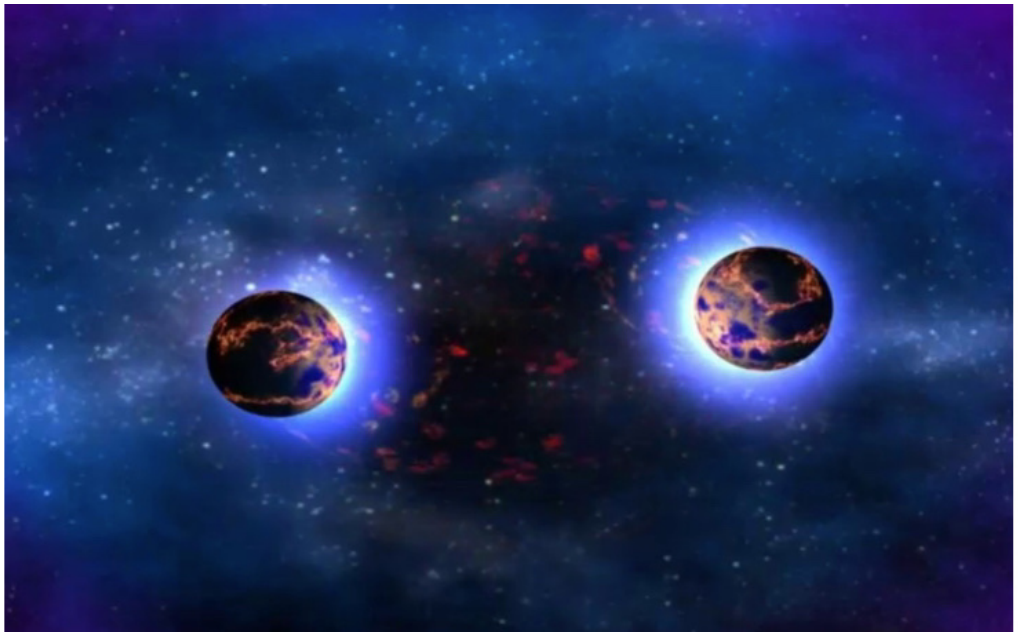
Hairy black holes and Kepler orbits

[De Neeling, DR, Seri, Waalkens - in progress]



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Diederik Roest
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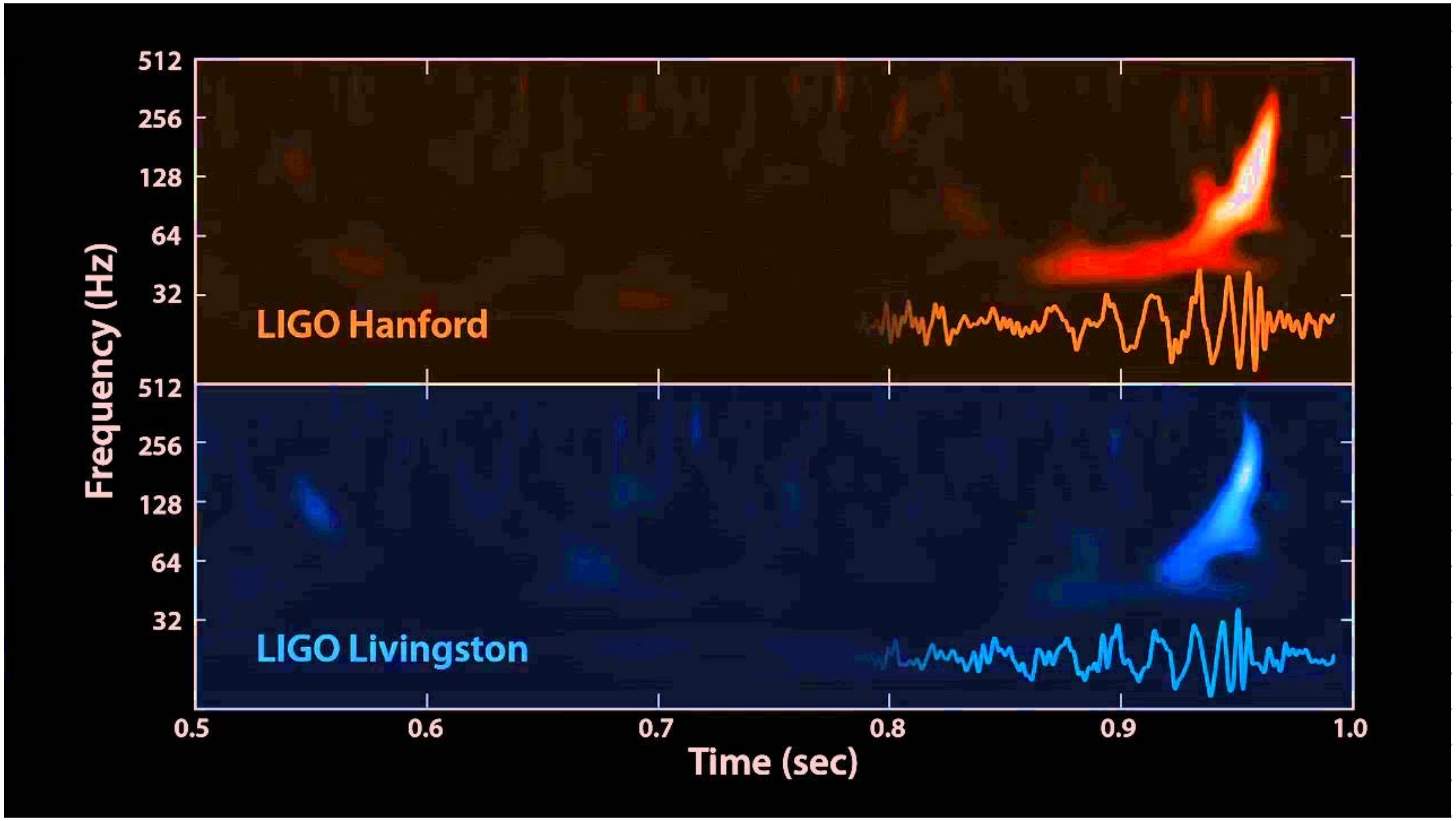
1. Introduction - motivation
2. Mechanics - Kepler orbits
3. Field theories - relativistic corrections
4. Synthesis - hairy black holes
5. Conclusion - outlook

Introduction



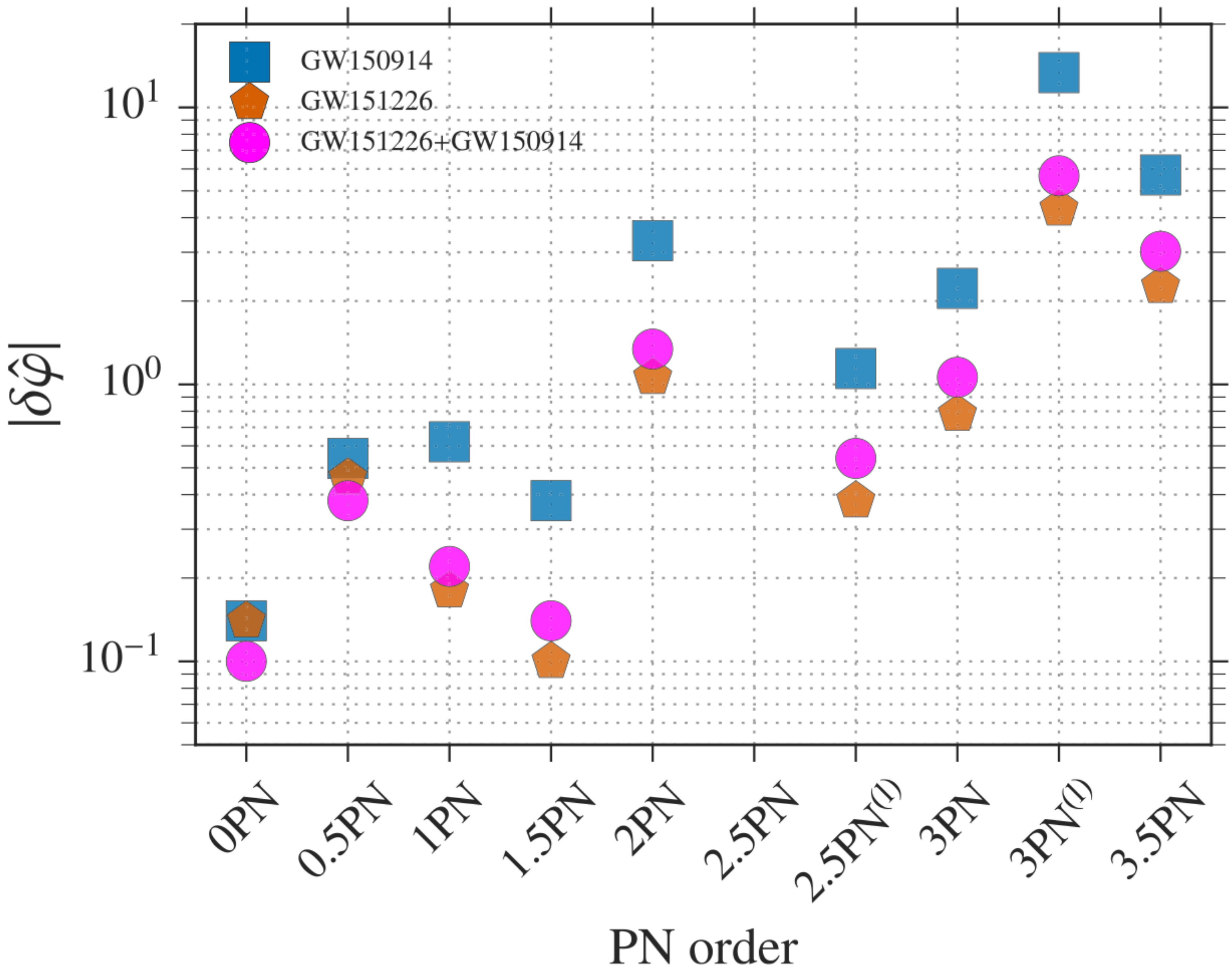


Figure 10. Two GW detections of inspiralling BH binaries were used to set bounds on possible deviations from their GR values of various PN coefficients that appear in the expression for the GW phase. Figure from Ref. [3].

