In search of the building blocks of the Universe

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15 September 2026 | Hot topics in Modern Cosmology | Spontaneous Workshop XVIII

Our Universe

What do we know?

Today we have so much information about the properties of our Universe and can test it over a large range of scales.

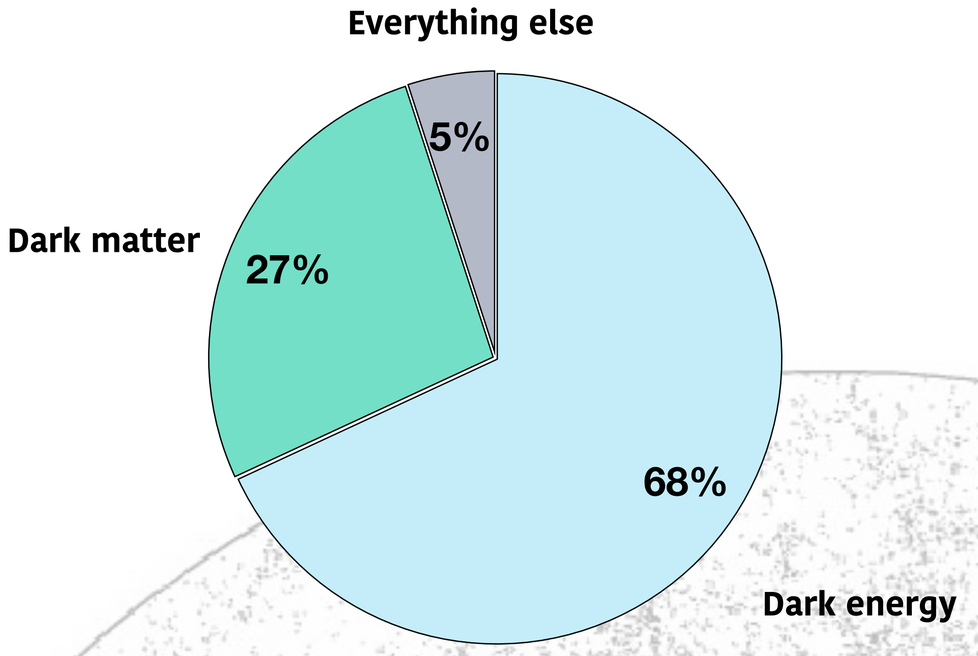
13.8 billion years old

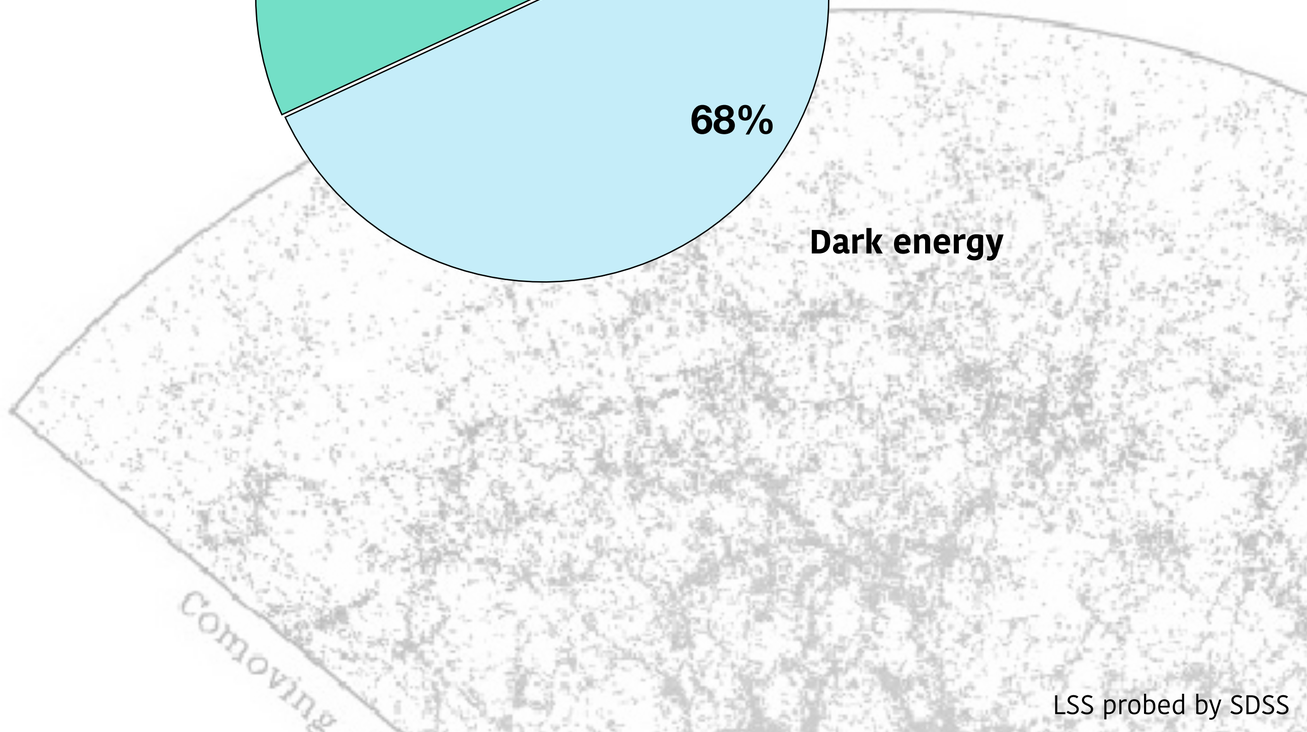
Spatially flat

Expanding; evolving in time

Homogeneous & isotropic on large scales

Large scale structure





LSS probed by SDSS

Our Universe

Theoretical basis

Einstein's relativity

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}R + S_m$$

Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\varepsilon + 3p) \qquad H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\varepsilon$$

Continuity equation

$$\dot{\varepsilon} = -3H(\varepsilon + p) \qquad \text{EoS} \quad p = w\varepsilon$$

Major theoretical challenge:
what is the theory that underlines this expansion and structure?

Dark matter

Dark energy: evolving with a phantom-like stage ?

GR is not power-counting renormalizable

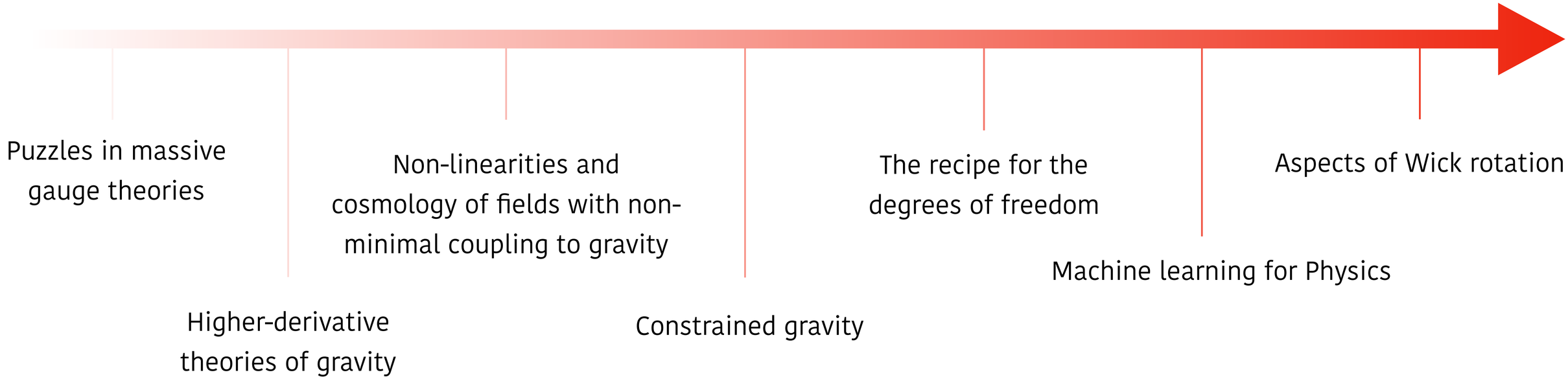
String theory/Quantum gravity suggests higher-order corrections.

Why not go beyond?

My research

addresses the fundamental questions by **uncovering building blocks of theories** in both linear and non-linear regimes.

