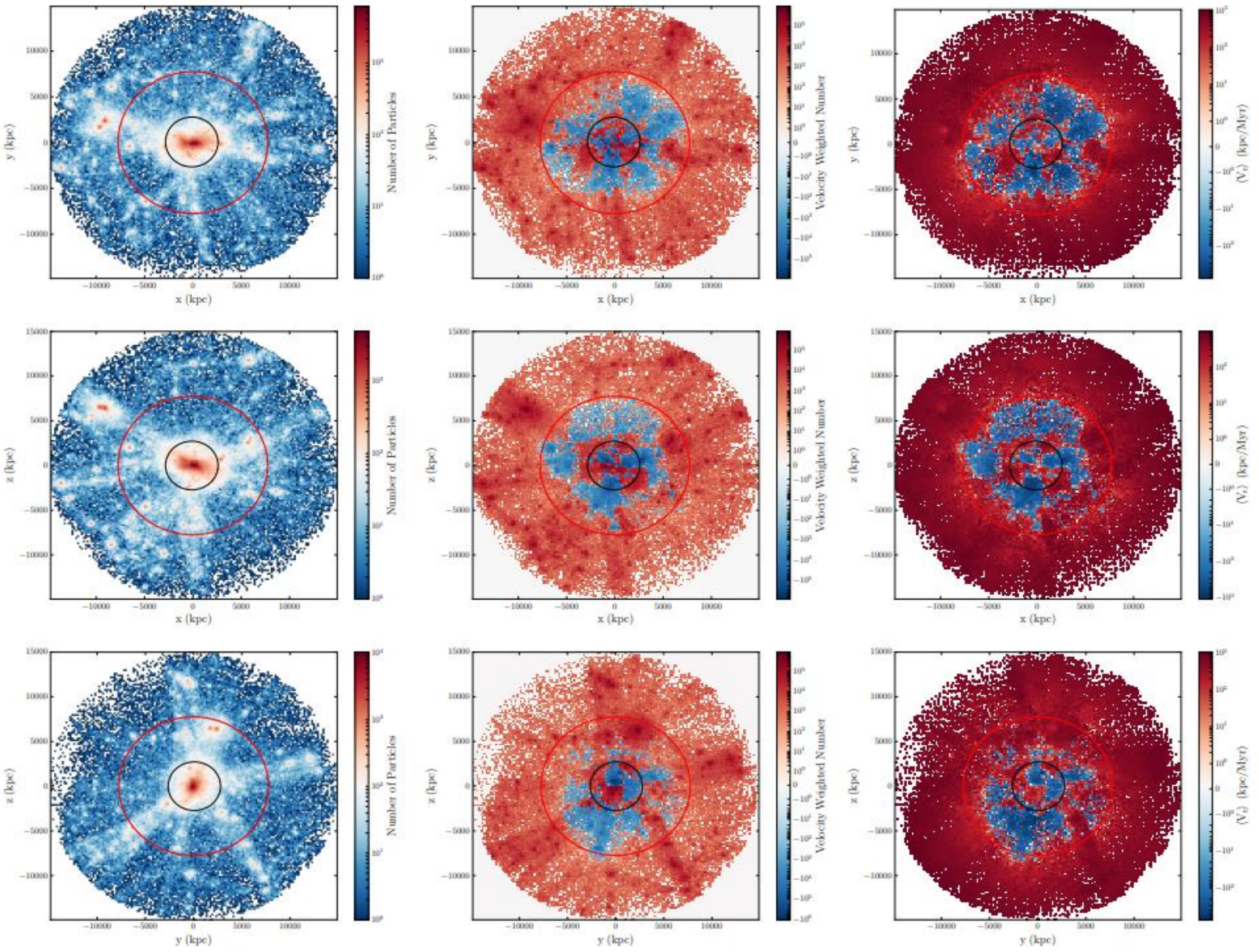
Mass function of galaxy clusters:

$$\frac{\mathrm{dN}}{\mathrm{d}\ln M} = \frac{\varrho_0}{M}\; f(\sigma_M)\left|\frac{\mathrm{d}\ln\sigma_M}{\mathrm{d}\ln M}\right|$$






