Ultraviolet completion of Starobinsky inflation

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Starobinsky model of inflation [15]

$$\hat{\mathcal{L}} = \frac{1}{2}\left(\hat{R} + \alpha \hat{R}^2\right)$$

Lagrange multiplier ϕ : $\hat{\mathcal{L}} = \frac{1}{2}(1+2\phi)\hat{R} - \frac{1}{2\alpha}\phi^2$ $\qquad \phi = \alpha \hat{R}$

Rescaling the metric to the Einstein frame $(\hat{g} \to g) \Rightarrow$ [10]

equivalent to a scalar field with exponential potential:

$$\mathcal{L} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - \frac{3}{4}m^2\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \qquad m^2 = \frac{1}{6\alpha} \quad \text{mass at } \phi = 0$$

- predictivity: one parameter $\Rightarrow$ fits the data

  amplitude of the power spectrum fixes $m \simeq 10^{-5} M_p \Rightarrow$ [7]

  $n_s \simeq 0.965 \quad r \simeq 0.0035$

- minimal coupling to matter with $\hat{g}$ fixes the reheating temperature

  $\mathcal{L} + \mathcal{L}_{\text{matter}} \Rightarrow \phi$ couples to scalar kinetic terms $\Rightarrow T_{\text{RH}} \simeq 3 \times 10^9$ GeV

Starobinsky model of inflation: problems

- validity of EFT at energies below $H = m/2$
  1. suppression of higher order operators e.g. R^3 etc
  2. large field super-Planckian excursion to guarantee ~ 60 e-folds
  3. microscopic origin
- deviation with present data?