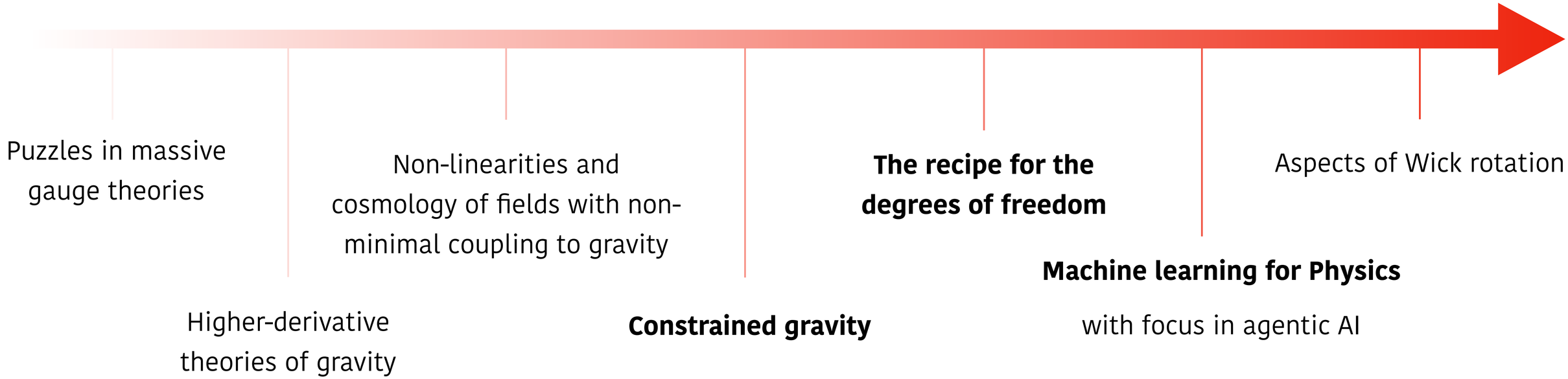
Timeline:



My research

addresses the fundamental questions by **uncovering building blocks of theories** in both linear and non-linear regime.

Timeline:



The building blocks of theories

The key challenge to study theories beyond standard models is to analyze their **degrees of freedom**.

$$\textbf{\# DOF} = \frac{\#\ initial\ conditions}{2}$$

They encode

Types of fields | forces

Stability

Theoretical consistency

Relation to observations.

If we get them wrong, we get the physics wrong!

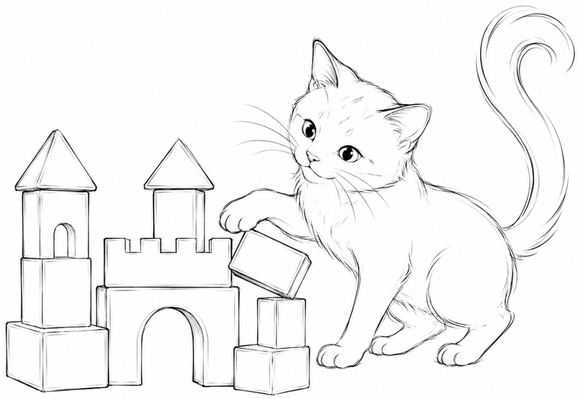


Image generated by ChatGPT.

The existing approaches to DOF

The Dirac-Bergmann algorithm

Based on the Hamiltonian formalism.

Applies the Poisson brackets to count all the propagating modes.

Focus in equations of motion

Analyze field equations directly based on jet bundles and formal power series.

Faddeev-Jackiw method

Uses the first-order action to identify the constraints

$$S = \int dt \left[p\dot{q} - H \right]$$

Manifestly covariant approach

Lagrangian based approach, commonly used in particle physics.

$$A_\mu = A_\mu + \partial_\mu \chi, \qquad \partial_\mu A^\mu = 0$$

The existing approaches to DOF

The Dirac-Bergmann algorithm

Great for finding the number of DOF for general background, but the behavior of modes can be somewhat obscure

Focus in equations of motion

Great start, but it needs gravity.

Mostly focused on the number of degrees of freedom.

Faddeev-Jackiw method

Words well, although can get a bit more technically long

$$S = \int dt \left[p\dot{q} - H \right]$$

Manifestly covariant approach

Easily misleading, and sometimes does not work.