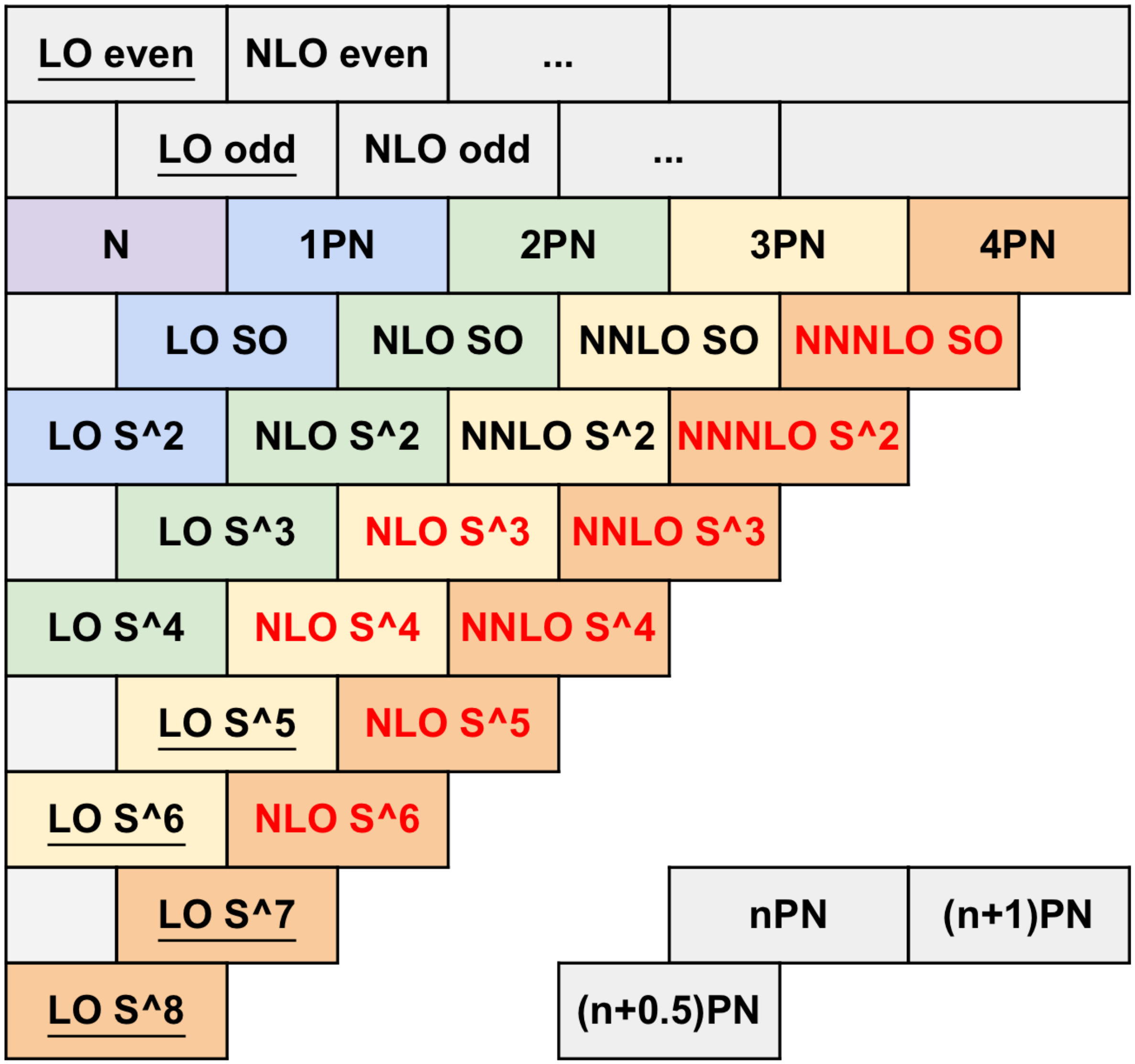


Figure 9. Contributions to the two-body Hamiltonian in the PN spin expansion, for arbitrary-mass-ratio binaries with spin induced multipole moments. Contributions in red are yet to be calculated. LO stands for "leading order", NLO for "next-to-leading order", and so on. SO stands for "spin-orbit". Figure from Ref. [996].

[BH and GW road map, 2018]

High Energy Physics - Theory

[Submitted on 5 Aug 2019 (v1), last revised 14 Feb 2020 (this version, v2)]

Black Hole Binary Dynamics from the Double Copy and Effective Theory

Zvi Bern, Clifford Cheung, Radu Roiban, Chia-Hsien Shen, Mikhail P. Solon, Mao Zeng

We describe a systematic framework for computing the conservative potential of a compact binary system using modern tools from scattering amplitudes and effective field theory. Our approach combines methods for integration and matching adapted from effective field theory, generalized unitarity, and the double-copy construction, which relates gravity integrands to simpler gauge-theory expressions. With these methods we derive the third post-Minkowskian correction to the conservative two-body Hamiltonian for spinless black holes. We describe in some detail various checks of our integration methods and the resulting Hamiltonian.

Comments:	134 pages, 32 figures
Subjects:	**High Energy Physics - Theory (hep-th)**; General Relativity and Quantum Cosmology (gr-qc); High Energy Physics - Phenomenology (hep-ph)
Journal reference:	JHEP 10 (2019) 206
DOI:	10.1007/JHEP10(2019)206
Report number:	CERN-TH-2019-128, CALT-TH 2019-026, UCLA/TEP/2019/103
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From: Mao Zeng [view email]

[v1] Mon, 5 Aug 2019 07:12:56 UTC (5,273 KB)

[v2] Fri, 14 Feb 2020 08:27:12 UTC (5,295 KB)

We will be addressing this problem from a different angle:

Is relativity necessarily complicated?
Possibility of tuning relativistic effects to regain simplicity?

Requires: more general relativity - inclusion of charges and scalar hair.

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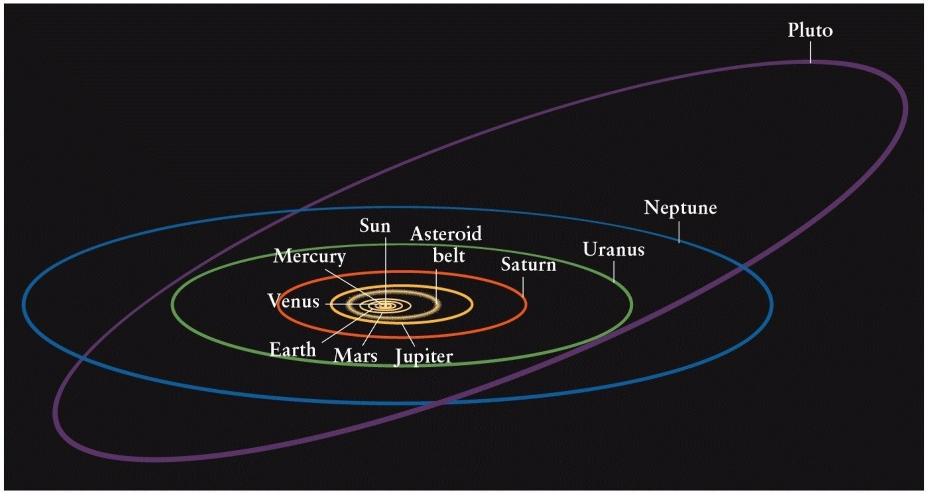
Mechanics

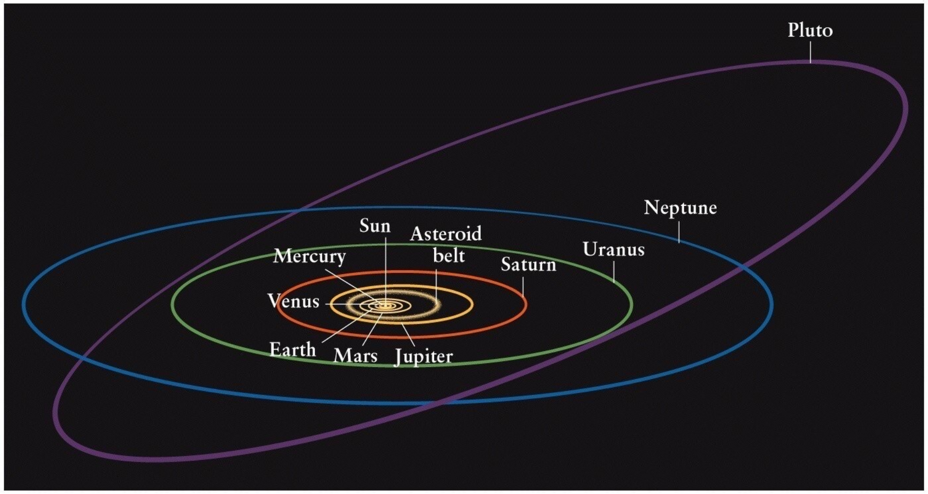
Newtonian dynamics and gravity results in closed, elliptical orbits.