1. study effects of R^3 or in general $f(R)$

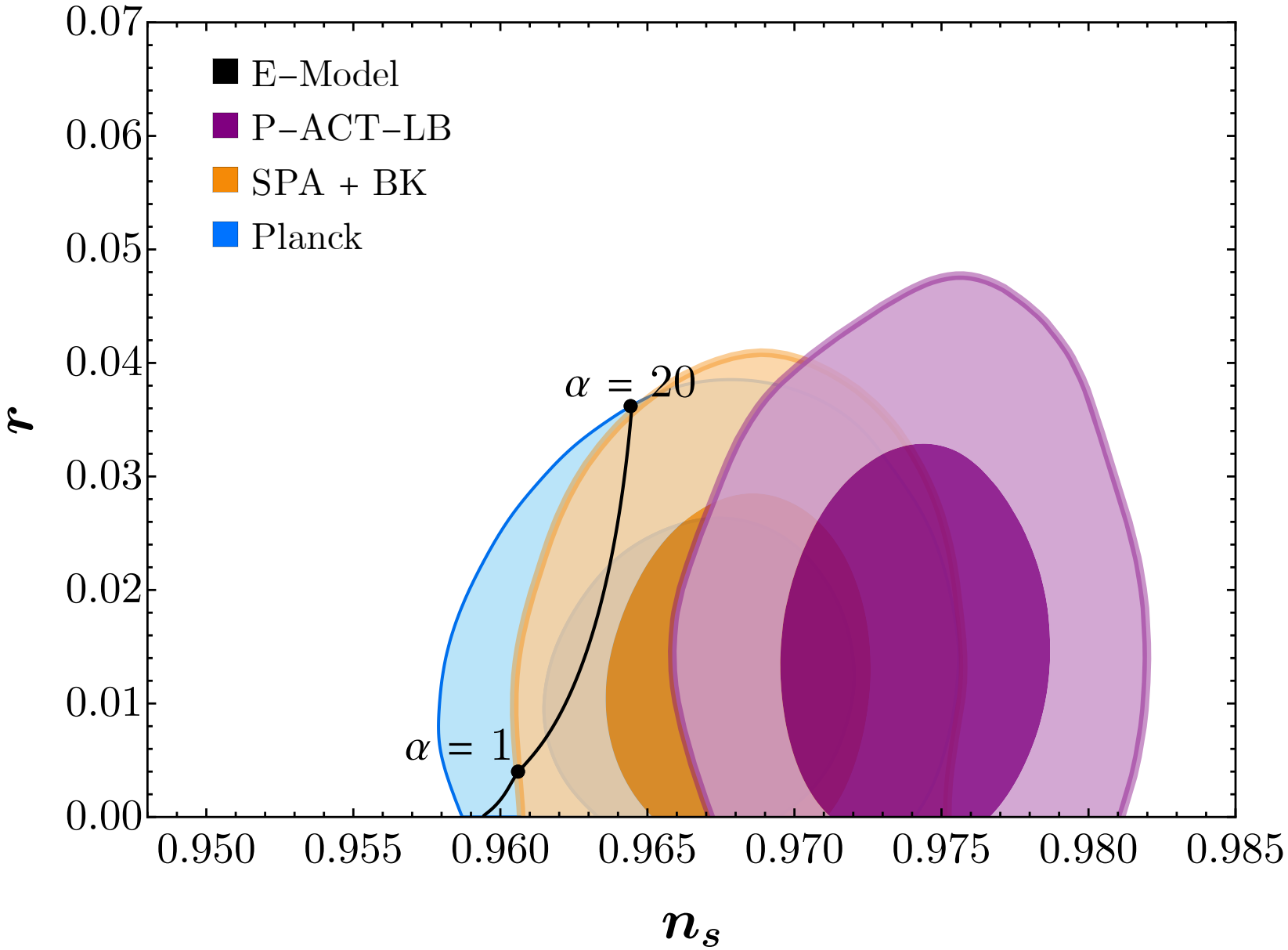
$\Rightarrow$ 2. use swampland program

3. supersymmetric extension and string embedding

Based on:

- I.A.-Nanopoulos-Olive 2410.16541 (JHEP)
- I.A.-Ellis-Ke-Nanopoulos-Olive 2504.12283 (JCAP)
- I.A.-Markou-Nanopoulos 2605.05295 (JCAP)
- I.A.-Ellis-Nanopoulos-Olive-Verver 2606.31975

Starobinsky model comparison: Planck, ACT, SPT



Swampland Program

Vafa '05, Ooguri-Vafa '06

Not all effective field theories can consistently coupled to gravity

- anomaly cancellation and unitarity/causality constraints are not sufficient

- consistent ultraviolet completion can bring non-trivial constraints

those which do not, form the 'swampland' - the rest form the 'landscape'

criteria $\Rightarrow$ conjectures

supported by arguments based on string theory and black-hole physics

Some well established examples:

- No exact global symmetries in Nature
- Weak Gravity Conjecture: gravity is the weakest force

  $\Rightarrow$ minimal non-trivial charge: $q \geq m$ in Planck units $8\pi G = \kappa^2 = 1$

  Arkani-Hamed, Motl, Nicolis, Vafa '06

Distance/duality conjecture (SDC)

At large distance in field space $\phi \Rightarrow$ tower of exponentially light states

$m \sim e^{-c\phi}$ with $c \sim \mathcal{O}(1)$ +ve parameter in Planck units

Tower of states

- provides a weakly coupled dual description up to the species scale

$$M_* = M_P/\sqrt{N}$$ Dvali '07

- tower can be either

1. a Kaluza-Klein tower (decompactification of d extra dimensions)

   $m \sim 1/R\,, \quad \phi = \ln R; \quad M_* = M_P^{(4+d)} = (m^d M_P^2)^{1/(d+2)}$

   $N = (M_* R)^d$

2. a tower of string excitations

   $M_* = m \sim$ the string scale $= g_s M_P\ ; \quad \phi = -\ln g_s\,, \quad N = 1/g_s^2$

emergent string conjecture Lee-Lerche-Weigand '19

Starobinsky inflation: large field → SDC applies

Inflaton potential:

$$V \sim \frac{3}{4} m^2 (1 - e^{-\sqrt{2/3}\,\phi})^2$$

pivot scale leading to the observed value of spectral index: $\Rightarrow \phi_* \simeq 5.3$

field excursion until the end of inflation:

$n_s \approx 1 - 2/N_*$ ← number of e-folds $\quad \Rightarrow \Delta\phi \simeq \sqrt{2\epsilon}\, N_* \approx 5$

estimate of the mass scale of the tower: $m_T \sim e^{-\Delta\phi} \simeq 10^{-2} M_p$

- Note: tower contains massive spin-2 fields (KK gravitons or string modes)