*Millenium project, Press & Schecter 1974*



# Maximum radius of structure: $R_{max} = \left(\frac{3GM}{\Lambda}\right)^{1/3}$

*Hansen et al JCAP (2020)*

# Spherically symmetric metric

$$ds^2 = -dt^2 + \frac{(R')^2}{1+2E}dr^2 + R^2 d\Omega^2$$

$$\dot{R}^2 - \frac{2GM}{R} = 2E \qquad\qquad 4\pi G\rho = \frac{M'}{R^2 R'}$$

$$E < 0 \quad , \quad R = -\frac{M}{2E}(1-\cos\mu) \quad , \quad \mu - \sin\mu = \frac{(-2E)^{3/2}}{M}(t - t_B)$$

$$E = 0 \quad , \quad R = \left(\frac{9}{2}M(t-t_B)^2\right)^{1/3}$$

$$E > 0 \quad , \quad R = \frac{M}{2E}(\cosh\mu - 1) \quad , \quad \sinh\mu - \mu = \frac{(2E)^{3/2}}{M}(t - t_B) \ ,$$

Hamiltonian constrain in the co-moving, synchronous gauge:

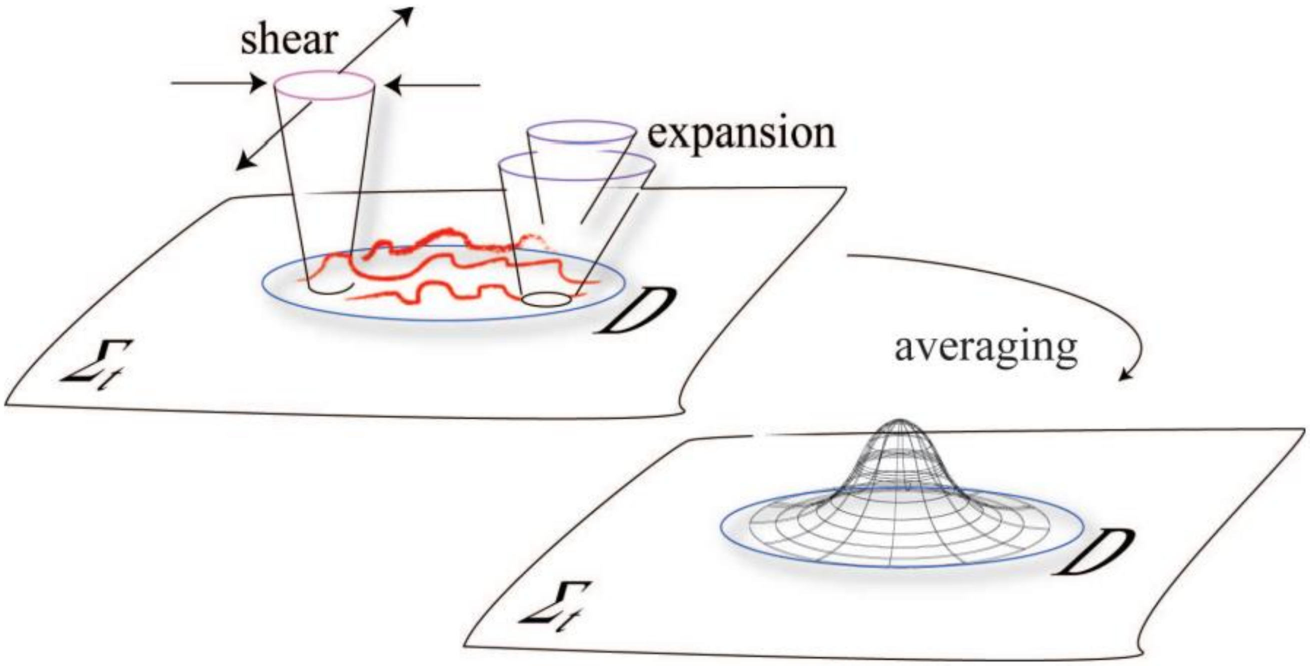
$$\frac{1}{3}\Theta^2 = 8\pi G\rho + \sigma^2 - \frac{1}{2}\mathcal{R} + \Lambda$$