$$L_0 = \frac{1}{2}m_1\mathbf{v_1}^2 + \frac{1}{2}m_2\mathbf{v_2}^2 + G\frac{m_1 m_2}{r}$$



Conserved quantities: angular momentum and Runge-Lenz vector

$$\mathbf{L} = \mu \mathbf{r} \times \mathbf{v} \qquad \mathbf{A} = \mathbf{p} \times \mathbf{L} - \alpha \frac{\mathbf{r}}{r} \; .$$

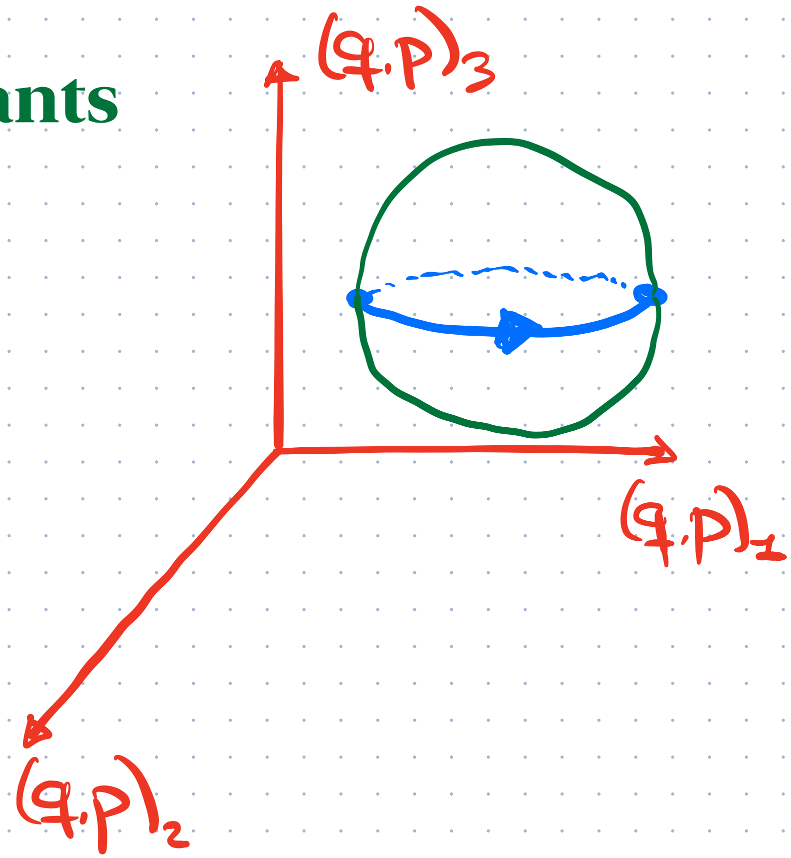


Enhances rotational SO(3) to hidden SO(4) symmetry:

$$\{L_i, L_j\} = \epsilon_{ijk} L_k \ ,$$
$$\{A_i, L_j\} = \epsilon_{ijk} A_k \ ,$$
$$\{A_i, A_j\} = -p_0^2\, \epsilon_{ijk} L_k \ .$$

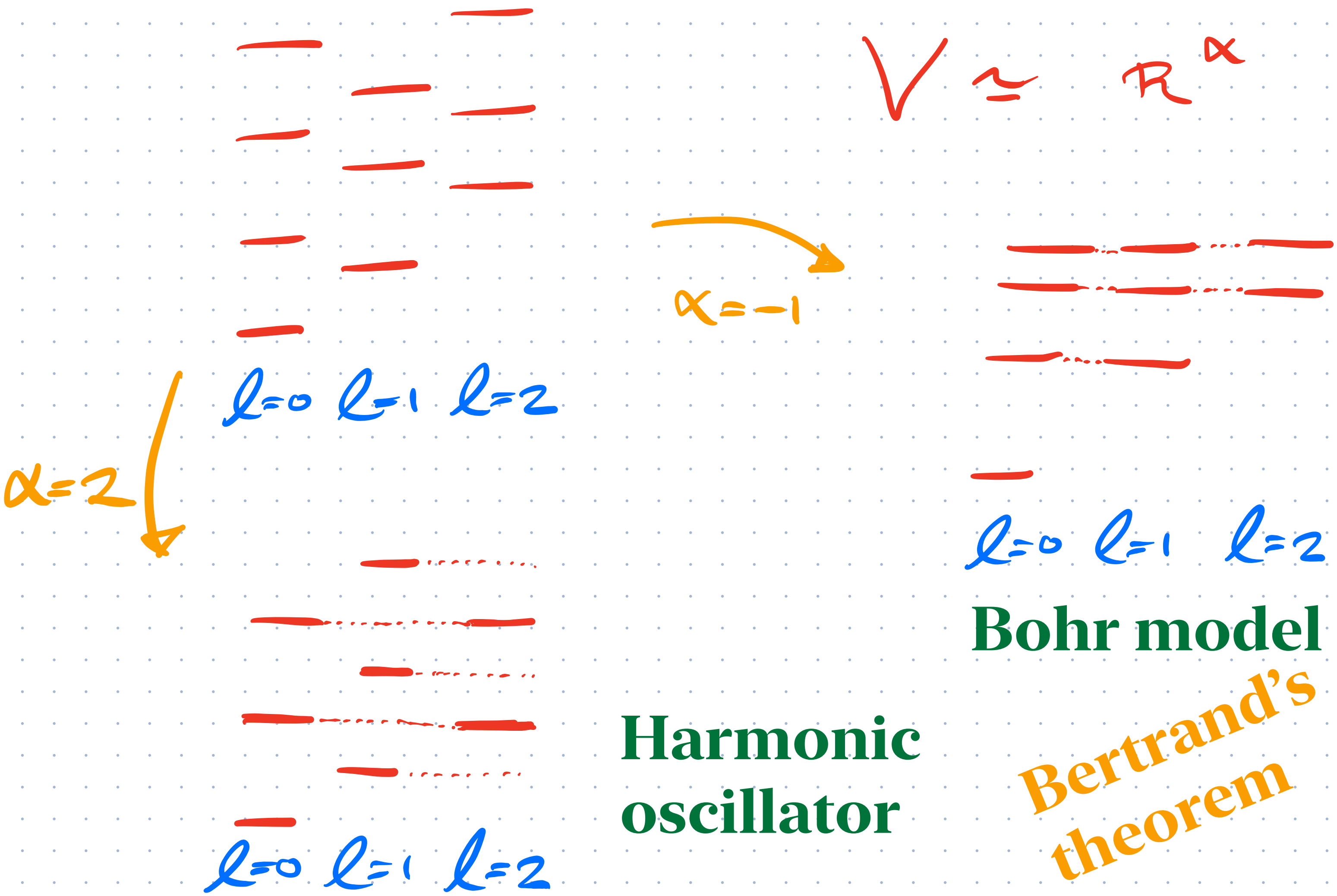
Five independent constants of motion: maximally superintegrable

Free motion on a three-sphere in phase space



[Moser, 1979, see also Nabet, Kol - 2014]

Similar in 3D quantum mechanics with spherically symmetric potential:



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Field theories

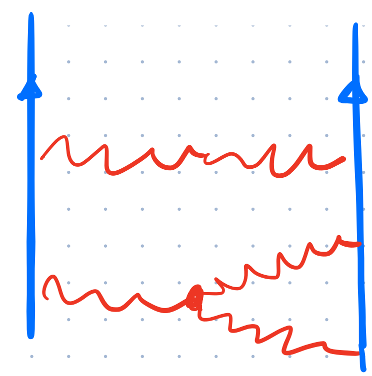
Back to solar system: Mercury & SR

Solved by General Relativity: metric theory of gravity

$$S = \int \mathrm{d}^4x \sqrt{-g} R\,.$$

Particle Lagrangian fixed by equiv principle

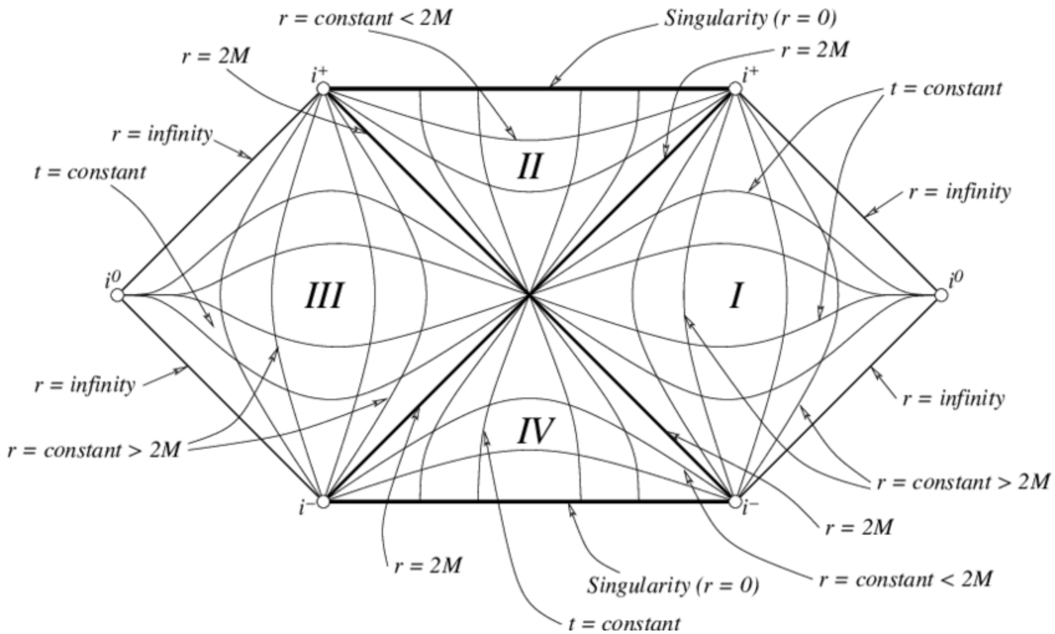
$$L_{pp} = \mathfrak{m}\sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}\,.$$