A. Hell, D. Lüst, G. Zoupanos, JHEP 02 (2024) 036

The building blocks of theories

The recipe for the degrees of freedom

In collaboration with E. Ferreira, D. Lüst & M. Sasaki
JHEP 03, 2026, 235

The number of DOF can drastically change depending on a background

$$S = \int d^4x \sqrt{-g} R^2$$

(A) Specify the background

(B) Prepare the perturbations

(C) Expand the action

(D) Analyze the theory

D.1) Assuming no higher derivatives, locate the non-propagating modes

D.2) Find their constraints, solve them, and substitute them back to the action

D.3) Does the determinant of the kinetic matrix vanish? If yes, repeat!

(E) Higher derivatives? Reduce!

The building blocks of theories

Example on flat spacetime

(A) Specify the background $\eta_{\mu\nu}$

(B) Prepare the perturbations

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$h_{00} = 2\phi$$

$$h_{0i} = S_i + \partial_i B \qquad \partial_i S_i = 0$$

$$h_{ij} = 2\psi\delta_{ij} + \partial_i\partial_j E + \partial_i F_j + \partial_j F_i + h_{ij}^{TT}$$

$$\partial_i h_{ij}^{TT} = 0 \qquad h_{ii}^{TT} = 0$$

(C) Expand the action

$$S = \int d^4x 4\beta \left(\Delta\phi + 3\ddot{\psi} - 2\Delta\psi\right)^2$$

(D) Analyze the theory

Phi does not propagate:

$$\Delta\phi = -3\ddot{\psi} + 2\Delta\psi$$

$$\Rightarrow \quad S = 0$$

The building blocks of theories

The recipe for the degrees of freedom

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JHEP 03, 2026, 235

The number of DOF can drastically change depending on a background

Example: $$S = \int d^4x\sqrt{-g}R^2$$

Flat spacetime has **0 degrees of freedom**.

(A) Specify the background

(B) Prepare the perturbations

(C) Expand the action

(D) Analyze the theory

D.1) Assuming no higher derivatives, locate the non-propagating modes

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The building blocks of theories

Example on curved spacetime

Jordan frame: $S = \int d^4x\sqrt{-g}R^2$

Let's go to the Einstein frame

Define: $\tilde{g}_{\mu\nu} = \frac{2\phi}{M_P^2} g_{\mu\nu}$

$$S = \int d^4x\sqrt{-g}\left(\phi R - \frac{1}{4}\phi^2\right)$$

$$\Rightarrow \quad S = \int d^4x\sqrt{-g}\left(M_p^2(\tilde{R} - 2\Lambda) - \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi\right)$$

$$\Rightarrow \quad \phi = 2R$$

$$\Lambda = \frac{M_P^2}{16} \qquad \Phi = \sqrt{\frac{3}{2}}\ln\frac{2\phi}{M_P^2}$$

The building blocks of theories

The recipe for the degrees of freedom

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JHEP 03, 2026, 235

The number of DOF can drastically change depending on a background

Example: $$S = \int d^4x\sqrt{-g}R^2$$

Flat spacetime has **0 degrees of freedom**.

Curved spacetime describes: **scalar mode | 2 tensor modes**

A. Hell, D. Lüst & G. Zoupanos JHEP 02 (2024) 036

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(B) Prepare the perturbations

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The building blocks of theories

Why do we need to know DOF?

The pure R2 theory is not a special case

$$S_{vgg} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}\nabla_\mu\sigma\nabla^\mu\sigma - \frac{\xi}{2}\sigma^n R^m - \lambda\sigma^s\right]$$

The case	Dof in S_{vgg}
General case	2 tensor modes, and 2 scalar modes
$\eta_{JF\mu\nu}$: $m > 1$, or $n > 1$	1 scalar mode
$\eta_{JF\mu\nu}$: $m = 1$, & $n = 1$	no modes
$g^{HI}_{\mu\nu}$: $m = 1$, & arbitrary n	1 scalar mode and 2 tensor modes

The number of DOF can drastically change depending on a background

The reason why we cannot see it in the Einstein frame is because the transformation between Jordan and Einstein become singular.

➔ **We need to use the Jordan frame to fully understand the theory, if this is the physical frame.**

Based on the work with D. Lüst JHEP 12 (2025) 091

LLM for Algorithmic Theoretical Physics

Application for degrees of freedom

Claude Opus 4.6 operating Maple™

In-context learning: Solved examples in Maple.