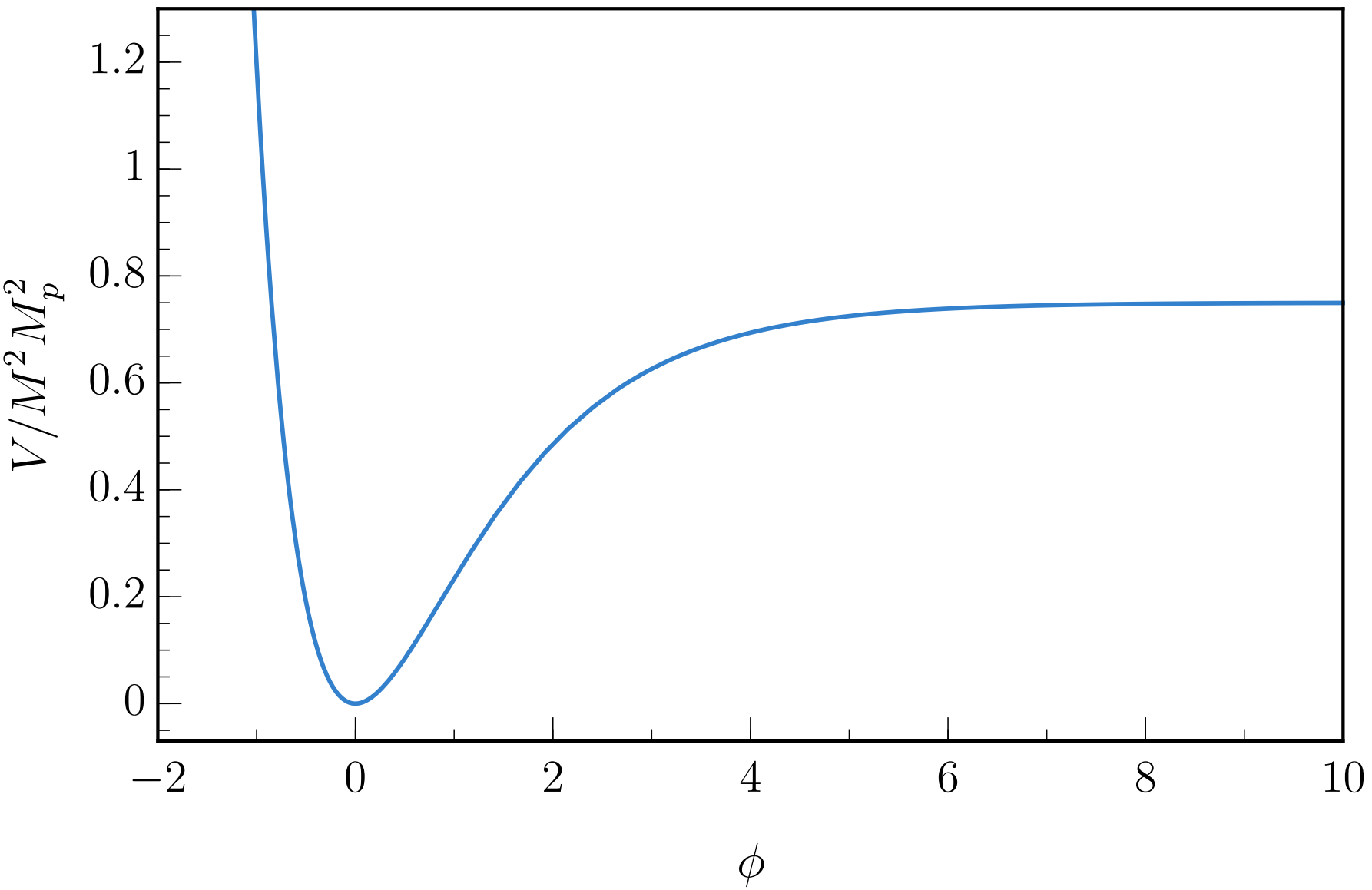
  unitarity in (approx) de Sitter: $m^2_{\text{spin}-2} \geq 2H^2$ Higuchi '87 [2]

  $\Rightarrow m_T \gtrsim m$

  max variation ($m_T \simeq m$): $m_{\max} \simeq 10^{-4} M_p \quad \Delta\phi_{\max} \simeq 10!$

  $\Rightarrow$ initial conditions: why $\phi_i \sim \phi_*$?

Starobinsky inflation: potential



Starobinsky model: supersymmetric extension [17]

- scalar fields in chiral multiplets $\Rightarrow$ complexification
- R upper component of chiral superfield $\mathcal{R} \Rightarrow \mathcal{R}^2$ does not contain R^2

  use $\mathcal{R}\bar{\mathcal{R}} \Rightarrow$ need two Lagrange multipliers

$\Rightarrow$ 2 chiral superfields: one $\ni$ the scalaron and the other the goldstino

Goldstone fermion of spontaneous supersymmetry breaking

$$K = -3\ln(T + \bar{T} - C\bar{C}) \quad ; \quad W = 3mC(T - \frac{1}{2})$$

Cecotti '87, I.A.-Dudas-Ferrara-Sagnotti '14

- $2T = e^{\sqrt{\frac{2}{3}}\phi} + ia$, $C = 0 \Rightarrow$

$$\mathcal{L}_{bos} = \mathcal{L}_{\text{Staro}}(\phi) - e^{-2\sqrt{\frac{2}{3}}\phi}\left(\frac{1}{2}(\partial a)^2 + m^2 a^2\right)$$ [11]