Gravitational interaction:

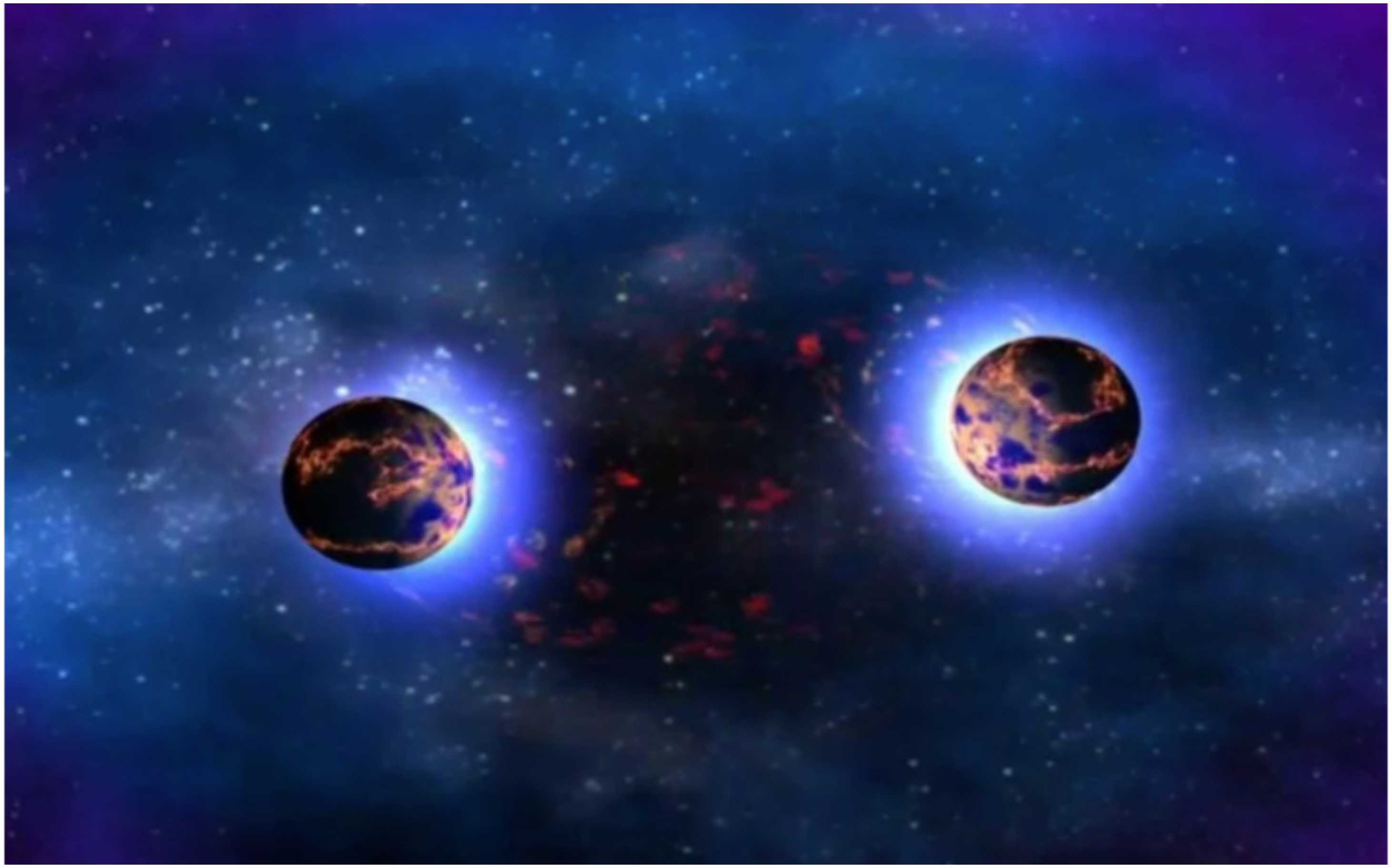
- **1/r^2 + corrections**
- **Attractive**
- **Couples to full energy-momentum tensor**

Schwarzschild black hole:



One-center version of Kepler:

Probe particle along geodesics of Schwarzschild geometry



Gravitational attraction for one center:

$$(\Delta\phi)_{\rm GR} = \frac{6\pi(G_N m_A m_B)^2}{L^2}.$$

Perfect for Mercury!

For BH binaries: two-body problem

Post-Newtonian expansion:

$$\mathcal{L} = L_0 + \frac{1}{c^2} L_{1PN} \ ,$$

$$L_0 = \frac{1}{2} m_1 \mathbf{v_1}^2 + \frac{1}{2} m_2 \mathbf{v_2}^2 + G \frac{m_1 m_2}{r} \ ,$$

$$L_{1PN} = \frac{1}{8} m_1 \mathbf{v_1}^4 + \frac{1}{8} m_2 \mathbf{v_2}^4 + G \frac{m_1 m_2}{2r} \left[3\mathbf{v_1}^2 + 3\mathbf{v_2}^2 - 8\mathbf{v_1} \cdot \mathbf{v_2} \right.$$
$$\left. + (\mathbf{v_1} \cdot \mathbf{v_2} - (\hat{\mathbf{r}} \cdot \mathbf{v_1})(\hat{\mathbf{r}} \cdot \mathbf{v_2})) \right] - G^2 \frac{m_1 m_2 (m_1 + m_2)}{2r^2} \ .$$

Einstein-Infeld-Hoffman terms

[Kol, Smolkin 2007]

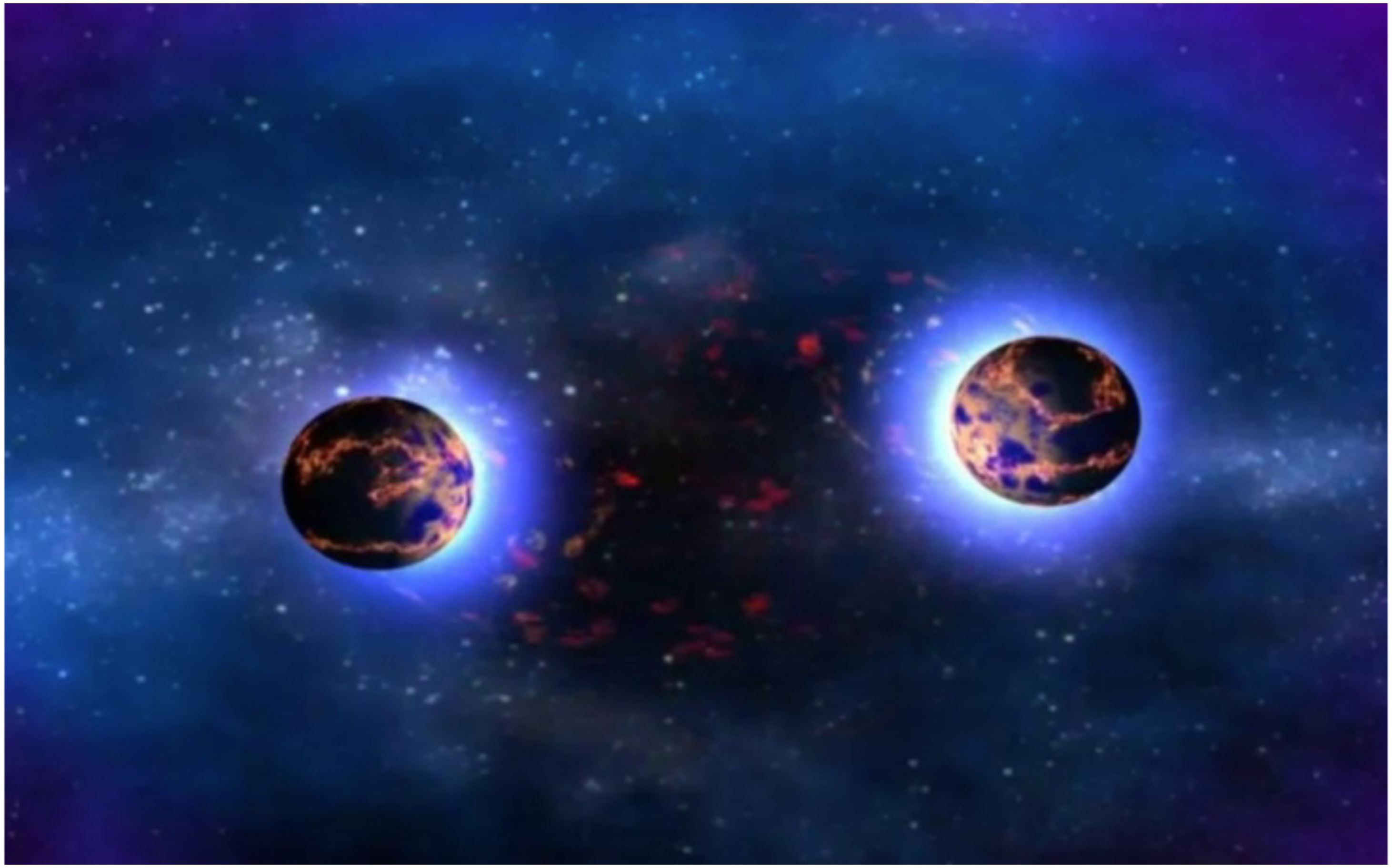
Hamiltonian in com frame:

$$H = H_0 + \frac{1}{c^2} H_{1PN}$$
$$= \left[\frac{\mathbf{p}^2}{2} - \frac{\alpha}{r}\right] + \frac{1}{c^2}\left[\frac{(3\nu-1)}{8}\mathbf{p}^4 - \frac{\alpha(3+\nu)}{2}\frac{\mathbf{p}^2}{r} - \frac{\alpha\nu}{2}\frac{(\mathbf{p}\cdot\hat{\mathbf{r}})^2}{r} + \frac{\alpha^2}{2r^2}\right] .$$

with $\nu = \frac{\mu}{m} = \frac{m_1 m_2}{(m_1+m_2)^2}$ and $\alpha = Gm = G(m_1 + m_2)$.

Break conservation of Runge-Lenz vector (and Kepler orbits and integrability...)

Note: symmetric mass ratio...



Gravitational attraction for two body:

$$(\Delta\phi)_{\mathrm{GR}} = \frac{6\pi(G_N m_A m_B)^2}{L^2}.$$

Independent of ratio!

Alternative relativistic completion: Maxwell

Based on gauge Symmetry:

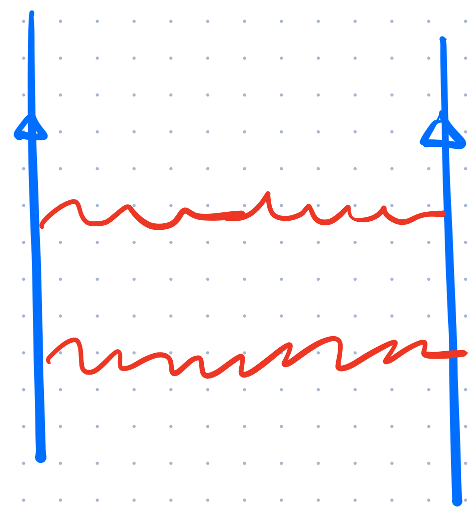
$$S = \int \mathrm{d}^4x \, (-F^2) \ .$$

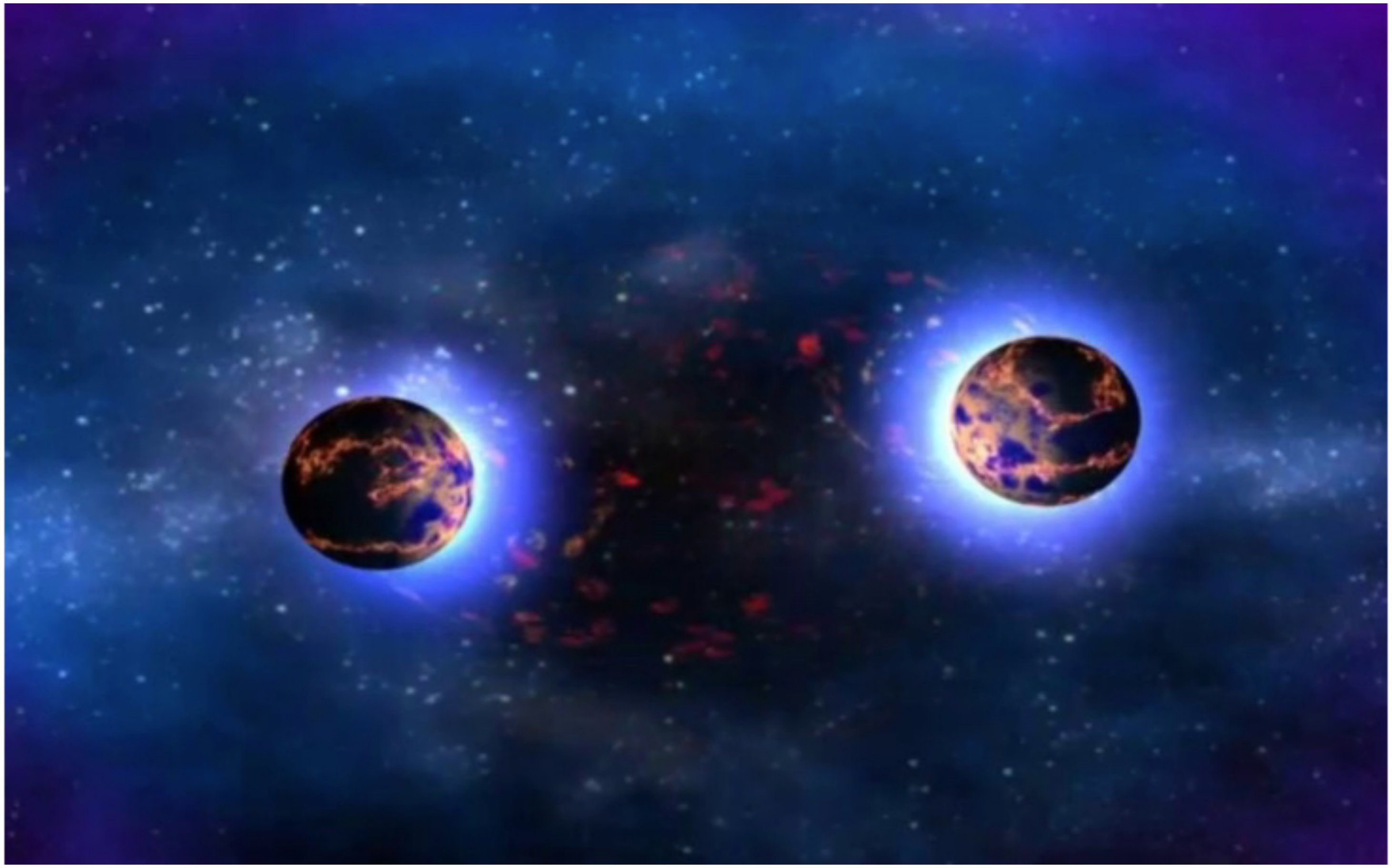
Particle Dynamics:

$$L_{pp} = \mathfrak{m}\sqrt{-\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + qA_\mu \dot{x}^\mu .$$

Electromagnetic interaction:

- spin-1
- 1/r^2 + corrections
- Repulsive
- Couples to EM charge





Electromagnetic attraction for one center:

$$\delta\phi = 2\pi \frac{q_1 q_2}{L^2}.$$

[Landau, Lifschitz classic textbook]

Finally: also possible with spin-0 theory

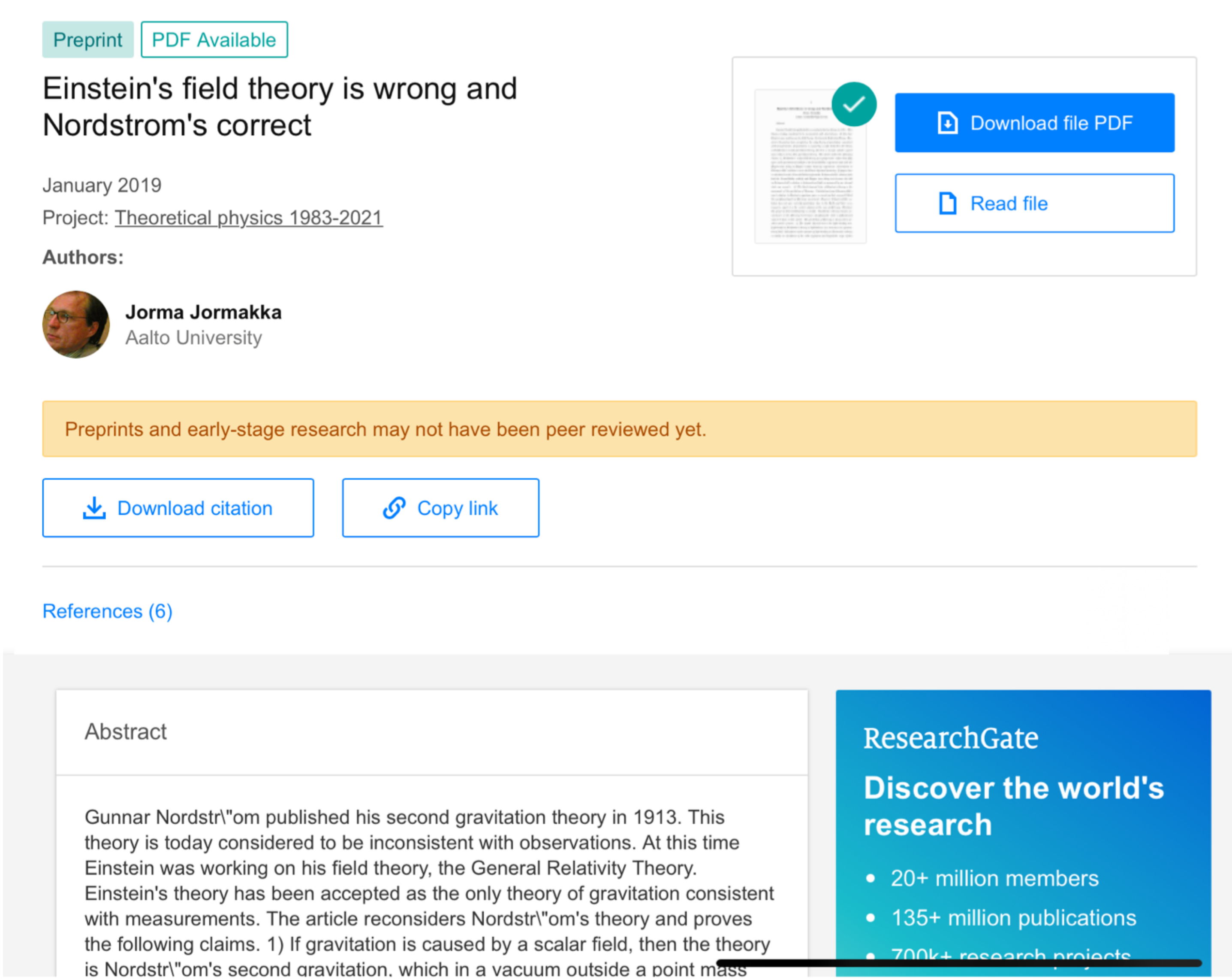
$$S = \int \mathrm{d}^4 x \left(-(\partial\phi)^2\right).$$