Maximum volume condition is equivalent to the zero averaged

$$3H_{\mathcal{D}} := \langle\theta\rangle_{\mathcal{D}} = \frac{4\pi}{3V}\int_0^{r_{\mathcal{D}}} \frac{\partial_r\left(\dot{R}R^2\right)}{\sqrt{1+2E}}\mathrm{d}r$$

$$V = \frac{4\pi}{3}\int_0^{r_{\mathcal{D}}} \frac{\partial_r\left(R^3\right)}{\sqrt{1+2E}}\mathrm{d}r$$

$$3H_{\mathcal{D}} = \frac{\partial_t V}{V}$$




*Buchert and Carfora, CQG (2008)*

$$a_{\mathcal{D}} = \left( \frac{V_{\mathcal{D}}(t)}{V_{\mathcal{D}_i}} \right)^{\frac{1}{3}} \qquad H_{\mathcal{D}} := \frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}$$

$$3\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}} + 4\pi G \langle \rho \rangle_{\mathcal{D}} = \mathcal{Q}_{\mathcal{D}} \quad , \quad H_{\mathcal{D}}^2 - \frac{8\pi G}{3} \langle \rho \rangle_{\mathcal{D}} = -\frac{\langle \mathcal{R} \rangle_{\mathcal{D}} + \mathcal{Q}_{\mathcal{D}}}{6}$$

$$\mathcal{Q}_{\mathcal{D}} = 2 \langle \mathrm{II} \rangle_{\mathcal{D}} - \frac{2}{3} \langle \mathrm{I} \rangle_{\mathcal{D}}^2 \qquad \mathrm{I} = \mathrm{tr}\ (\Theta_{ij}) \quad , \quad \mathrm{II} = \frac{1}{2}\left( (\mathrm{tr}\Theta_{ij})^2 - \mathrm{tr}\left( (\Theta_{ij})^2 \right) \right) \quad , \quad \mathrm{III} = \det\left(\Theta_{ij}\right)$$

# Relativistic Zel'dovich approximation

$$ {}^{\mathrm{RZA}}\mathcal{Q}_{\mathcal{D}} = \frac{\dot{\xi}^2\left(\gamma_1 + \xi\gamma_2 + \xi^2\gamma_3\right)}{\left(1 + \xi\left\langle \mathrm{I_i}\right\rangle_{\mathcal{I}} + \xi^2\left\langle \mathrm{II_i}\right\rangle_{\mathcal{I}} + \xi^3\left\langle \mathrm{III_i}\right\rangle_{\mathcal{I}}\right)^2}\,, \quad \text{with :} $$

$$ \begin{cases} \gamma_1 := 2\left\langle \mathrm{II_i}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{I_i}\right\rangle_{\mathcal{I}}^2 \\ \gamma_2 := 6\left\langle \mathrm{III_i}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{II_i}\right\rangle_{\mathcal{I}}\left\langle \mathrm{I_i}\right\rangle_{\mathcal{I}} \\ \gamma_3 := 2\left\langle \mathrm{I_i}\right\rangle_{\mathcal{I}}\left\langle \mathrm{III_i}\right\rangle_{\mathcal{I}} - \frac{2}{3}\left\langle \mathrm{II_i}\right\rangle_{\mathcal{I}}^2 \end{cases} $$

$$ \xi(t) := [q(t) - q(t_{\mathbf{i}})]/\dot{q}(t_{\mathbf{i}}) $$

$$ \ddot{q}(t) + 2H\dot{q}(t) - 4\pi\varrho_H q(t) = 0 $$

$$ \langle \mathcal{A}\rangle_{\mathcal{D}} = \frac{\langle \mathcal{A}J\rangle_{\mathcal{I}}}{\langle J\rangle_{\mathcal{I}}}\,, \quad \langle J\rangle_{\mathcal{I}} = a_{\mathcal{D}}^3 $$

$$ \mathcal{J}(t, \mathbf{X}) := 1 + \xi(t)\mathrm{I_i} + \xi^2(t)\mathrm{II_i} + \xi^3(t)\mathrm{III_i} $$