  $m_a^2 = 8H^2$ decouples during inflation

- SUSY is broken during inflation with $C \sim \mathcal{R}$ the goldstino superfield

  however C is unstable $\Rightarrow$ modify K

string dilaton cures the s-goldstino instabiity

$$\mathcal{L}_{\text{string frame}} = e^{-2\varphi}\left\{\frac{1}{2}R[G] + 2(\partial\varphi)^2 + \mathcal{L}_{\text{Staro}}(\phi)\right\} \qquad \langle e^{\varphi}\rangle = g_s$$

Einstein frame:

$$G = e^{2\varphi}g \Rightarrow \mathcal{L}_{\text{string,E}} = \left\{\frac{1}{2}R[G] - (\partial\varphi)^2 - \frac{1}{2}(\partial\phi)^2 - e^{2\varphi}V_{\text{Staro}}(\phi)\right\}$$

Jordan frame: $g \to \hat{g}$ [2], then string Jordan frame: $\hat{g} = e^{-2\varphi}\hat{G}$

$$\Rightarrow \hat{\mathcal{L}}_{\text{string frame}} = e^{-2\varphi}\left\{\frac{1}{2}R[\hat{G}] + 2(\partial\varphi)^2 + \alpha\left[\hat{R} - 2(\partial\varphi)^2\right]^2\right\}$$

SUSY: $S = e^{-2\varphi} + ib$ and $K \to K + \delta K$ with $\delta K = -\ln(S + \bar{S})$

$\Rightarrow m_C^2 > 0$ for any T: C can be set to zero!

assuming a mechanism of dilaton stabilisation

string realisation: flipped $SU(5) \times U(1)$ model

I.A.-Nanopoulos-Rizos '21, '22

$$K_{\text{string}} = -\ln(S+\bar{S}) - 2\ln\left(1-|y|^2\right) - 2\ln\left(1-\frac{1}{2}|z|^2\right)$$

$$W = M\, z\, y(1-\lambda y) \quad [17]\ [18]$$

- K_{string}: exact in α'-expansion
- inflation scale M: generated at 6th order

  via singlet VEVs cancelling the anomalous $U(1)_A$ contribution in D-terms

Field redefinition: $y = \frac{2T-1}{2T+1} \Rightarrow$ $\Rightarrow\ M \sim 10^{-6} M_p \simeq 10^{12}$ GeV

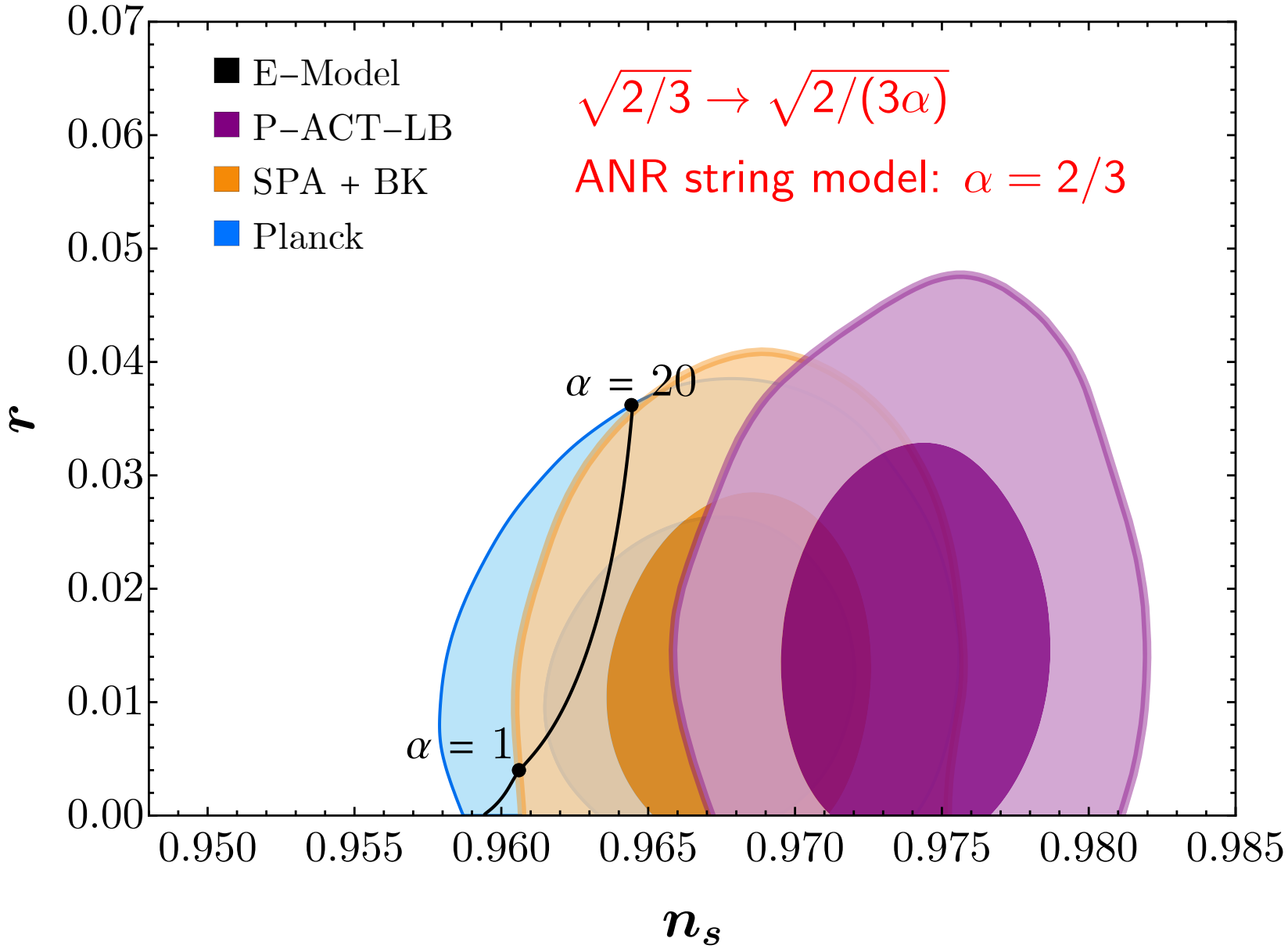
$$K_{\text{string}} = -\ln(S+\bar{S}) - 2\ln\left(1-|z|^2/2\right) - 2\ln(T+\bar{T})$$