$$L_{pp} = \mathfrak{m} e^{\phi} \sqrt{-\eta_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}\,.$$

Dilaton interaction:

- Spin-0
- 1/r^2 corrections & attractive
- "Scalar version" of gravity - couples to trace of energy-mom tensor.
- Equivalence principle.
- Spontaneous breaking of conformal symmetry.
- Introduced by Nordstrom in 1913.

[Sundrum 2003, Deruelle 2011]


Preprint
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Einstein's field theory is wrong and Nordstrom's correct
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Project: Theoretical physics 1983-2021
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Abstract
Gunnar Nordstr\"om published his second gravitation theory in 1913. This theory is today considered to be inconsistent with observations. At this time Einstein was working on his field theory, the General Relativity Theory. Einstein's theory has been accepted as the only theory of gravitation consistent with measurements. The article reconsiders Nordstr\"om's theory and proves the following claims. 1) If gravitation is caused by a scalar field, then the theory
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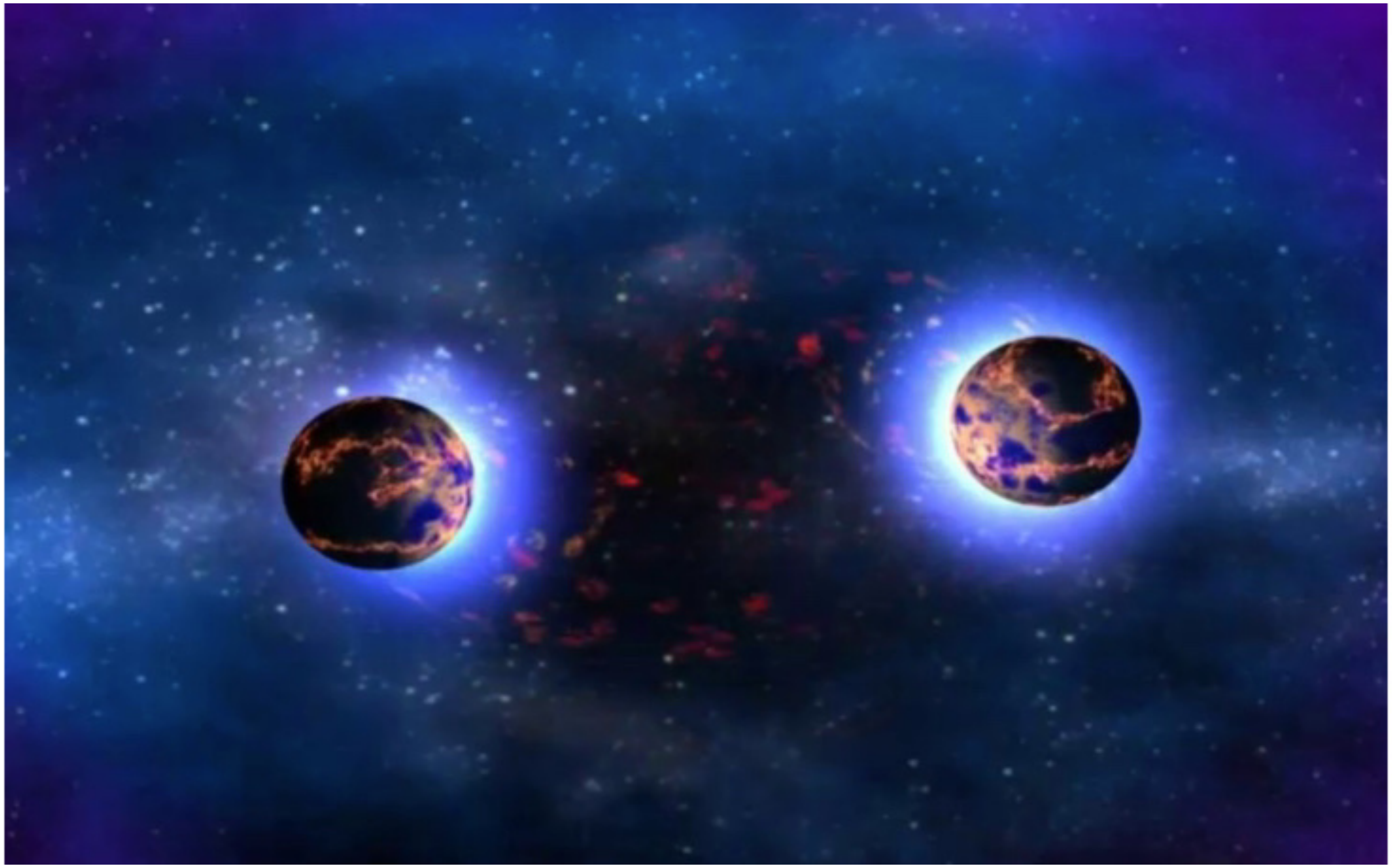

Dilaton interaction for one center:

$$\delta\phi = -2\pi \frac{M^2\mu^2}{L^2}.$$

Opposite sign!

[Deruelle 2011, Brax, Davis 2018]

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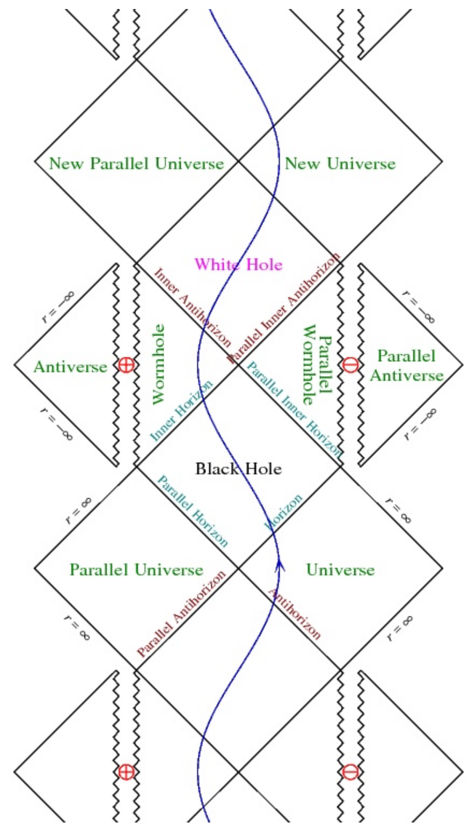
Synthesis

