A new standard for cosmology: the axially symmetric Szekeres model

Marie-Noëlle Célérier (LUX)



MNC PRD 110, 123526 (2024)

MNC PRD 114, 023510 (2026)

Hot topics in modern cosmology – Cargèse – September 2026

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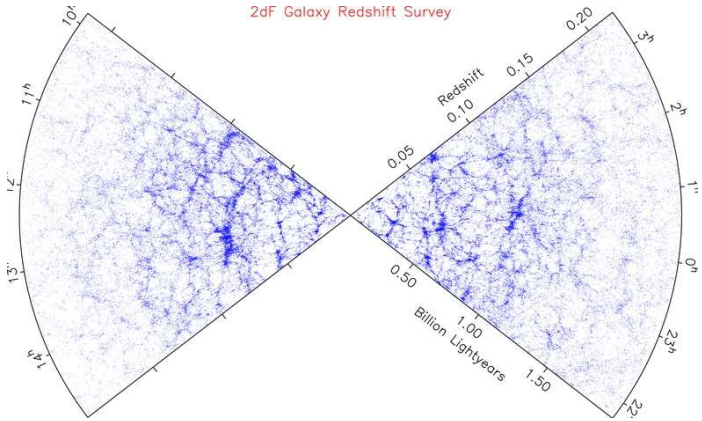
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Outline

1. Motivations and assumptions
2. The cosmological Szekeres model
3. Axial symmetry
4. Chebyshev parametrization
5. Conclusion

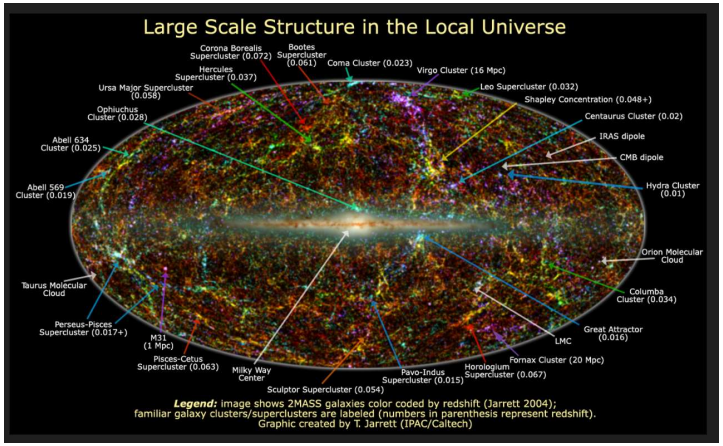
Motivations and assumptions

(In)homogeneity of the Universe



At large scales, the Universe is quasi-homogeneous

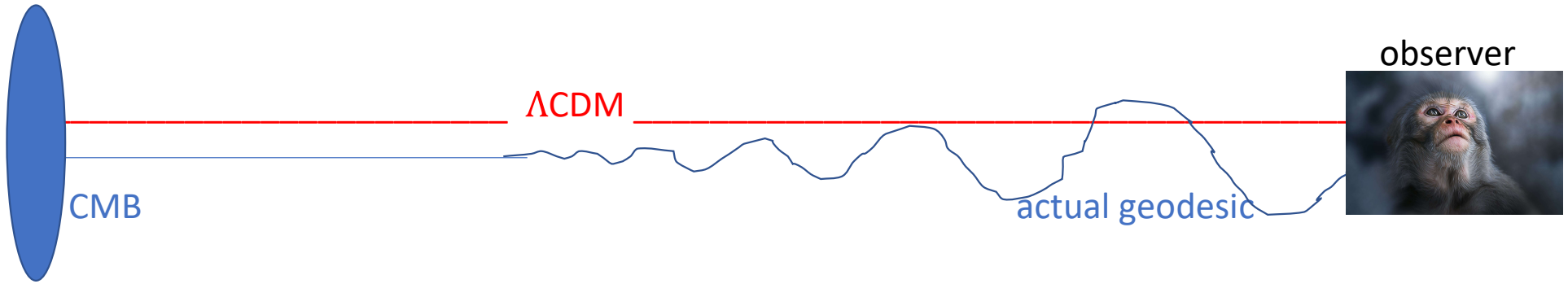
At small scales it is not



Cosmological tensions: $H_0 and\ S_8$

confirmed: H_0 * more controversial: S_8 - σ_8 / voids / local flow anomalies

Mostly due to discrepancies between large scale measurements transported to the observer through an homogeneous standard model and local measurements realized in a lumpy region of the Universe