$$W = Mz\left[(T-1/2)+(1-\lambda)(T-1/2)^2\right] \quad [13]$$

$\Rightarrow\ V(z=0)|_{\lambda=1} = V_{\text{Staro}}(\phi)$ with $\sqrt{2/3} \to 1$ [9] [14]

normalised $\phi = \ln(\text{Re}\, T)$ large $\Rightarrow$ decompactification to 6 dims $(y \to 1)$

α-Starobinsky models vs Planck, ACT, SPT



Inflaton excursion implies KK tower of 2 dims

Kinetic terms, $T \equiv R^2 + ib$ in string units:

$$-2\frac{\partial T \partial \bar{T}}{(T+\bar{T})^2} = -2\left(\frac{\partial R}{R}\right)^2 - \frac{1}{2}\frac{(\partial b)^2}{R^2}$$

Dimensional reduction from $(4+d)$ dimensions:

$$\frac{1}{2}\mathcal{R}^{(4+d)} \longrightarrow \frac{1}{2}\mathcal{R}^{(4)} - \frac{d(d+2)}{4}\left(\frac{\partial R}{R}\right)^2$$

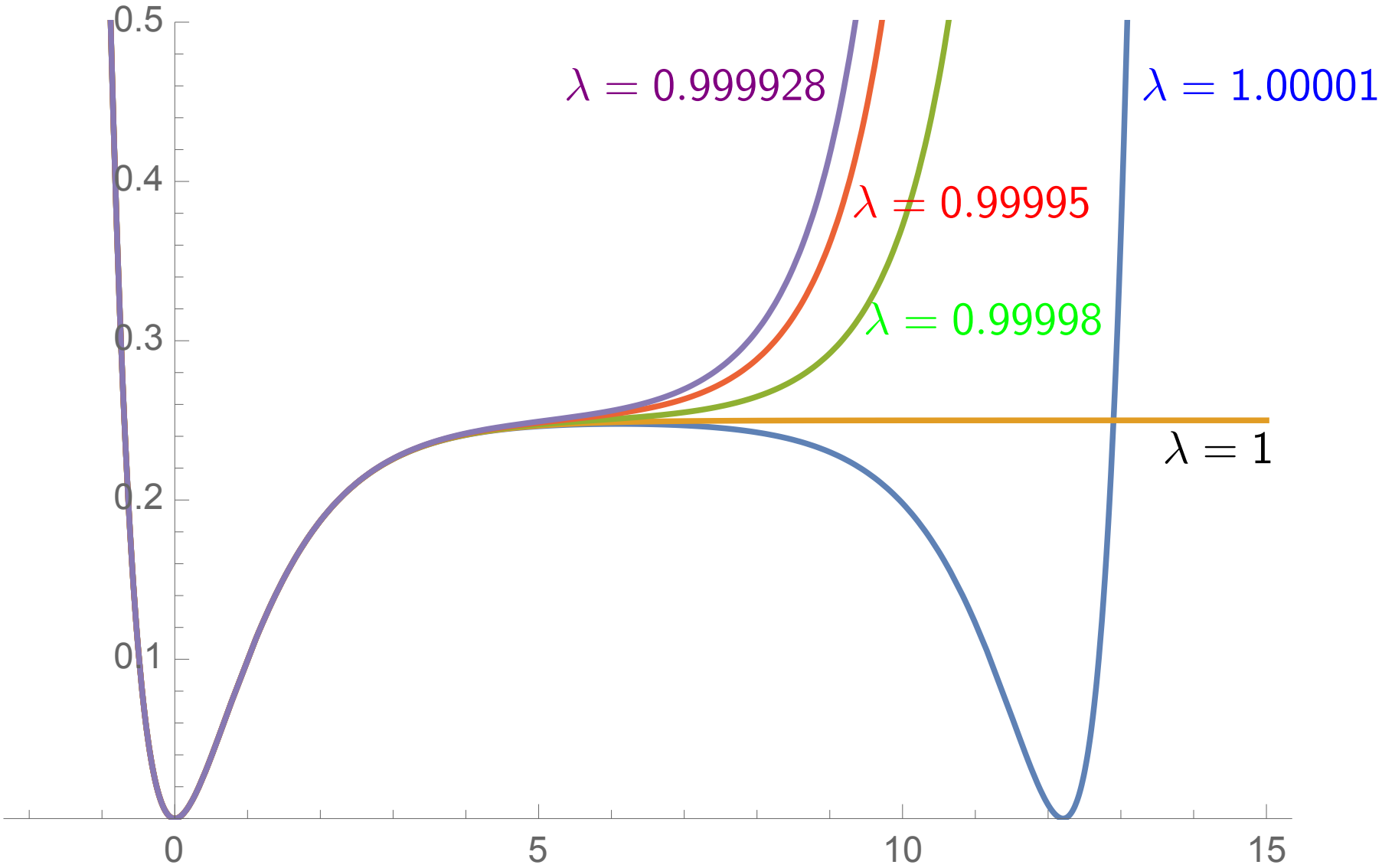
agreement $\Rightarrow d = 2$ and $RM_s \sim e^{\phi_*/2} \simeq e^{2.5}$ $\quad M_s =$ string scale