Procedure: Claude tries to solve an entirely new task by independently programing it in a Maple sessions whose intermediate results inform further computation.

Ensure that tasks have never been studied in the literature:

Framework of **constrained gravity**

A. Hell, M. Sasaki, e-print 2507.00986, JCAP 07 (2026) 034

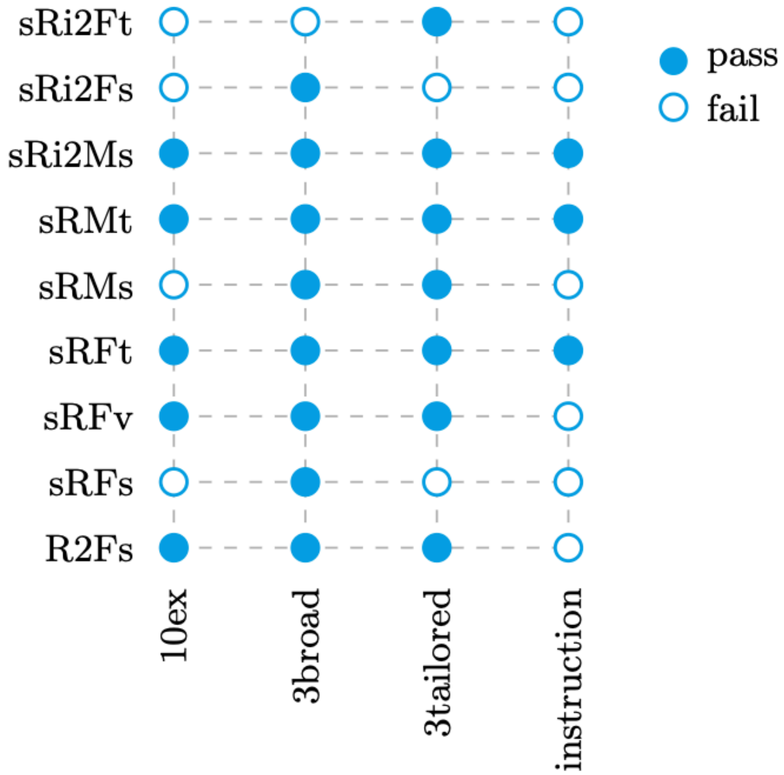
Collaboration with L. Thiele, accepted to ICML 2026 e-Print: 2605.08212

LLM for Algorithmic Theoretical Physics

Application for degrees of freedom

Task example:

sRFv Consider $\frac{M_P^2}{2}(R-2\Lambda)+\frac{1}{2}\sigma^2(m^2+f(R))$ gravity on a cosmological background, where M_P is the Planck mass, Λ is the cosmological constant, σ is the scalar field, m^2 is the mass, and $f(R)$ is a function of Ricci scalar. Find the quadratic action for the vector perturbations around this background, and express it in terms of the propagating modes.



A carefully chosen set of three examples is better than a broad set of 10 examples.

A. Hell, L. Thiele,
accepted to ICML 2026 e-Print: 2605.08212

LLM for Algorithmic Theoretical Physics

Application for degrees of freedom

Agent behaves like a first-year PhD student.

(But supplement the examples.)

Why is this useful in MG research?

There are many paths to search for degrees of freedom, and the computation time can significantly differ.

Useful as an education material.

A working framework whenever we have an algorithmic problem in physics

DOF:

More complicated theories including vector, p-form or tensor fields | more complicated backgrounds.

More general:

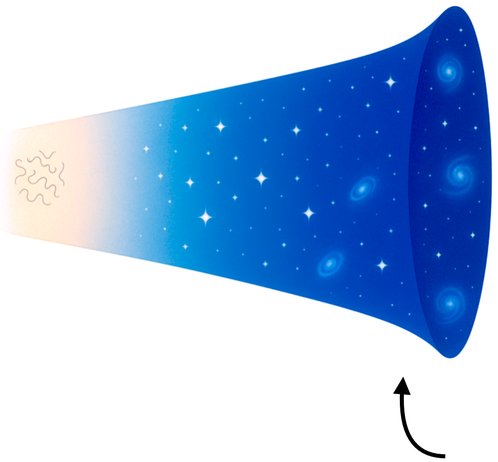
Non-perturbative solutions in field theories | Comprehensive analysis of models | gravitational particle production | ...