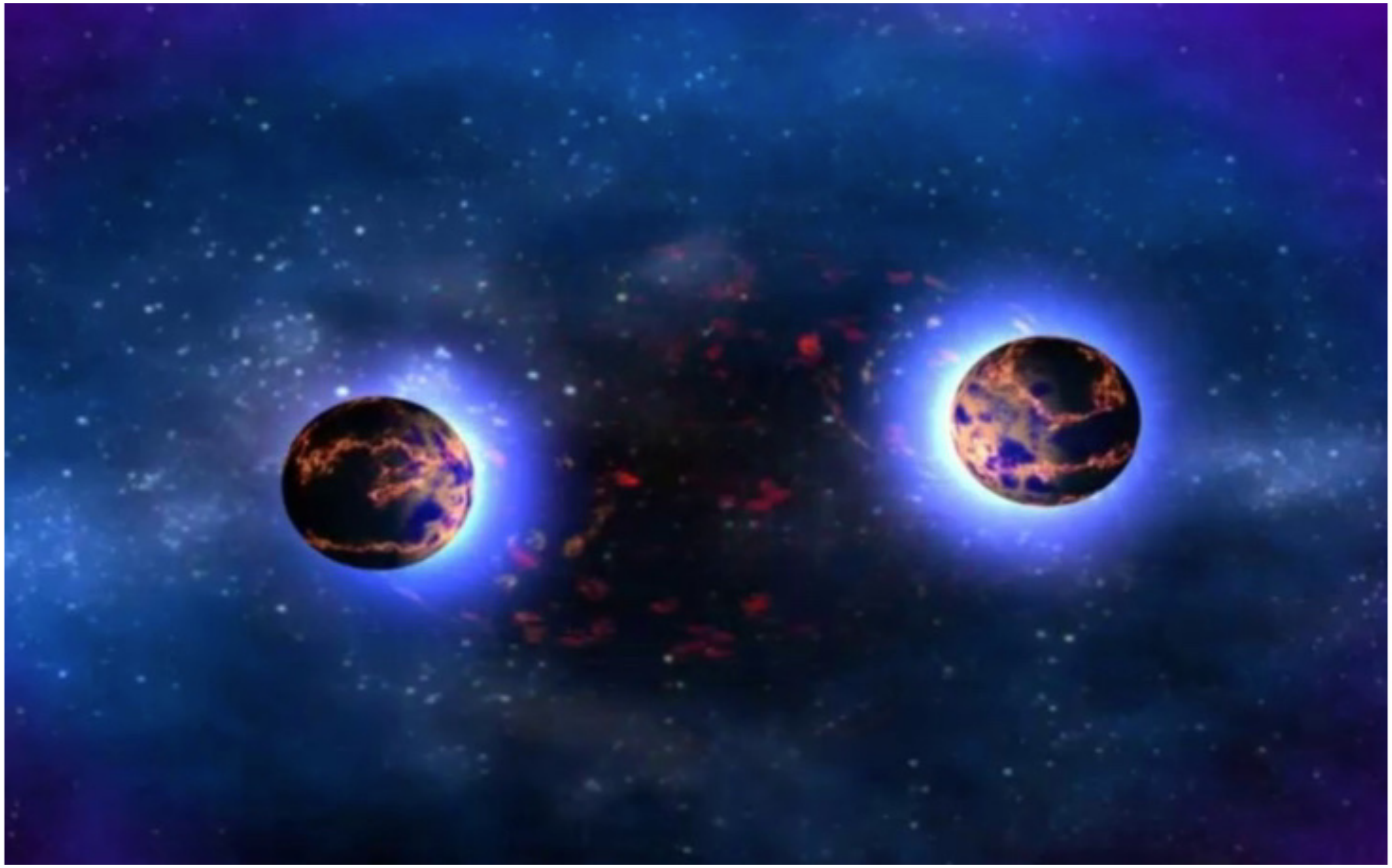
Combine relativistic effects to regain simplicity? Kepler - RL - integrability?

First include charges:

Reissner-Nordström black hole with mass M and charge Q.

Note: no scalar hair





Two-body system with mass m and charge e:

$$\delta_{RN} = \pi \frac{M^2\mu^2}{L^2}\left(6(1-e_1e_2)+e_1^2e_2^2-\frac{(m_1e_1^2+m_2e_2^2)}{M}\right),$$

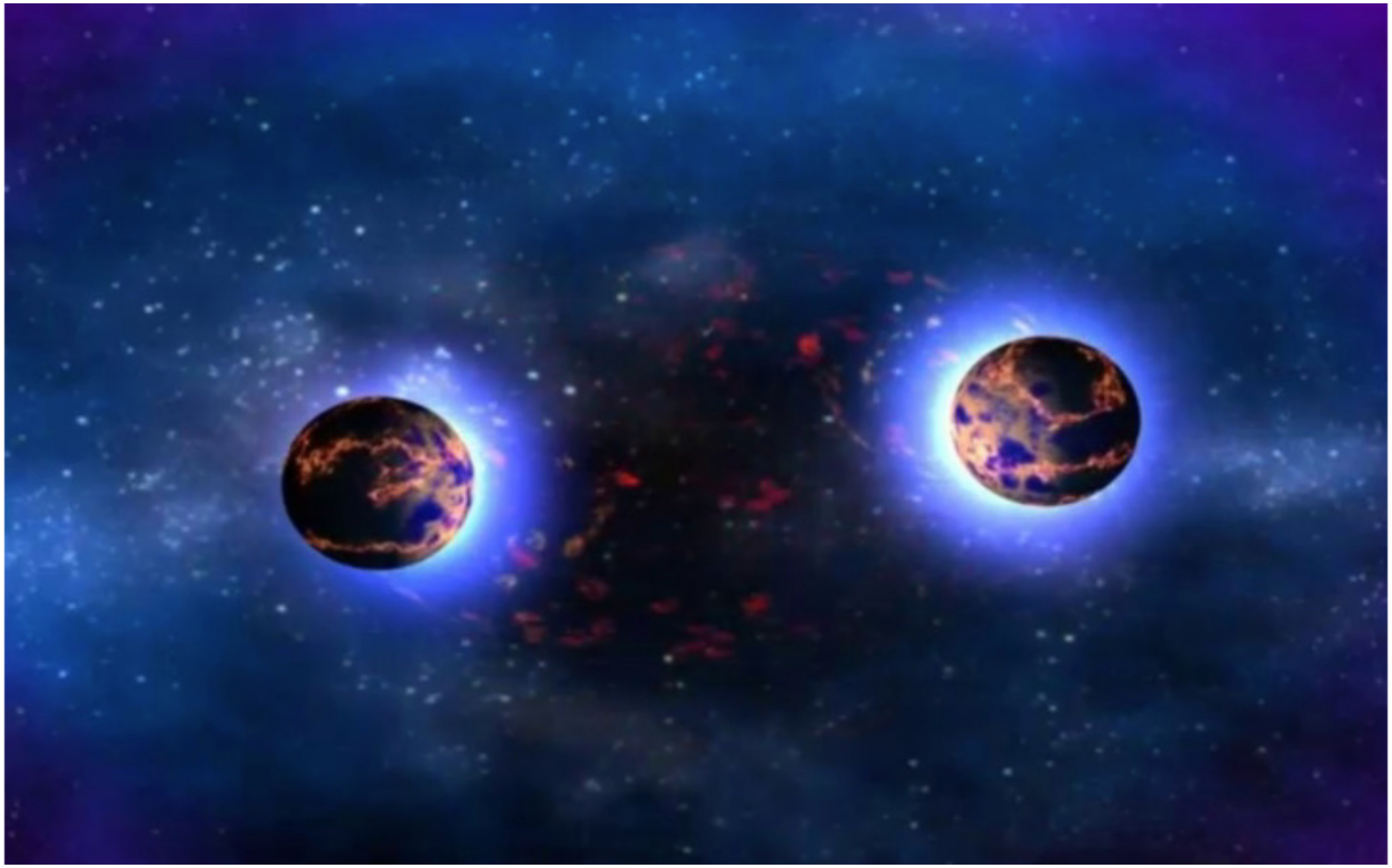
symmetric mass ratio

Distinguish between

- **subextremal $|Q| < M$**
- **extremal $|Q| = M$**

Latter are QM stable with

- **Zero Hawking temperature**
- **Often extra symmetry**
- **Vafa-Strominger entropy counting**
- **No force condition between extremal**
- **Consider extremal + anti-extremal**



Extremal - anti-extremal two-body system:

$$\delta\phi = 12\pi\frac{M^2\mu^2}{L^2}.$$

Twice GR result...

Final ingredient: coupling to scalar.

No additional mass/charge parameter but coupling strength in bulk action:

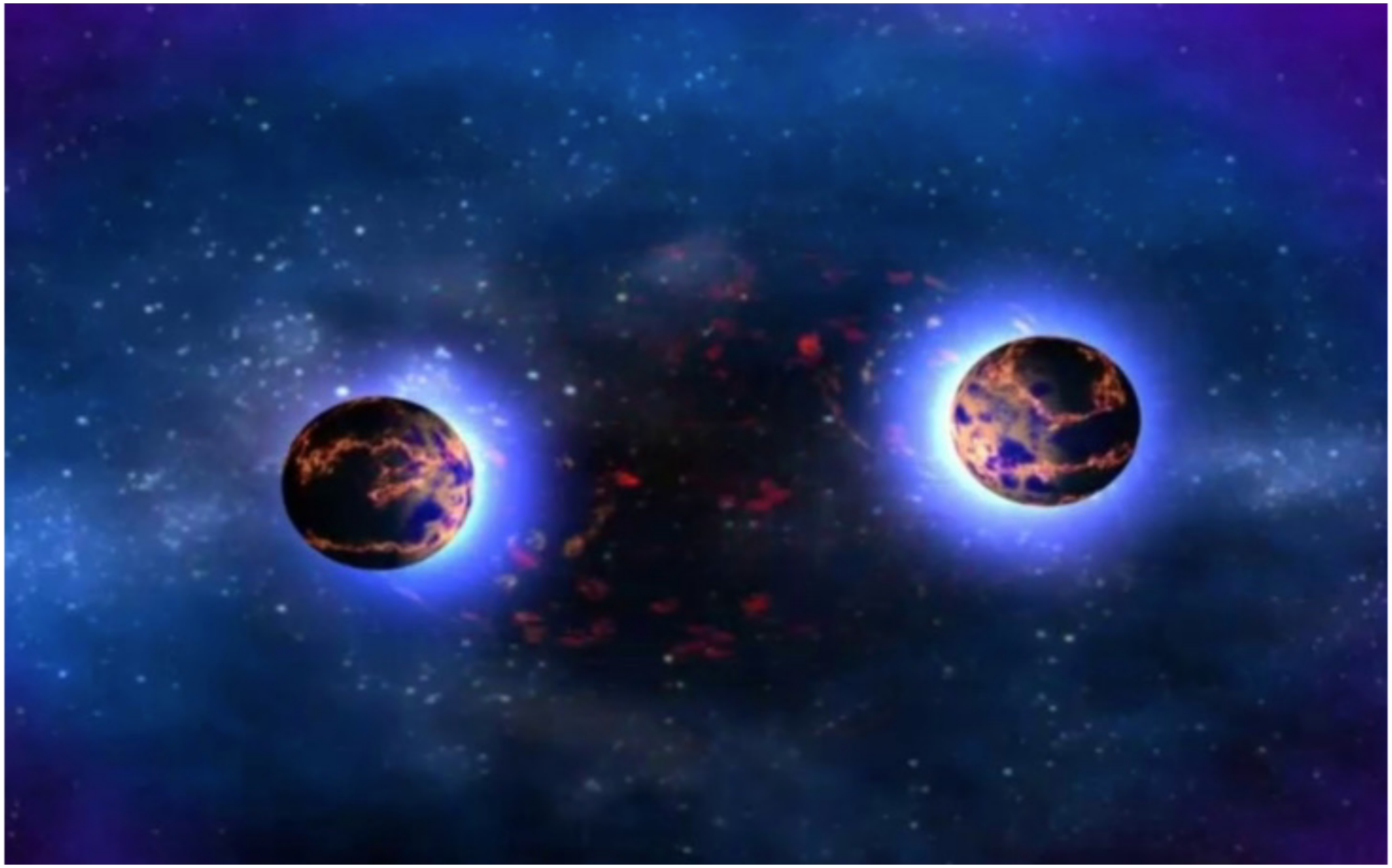
$$S = \int \mathrm{d}^4x \sqrt{-g} \left(R - 2(\partial\phi)^2 - e^{-2a\phi} F^2 \right)$$