$$ J \approx {}^{\mathrm{RZA}}J := a^3\mathcal{J} $$

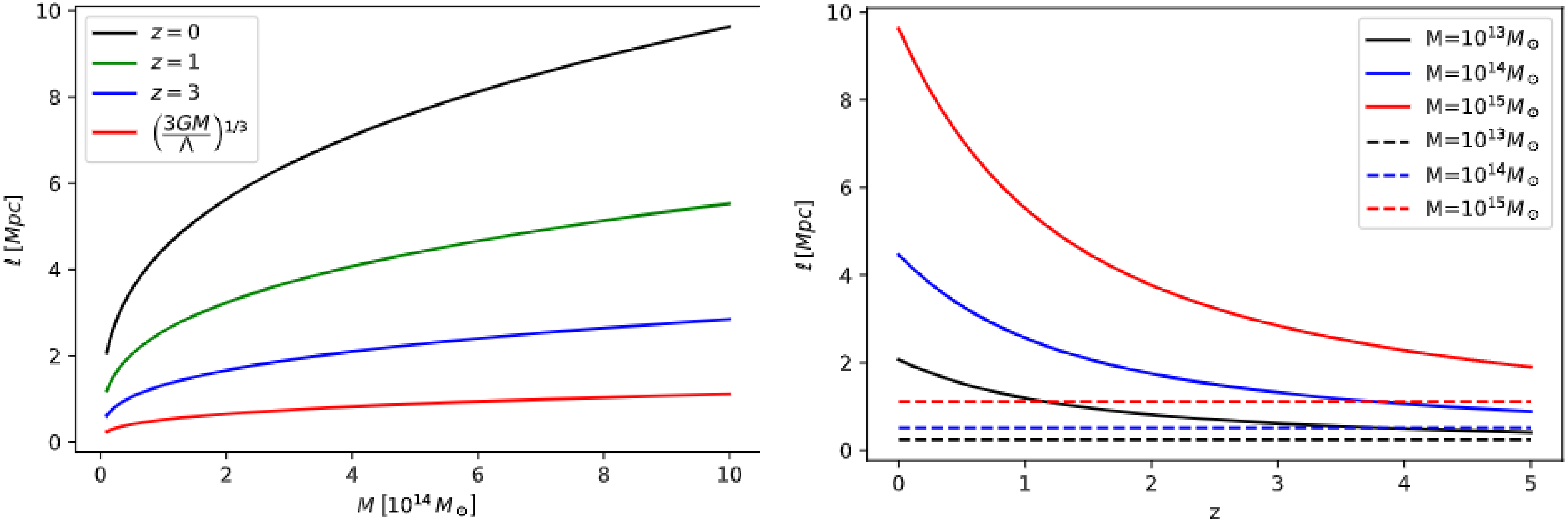
*Buchert et al, PRD (2013)*

Using the condition for the turnaround:

Turnaround: $$3H_{\mathcal{D}} = \langle \Theta \rangle_{\mathcal{D}} = \frac{\dot{V}_{\mathcal{D}}}{V_{\mathcal{D}}} = 0 \implies \dot{V}_{\mathcal{D}} = 0$$

averaged equations and the Relativistic Zel'dovich approximation as a closure condition, we get:

$$V_{\mathcal{D}}(t_{ta}) = \frac{M}{\rho_H \left(1 + 3H \left(\frac{\dot{q}}{q}\right)^{-1}\right)}$$



$z=0$
$z=1$
$z=3$
$\left(\frac{3GM}{\Lambda}\right)^{1/3}$
$\ell$ [Mpc]
$M$ [$10^{14} M_\odot$]
M=$10^{13}M_\odot$
M=$10^{14}M_\odot$
M=$10^{15}M_\odot$
M=$10^{13}M_\odot$
M=$10^{14}M_\odot$
M=$10^{15}M_\odot$
$\ell$ [Mpc]
z

# Conclusions

- Non-linear, relativistic approach to the cosmological structure formation can guide us towards solving the averaging problem
- Extreme cosmological structures provide a useful test for different cosmological backgrounds and/or alternative theories of gravity
- Further details can be found in: *Bolejko & Ostrowski PRD (2020), Roukema & Ostrowski JCAP (2020), Ostrowski AppB (2020), Ostrowski & Delgado Gaspar JCAP (2022)*