Black-hole-like objects, Saturons, in an integrable model†

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IESC Cargèse, May 13

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† Based on Dvali, Sakhelashvili Arxiv 2111.03620

Unitarity in QFT and entropy[†]

- At tree level pertubative series break

$$N\alpha \sim 1$$

- Baryons, Solitons and etc are formed from N quanta

$$M \propto \frac{1}{\alpha}$$

Entropy:

$$S_\alpha = \frac{1}{\alpha}$$

Maximal

$$S \leq S_\alpha$$

[†]Dvali, Arxiv 1906.03530

Area Law and Saturons†

- Baryons with a lot of flavours N_f

Classically can not distinguish

$$S \sim S_{max}$$

$$S = (Rf)^{d-1}$$
$$t_p = \frac{R}{\alpha} = SR$$

$$S = R^2 f_\pi^2 \sim \frac{Area}{G}$$
$$S = \frac{1}{\alpha}, \ \alpha = \frac{q^2}{f_\pi^2}$$

Bonus:

$$S_{BEK} = 2\pi MR = S = S$$

† Dvali, Arxiv 1907.07332

Scattering and Saturon[†]

Macro same

$n = \sum_j n_j \leftarrow j$ flavor

$n_{st} \sim \binom{n + n_f}{n_f}$

$S = \log(n_{st})$

Micro flavor f

$\sigma = \sum_{n_s t} \sigma_{2\to n} =$

$\sigma_{2\to n} n_{st} = \sigma_{2\to n} e^S$

$\sigma_{2\to n} \propto e^{-\frac{1}{\alpha}}$

$S = \frac{1}{\alpha} \leftarrow$ Saturon

[†]Dvali, Arxiv 2003.05546

The Gross-Neveu (GN) model[†]

$$S = \int d^2x \bar{\psi} i \partial\!\!\!/ \psi + \frac{\alpha}{2} \left(\bar{\psi}\psi\right)^2$$

$\psi_j \rightarrow$ Color/flavor

$\psi \rightarrow \gamma_5 \psi$

$$\mathcal{L}_\sigma = \bar{\psi} i \partial\!\!\!/ \psi - \frac{1}{2\alpha}\sigma^2 - \sigma\bar{\psi}\psi$$

$\psi \rightarrow \gamma_5 \psi$

$\sigma \rightarrow -\sigma$

similar model in $1+3$ [‡]

[‡]Dvali, Kaikov, Valbuena Bermúdez, Arxiv 2112.00551

[†]Phys. Rev. D10, 3235

Large N in GN

$$\lambda = \alpha N$$

λ fixed $N \to \infty$

$$\mathcal{L}_{eff}(\sigma, \lambda, N) = N\tilde{\mathcal{L}}(\sigma, \lambda)$$

$$V = \frac{N}{2\lambda}\sigma^2 - \frac{N}{4\pi}\sigma^2 \left(\log\left(\frac{\sigma^2}{\mu^2}\right) - 1\right)$$

$\sigma \to -\sigma$ is still symmetry $\sigma_0 = \pm\mu e^{-\frac{2\pi}{\lambda}}$ SSB!

Asymptotic freedom:

$$\beta(\lambda) = -\frac{\lambda}{2\pi}$$

$$m_f = \sigma_0$$

Bound states in GN[†]

$$\psi_j = \frac{1}{\sqrt{2}} \left(\psi_j^1 + i\psi_j^2\right)$$

ψ_j^α **Real**

$SO(2N)$ **fundamental**

What about n fermions?

Irreducible tensors

n different fermions

$n_{st} = \frac{(2N)!}{n!(2N-n)!}$

$M_n = m_f \frac{2N}{\pi} \sin\left(\frac{n}{N}\frac{\pi}{2}\right)$

$0 < n < N$

Particle multiples

[†] Phys. Rev. D12, 2443

Saturons in GN

- **Saturation point**
  $\lambda \sim 1$
  $\alpha \propto \frac{1}{N}$
- **Entropy**

$$S = \log(n_{st}) = \log \binom{2N}{n}$$

Let us take a Saturon $n \sim N$

$$S = 2N\log(2) + O(\log(N))$$

Area Law/Unitarity in GN

Saturons entropy:

$$S \propto 2N \sim \frac{2}{\alpha}\,(\lambda \sim 1)$$

$$S \propto \frac{Area}{G} = \frac{1}{\alpha}$$

$R^0 \sim 1$

Area in 2D

"Newtons Constant"

$G \sim \alpha \sim \frac{1}{N}$

A Bonus:

$R = \frac{1}{2m_f}$†

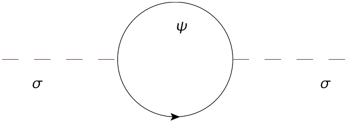
$M_N = 2m_f\frac{N}{\pi}$

$$S_{BEK} = 2\pi M_N R = 2N$$

† 2 fermions, see next slides

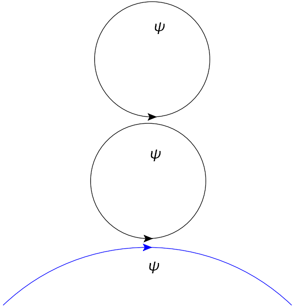
Scattering amplitudes in GN - I

$$\mathcal{L} \supset \sigma \bar{\psi}\psi$$



$$\propto \alpha N = \lambda$$

$$\mathcal{L} \supset \frac{\alpha}{2} \left(\bar{\psi}\psi\right)^2$$



$$\propto \lambda^2$$

Scattering amplitudes in GN - II

$2_f \to$ *Saturon or*
2 Saturons?

$i \ \psi$
$j \ \bar{\psi}$

$$\langle i_n, j_n| \frac{1}{n!} \frac{1}{2^n} \alpha^n \left(: \bar{\psi}\psi\bar{\psi}\psi :\right)^n |0\rangle$$

i contraction, $n!$(*each vertex*) $\times 2^n \leftarrow 2$ *different* $\bar{\psi}$.

$$: \mathcal{L}_I :: \mathcal{L}_I : ... : \mathcal{L}_I :: \mathcal{L}_I :$$

$n!$ contraction. No factor for j contraction (flavor conservation)!

$$amp \propto \frac{1}{n!} \frac{\alpha^n}{2^n} n! 2^n n! \propto \alpha^n n!$$

We have factors from Order, coupling, i contraction, $\mathcal{L}_I$ contraction.

GN and Saturons

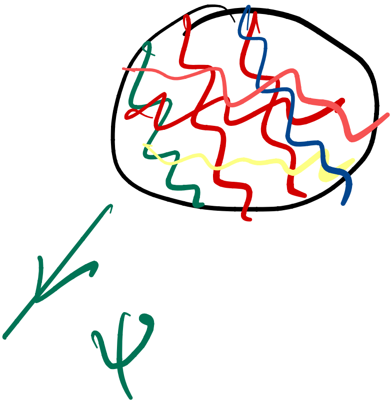
Let's take a Saturon $n \sim N$, at $\lambda \sim 1 \to \alpha \sim \frac{1}{N}$.

$$|Amp|^2 \propto \alpha^{2n}(n!)^2 \propto \left(N!N^{-N}\right)^2 \simeq e^{-2N} = e^{-S^*} = e^{-\frac{2}{\alpha}}$$

Micro entropy	$S = \log n_{st} = 2N \log(2)^*$
Coupling entropy	$S = \frac{2}{\alpha} = 2N$
Amplitude scaling	$\lvert Amp\rvert^2 \propto e^{-2N}$
Cross section	$\sigma \propto e^{\frac{2}{\alpha}} n_{st} \sim 1$
Bekenstein entropy	$S = M_N R = 2N$
Area Law entropy	$S = 2N$

*Remark $C_\lambda = 1 - \log(2) \simeq 0.3$ (Off-shell)

GN and Page's time



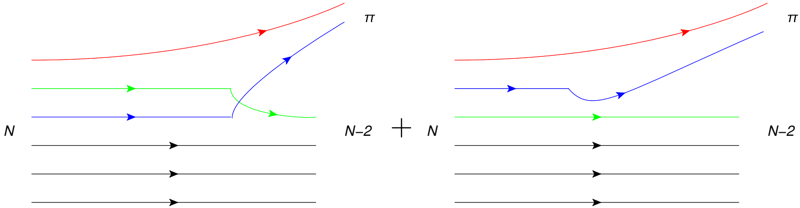
$$\Gamma \propto |Amp|^2 \propto \alpha^2 m_f \sim \frac{m_f}{N}$$

$$t_p \sim \frac{1}{\Gamma} \sim \frac{N}{m_f} \sim SR$$

Hawking evaporation in GN

Pion Field $\pi_{ij} = -\pi_{ji}$, $SO(2N)$ indices.

$$\mathcal{L} \supset \frac{m_f}{f_\pi} \pi_{ij} \bar{\psi}_i \gamma_5 \psi_j$$



$f_\pi \sim \sqrt{N}$ t'Hooft.

$$\Gamma_\pi \propto \frac{1}{f_\pi^2} m_f N \alpha^2 N^2 \propto m_f \propto \frac{1}{R} \propto T$$

The Boltzmann factor

$$amp^2 \propto e^{-2N} = e^{-M_N R} = e^{-\frac{M_N}{T}}$$

Thanks for listening