$$M_p^2 = \frac{1}{g_s^2} M_*^2 (RM_*)^d \quad \Rightarrow \quad RM_p = \frac{1}{g_s}(RM_*)^{2+d/2} \sim 10^4 \text{ for } d = 2$$

- KK tower starts at $m_{\mathsf{T}} \sim 10^{14}$ GeV $>$ inflation scale $M \sim 10^{12}$ GeV

note: $\phi_* \simeq 6 \Rightarrow m_{\mathsf{T}} \sim M$! [11]

initial conditions of inflation and $\lambda \neq 1$



what is λ?

higher order corrections: $F(R) = R + Rf(R)$ [17]

$$\hat{\mathcal{L}} = \frac{1}{2}F(\hat{R}) \qquad \text{Lagrange multiplier } \hat{\phi} \equiv F'(X) \text{ with } X = \hat{R} \Rightarrow$$

$$\hat{\mathcal{L}} = \frac{1}{2}\left[F(X) + F'(X)(\hat{R} - X)\right] = \frac{1}{2}\left[\hat{\phi}\hat{R} - \hat{\phi}X(\hat{\phi}) + F(X(\hat{\phi}))\right]$$

Einstein frame: $\hat{g} = \frac{1}{\hat{\phi}}g$ and normalised field: $\hat{\phi} = e^{\sqrt{2/3}\phi} \Rightarrow$ [2]

$$V(\phi) = \frac{X(\phi)F'(X(\phi)) - F(X(\phi))}{2\left[F'(X(\phi))\right]^2} = \frac{X(\phi)^2 f'(X))}{\left[1 + f(\phi) + X(\phi)f'(X)\right]^2}$$