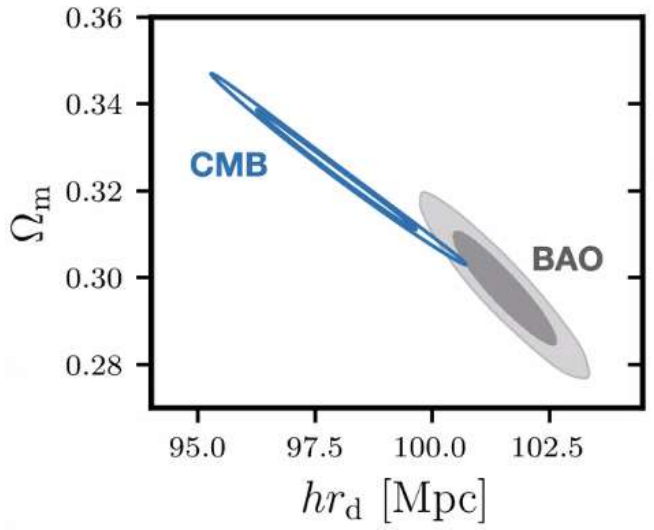


*See: H0DN Collaboration, A&A 708 (2026) A166 (7.1 σ H_0 tension)

CMB-BAO tension

CMB and BAO (DESI) data both used to infer the highly correlated parameters Ω_m and hr_d

CMB and BAO results differ at 2-4 σ



credit Camphuis et al. 2025

Why keep GR as a gravitation theory?

GR = the most successfully tested gravitation theory

1) weak field in the solar system: MICROSCOPE satellite (universality of free fall), precision: 1.3×10^{-15}
Touboul et al. PRL 129, 121102 (2022) +++

2) strong field at the Galaxy scale:
- the Double Pulsar (universality of free fall), precision: 6.3×10^{-5} Kramer et al. PRX 11, 041050 (2021)
- Pulsar PSR J0337+1715 in a triple stellar system (strong equivalence principle), precision: $\sim 10^{-6}$ Voisin et al. A&A 693, A143 (2025)

3) Inspiral Merger Ringdown (IMR) consistency test: nonlinear strong-gravity regime of black-hole mergers
Ghosh et al. CQG 35, 014002 (2018)

4) cosmological scale tests (MG parameters):
- a series of positive results by the DESI collaboration, the more precise tests at cosmological scales to date
Adame et al. JCAP 07(2025)028, Ishak et al. JCAP 09(2025)053, Gonzalez et al. arXiv: 2604.26915

Main observations

Mass dipole:

Rubart, Schwarz (2013)
Colin, Mohayaee, Ramez, Sarkar (2017)
Bengali, Maartens, Santos (2018)
Singal (2019a, 2019b, 2024)
Secrest et al. (2021, 2022, 2025a, 2025b)
Siewert, Schmidt-Rubart, Schwarz (2021)
Wagenveld, Klöckner, Schwarz (2023)
Böhme, Schwarz, Tiwari et al. (2025)
Wagenveld et al. (2025)

Axial expansion:

Bolejko, Nazer, Wiltshire (2016)
Colin, Mohayaee, Rameez, Sarkar (2019)
Krishnan, Mohayaee, O Colgain et al. (2022)
Luongo, Muccino, O Colgain et al. (2022)
Rameez (2025)
Sah, Rameez, Sarkar, Tsagas (2025)
Kalbouneh et al. (2026)

We consider only the late time inhomogeneities without modifying the earlier Universe

The cosmological Szekeres model

The Szekeres metric

Exact solution of GR for an irrotational dust sourced spacetime with no symmetry = matter dominated region of the Universe. In comoving and synchronous projective coordinates:

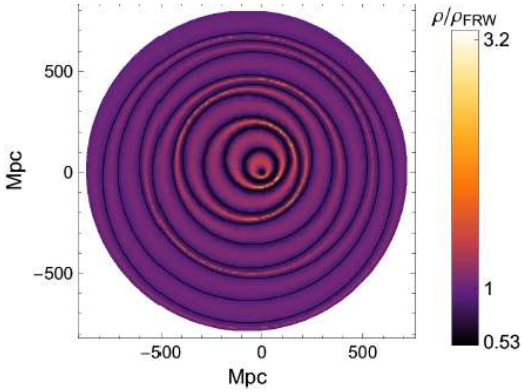
$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{\left(\Phi_{,r} - \Phi E_{,r}/E\right)^2}{\epsilon - k}\mathrm{d}r^2 + \frac{\Phi^2}{E^2}(\mathrm{d}p^2 + \mathrm{d}q^2)$$

$$\Phi(t,r) \qquad\qquad k(r)$$

$$E(r,p,q) = \frac{S}{2}\left[\left(\frac{p-P}{S}\right)^2 + \left(\frac{q-Q}{S}\right)^2 + \epsilon\right]$$

$$S(r), \quad P(r), \quad Q(r)$$

Szekeres geometry and mass dipoles



credit Buckley and Schlegel

- Quasi-spherical solutions ($\epsilon = +1$) = a set of non-concentric evolving spheres, with a dipole distribution of the energy density around each sphere

- Quasi-hyperbolic solutions ($\epsilon = -1$) = a set of evolving hyperboloids, with a pseudo-spherical dipole with possibly infinite magnitude

The strenght and orientation on each comoving slice in both classes are determined by $S(r), P(r), Q(r)$