Particle action fully fixed in terms of mass and charge:

$$L_{pp} = \mathfrak{m}(\phi)\sqrt{-g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} + qA_\mu \dot{x}^\mu .$$

$$\mathfrak{m}(\phi) = \sqrt{\mu^2 + \frac{1}{2} q^2 e^{2\phi}}, \qquad \mu^2 = m^2 - \frac{q^2}{2},$$

[Julie 2017 & Khalil, Sennett, Steinhoff, Vines, Buonnano - 2018]



(Anti-)extremal BHs with dilaton hair:

$$\delta\phi = 4\pi(1+a^2)(3-a^2)\frac{M^2\mu^2}{L^2}.$$

Now we're talking!

Structure of two-body system

$$H_{1PN} = -\frac{H_0^2}{\mu}(3A + 4B + 2C) + \{H_0, Y\} + \frac{\gamma^2}{\mu^2 r^2}(A + 2B + C + D)$$

$$Y = 2H_0(\mathbf{p}\cdot\mathbf{r})(A + B + C) - \gamma\{H_0, r\}C.$$

$$\mathbf{A}_1 = \{\mathbf{A}_0, Y\},$$

Hamiltonian and Runge-Lenz vector

$$\begin{aligned}\{\mathbf{A}, H\} &= \{\mathbf{A}_0, H_1\} + \{\mathbf{A}_1, H_0\} \\ &= \left\{\mathbf{A}_0, \frac{\gamma^2}{\mu^2 r^2}\right\}(A + 2B + C + D) + \{\mathbf{A}_0, \{H_0, Y\}\} + \{\{\mathbf{A}_0, Y\}, H_0\} \\ &= \left\{\mathbf{A}_0, \frac{\gamma^2}{\mu^2 r^2}\right\}(A + 2B + C + D) - \{Y, \{\mathbf{A}_0, H_0\}\},\end{aligned}$$

[Caron-Huot, Zahraee - 2018]

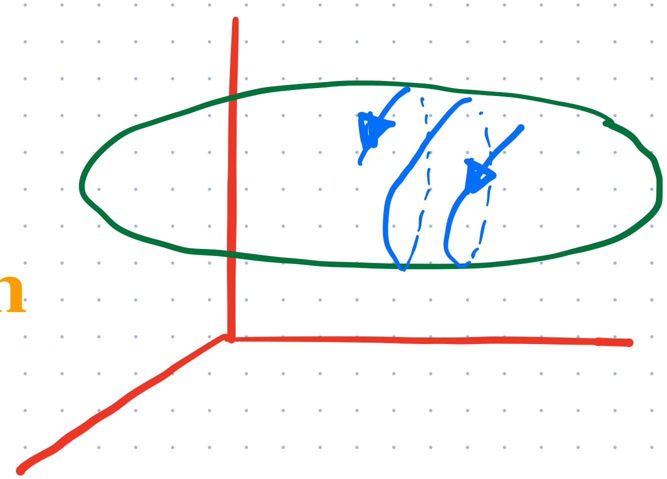
Structure of one-center system

$$\mathcal{H} = \frac{\mathbf{p}^2}{2} - 2E^2\frac{M}{r} + \\ + \varepsilon\left[-\frac{E^2 M}{2r} + \frac{\mathbf{p}^2}{8}\log\left[1+\frac{4M}{r}\right]\right] + \qquad + \mathcal{O}(\varepsilon^2),$$

where we expanded in the deviation $\varepsilon = 3 - a^2$ from the Kepler case.

Deviation away from a^2 = 3 introduces curved metric for kinetic term - all extremal one-center systems are curved Kepler

Four constants of motion, super integrable. Free motion on deformed three-sphere?



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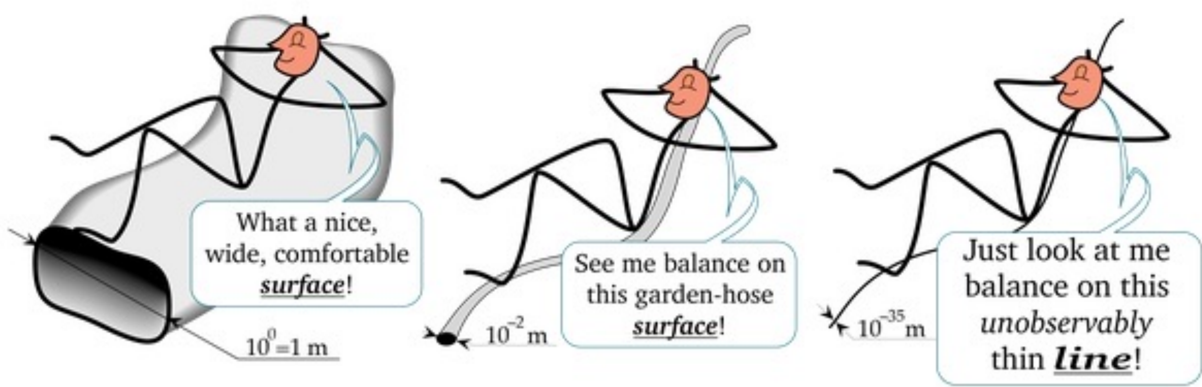
Hidden symmetry?

In the classical mechanics case: SO(4) symmetry. What about relativistic case?

Remarkably: the a^2 = 3 case follows from a 5D Lagrangian...



Hidden symmetry?

Similar phenomenon in much more complicated setting: N=8 supergravity (4D limit of string theory). Also vanishing perihelion shift for (anti-)extremal

Underlying symmetry from higher-dim origin?

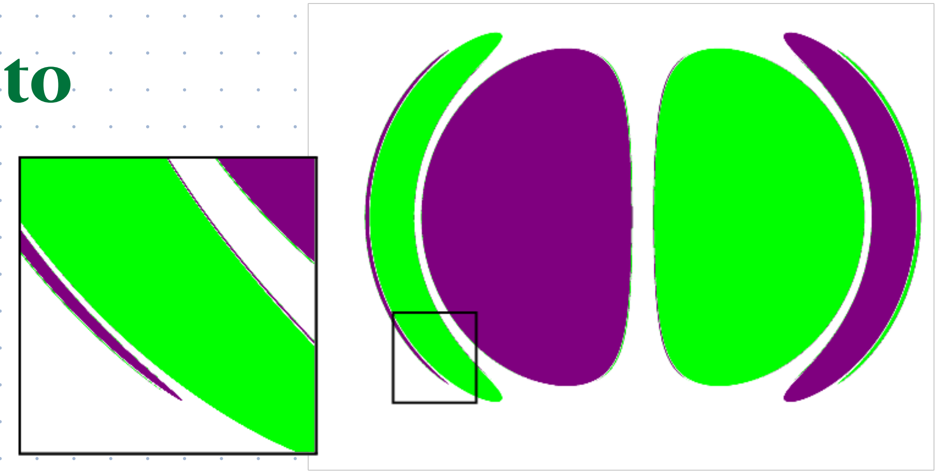
[Caron-Huot, Zahraee - 2018]

Two-center case

Consider multiple BH (no-force condition)

Two-center case: allows for mapping onto Kepler case when a^2 = 3.

Interesting relations to black hole shadows - chaotic and regular behavior



[Cornish & Gibbons - 1996, Shipley & Dolan - 2017]

Summary

- Relativistic corrections to Kepler
- Special case: extremal BH with dilaton hair and $a^2 = 3$
- Maps onto Kepler at 1PN of two-body and at all PN of one-center
- What about higher orders?
- Origin of hidden symmetry?
- What happens to phase space structure away from special case?
- Two-center case maps onto Newtonian

Many thanks to all of you!