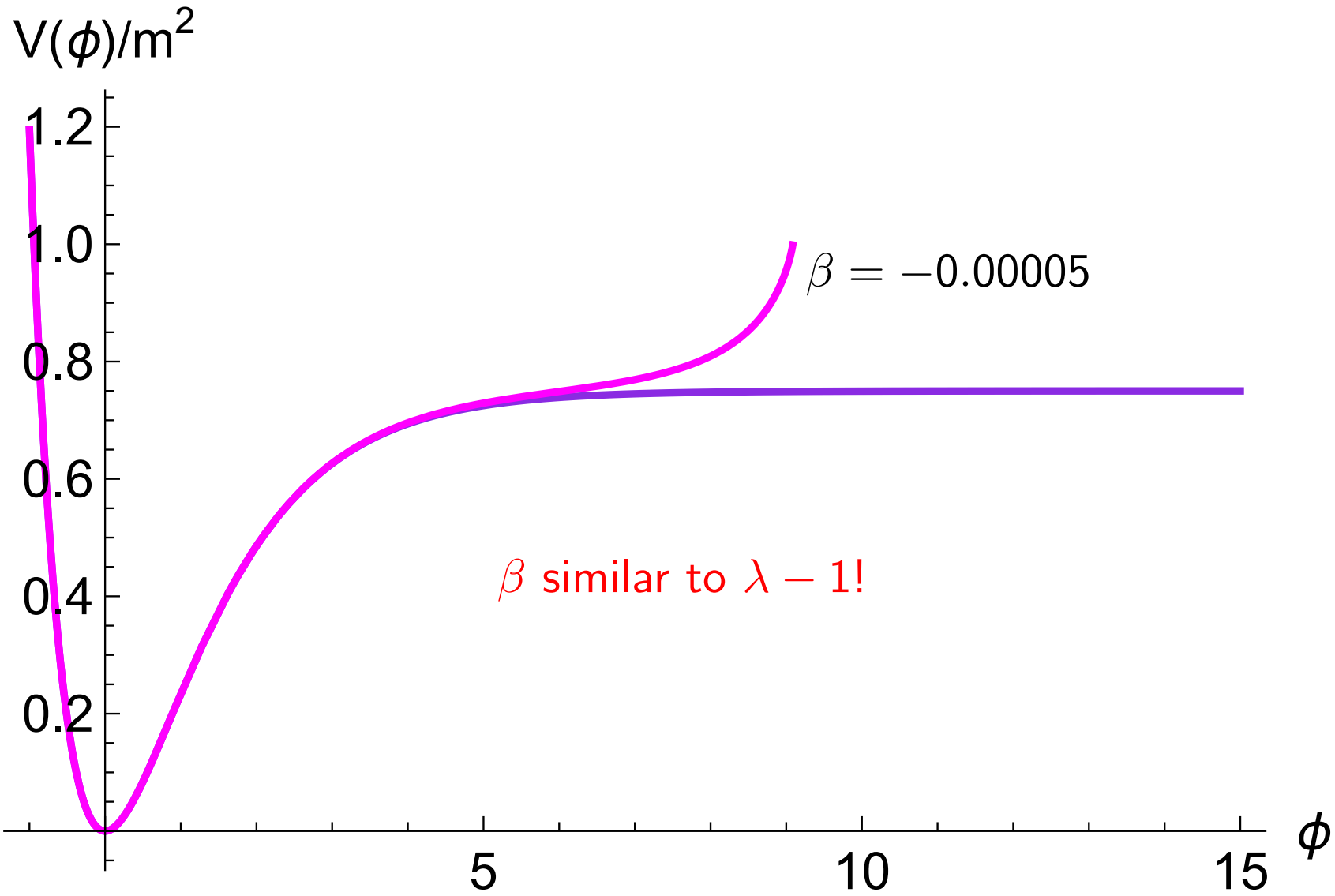
$F' > 0, F'' > 0$ (no ghosts and tachyons); $\quad V > 0 \Rightarrow f' > 0$

example: $F(R) = R + \alpha R^2 + 4\alpha^2\beta R^3 = R + \frac{1}{6m^2}R^2 + \frac{1}{9m^4}\beta R^3$

m: EFT scale; $\quad \beta$: dimensionless parameter

Gialamas, Katsoulas, Tamvakis '25

R^3 deformation



$F(R)$ supergravity [15] [9]

$$K = -3 \ln \left\{ T + \bar{T} - C\bar{C} \left[f'(X) + \bar{f}'(\bar{X}) \right] \right\}$$

$$W = \sqrt{3} C \left\{ T - \frac{1}{2} - \frac{1}{2} \left[f(X) - X f'(X) \right] \right\}$$

positivity of $C\bar{C}$ matrix $\Rightarrow \mathrm{Re} f' > 0$

add string dilaton $\Rightarrow$ no C instability $\rightarrow \langle C \rangle = 0 \Rightarrow X$ is auxiliary:

$$1 - 2T + f(X) + X\bar{f}'(\bar{X}) = 0 \quad \Rightarrow \quad F'(X) = 2T - 1$$