- Quasi-planar solutions ($\epsilon = 0$) have no dipole-like structure

Einstein's equations with a cosmological constant

$$\Phi_{,t}^2 = \frac{2M}{\Phi} - k + \frac{\Lambda}{3}\Phi^2$$

integrated as

$$t - t_B(r) = \int_0^{\Phi} \frac{\mathrm{d}\tilde{\Phi}}{\sqrt{\frac{2M}{\tilde{\Phi}} - k + \frac{\Lambda}{3}\tilde{\Phi}^2}}$$

$$4\pi\rho(t,r,p,q) = \frac{M_{,r} - 3ME_{,r}/E}{\Phi^2(\Phi_{,r} - \Phi E_{,r}/E)}$$

A given Szekeres model is a priori determined by 6 functions of r: $k(r), S(r), P(r), Q(r), M(r), t_B(r)$, one quasi-constant parameter ϵ , a cosmological constant Λ, the location of the observer. The number of independent functions can be reduced to 5 by rescaling r through, e. g., a choice of any arbitrary function.

Essential property of the Szekeres solutions: they possess the FLRW model as a homogeneous limit

$S = 2\epsilon,\ P = Q = 0,\ \ \Phi = ra(t), k = k_0 r^2,\ \ r$ freedom $\Rightarrow\ t_B = const. = 0$.

$\Downarrow$

A new parameter: the homogeneity-inhomogeneity transition scale z_{trans}, depending on the desired accuracy.

Axial symmetry

Axially symmetric quasi-spherical Szekeres model

Axial symmetry => $P(r) = Q(r) = 0$

r -freedom => $t_B\,(r) = 0$

Finite dipole <=> finite hypersurface <=> quasi-spherical Szekeres (QSS) model

Model determined by 3 functions of r + the observer's location (t_0, r_0) on the symmetry axis + a cosmological constant Λ

To which we add the transition scale to the FLRW homogeneous limit: r_{trans} (fixed a priori depending on the data set depth and on the desired precision)

Chebyshev parametrization

From functions to parameters: Chebyshev expansions

Fitting functions = difficult inference problem

Spectral approximation problem: approximate an unknown function on a compact interval with boundary constraints

$$M(r) = M_0 r^3 [1 + \sum_{n=1}^{N} m_n (T_n \left(2 \frac{r}{r_{trans}} - 1\right) - 1)] \qquad k(r) = k_0 r^2 [1 + \sum_{n=1}^{N} k_n (T_n (2 \frac{r}{r_{trans}} \text{-1}) \text{-1})]$$

$$S(r) = 2\,\epsilon\,[1 + \sum_{n=1}^{N} s_n (T_n (2 \frac{r}{r_{trans}} \text{-1}) \text{-1})]$$

ΛCDM correspondance: $\qquad M_0 = \frac{\Omega_m H_0^2}{2} \qquad k_0 = H_0^2 (\Omega_m - 1) + \frac{\Lambda}{3}$

Chebyshev polynomials of the first kind are known to be optimal for approximating smooth functions on finite interval

Solution = truncated Chebyshev polynomial expansion

Future work

First step: truncate the expansion at second order => 3 functions replaced by a set of 6 constant parameters + ϵ + the location of the observer (t_0, r_0) + the cosmological constant Λ

WE ARE LEFT WITH 10 CONSTANT PARAMETERS TO FIT THE DATA

The transition scale, $z_{trans} \Rightarrow r_{trans}$, is fixed by the depth of the data

Second step: increase z_{trans} (following the depth of the data) => increase the precision

Later on if needed: go to higher order in the Chebyshev expansion => increase the number of parameters and also the precision

Data processing

Standard method

Warning: current data catalogs are not agnostic (not cosmological model independent)

LSS catalogs are obtained by converting for every galaxy (RA, Dec, z) into Cartesian coordinates using the transverse comoving distance $D_M(z)$

obtained from the ΛCDM expansion history $H(z)$: $D_M = D_C = \int_0^z H(z')dz'$

Szekeres

not one $H(z)$ but two: $H_{\parallel}$ and $H_{\perp}$ $\Rightarrow$ more involved

Redshift/distance requires solving null geodesics

Unlike FLRW, there's **no global flat comoving 3-space** to embed arbitrary galaxy pairs into => use a local approximation with position- and-direction-dependent rates

Take-home messages

- We have currently reached the point where ΛCDM + perturbation theory can no more satisfy the requirements of precision cosmology
- Szekeres inhomogeneous models, including FLRW as homogeneous limits, appear as a promising GR alternative, able to keep the most robust early universe predictions of the standard model
- Axial symmetry as found in expansion analyses can be used to simplify the model
- After a truncated Chebyshev expansion of the metric functions, the 10 constant parameters of the Szekeres model can be measured through a fit to current and in-coming cosmological data sets
- This model exhibits an intrinsic mass dipole which has been identified in recent survey analyses
- If the late Universe is FLRW (ΛCDM) this will appear from the Szekeres fitting to the data (vanishing Szekeres parameters) => quasi-agnostic cosmological methodology
- Since the current reduction processes all involve ΛCDM priors, an adaptation to Szekeres of the data in all cosmological surveys is essential to avoid systematic bias

THANK YOU FOR YOUR ATTENTION