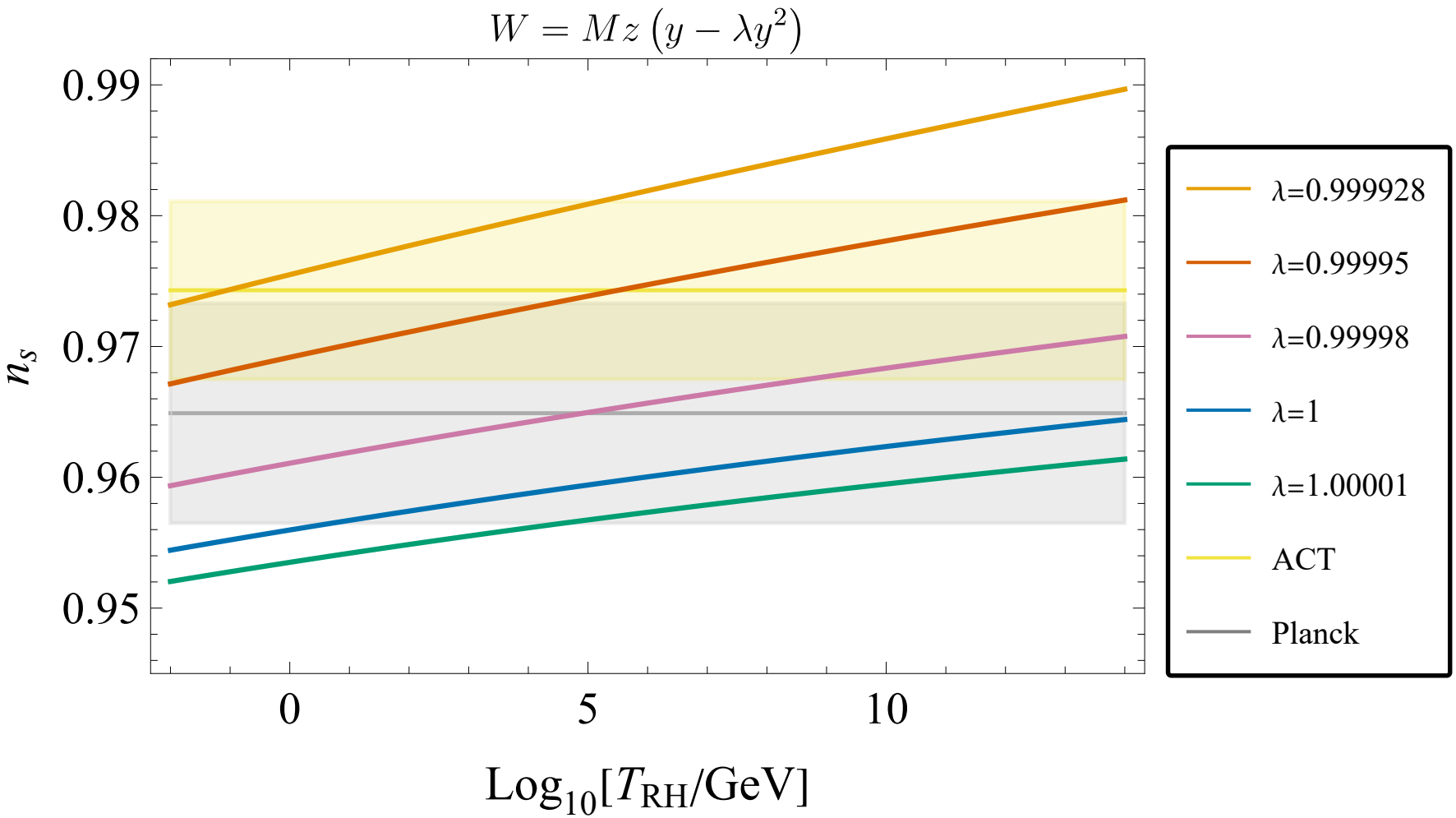
$\mathrm{Im}\, T$ heavy $\Rightarrow X^2 f'''(X) < 2 f'(X)$ (for X real) $\rightarrow \mathrm{Im}\, T = 0$

$C = \mathrm{Im}\, T = \mathrm{Im} X = 0 \Rightarrow V_{\mathrm{sugra}}(X) = V$ of $F(R)$ gravity with $\hat{\phi} = 2T$

string model effective SUGRA: $\lambda - 1 = 2\beta +$ series of R^n corrections [11]

power series of R in $f(R)$ $\leftrightarrow$ power series of $(T - 1/2)$ in $W(T)$

comparison with observations [11]



Conclusions

Ultraviolet completion of Starobinsky model:

- Swampland distance conjecture addresses the large inflaton variation

  but brings a problem of initial conditions
- SUSY extension: string dilaton cures the s-goldstino instability
- string theory embedding brings a dimensionless deformation parameter

  it explains the initial conditions if its magnitude is $\sim \mathcal{O}(10^{-5})$
- the deformation can be described as an R^3 correction
- construction of $F(R)$ supergravity
- the deformation allows better agreement with the recent data

  but calls for an explanation of its small value