Collaboration with L. Thiele,
accepted to ICML 2026 e-Print: 2605.08212

The central challenge

of theories of gravity

Important in the light of **recent preliminary observations**



DESI BAO+CMB indicate that **dark energy** evolves in time and has a phantom-like stage.

Ghosts

pathological modes that lead to instability, and violation of unitarity.

Standard approach: scalar field has to be a ghost to account for such a stage.

Constrained gravity

Framework in which gravity if modified through non-propagating external fields.

$$\sigma\left(\xi R - m^2\right) = 0$$

$$\sigma \neq 0 \qquad \sigma = 0$$

Example: **The constrained scalar**

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}(R - 2\Lambda) - \frac{m^2}{2}\sigma^2 + \frac{\xi}{2}R\sigma^2\right]$$

Usually the scalar fields have kinetic terms.
Here, the scalar field is constrained.

The constrained scalar gives rise to a **non-trivial cosmological background**:

$$\dot{H} = \frac{12\xi H^2 - m^2}{-6\xi}$$

$$(\dot{\sigma})^2 = \frac{1}{6H\xi}\left[(m^2 - 6\xi H^2)\sigma^2 + 2M_p^2(\Lambda - 3H^2)\right]$$

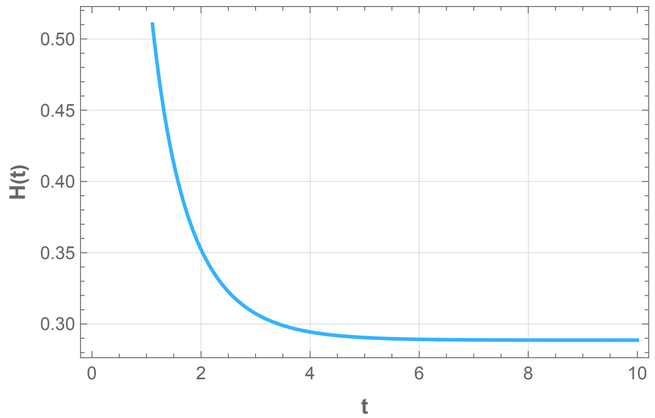
A. Hell, M. Sasaki, JCAP 07 (2026) 034, e-print 2507.00986,

Funded by KAKENHI

Constrained gravity

Free theory

Radiation dominated to exponential expansion

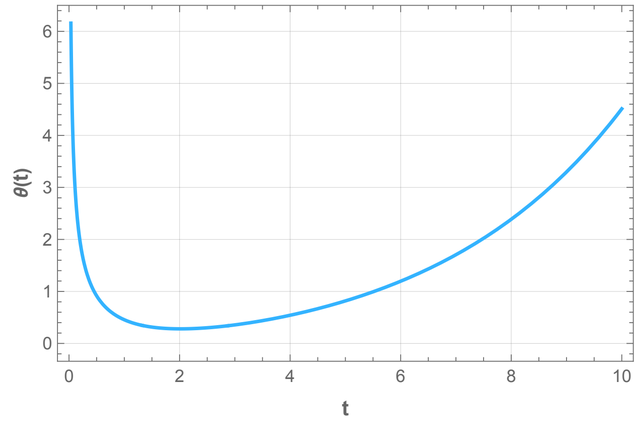


Early times

$$H \sim \frac{1}{2t}$$

Late times

$$H \to H_{lt} = \frac{m}{2\sqrt{3\xi}}$$



$$\Theta = \sigma^2$$

Scalar modes

$$\mathscr{L}_S = c_1(t)\delta\dot{\sigma}_f\delta\dot{\sigma}_f - c_2(t)\delta\sigma_f\delta\sigma_f$$

$$c_1(t) = \frac{3H^2a^5\xi^2\sigma^2\left(\xi\sigma^2 + M_p^2\right)}{\left(\dot{\sigma}\xi a\sigma + \left(\xi\sigma^2 + M_p^2\right)Ha\right)^2} > 0 \qquad c_\sigma^2 = 1$$

Tensor modes

$$\mathscr{L}_T = \frac{a(\xi\sigma^2 + M_p^2)}{8}\left(a^2\dot{h}_{ij}^T\dot{h}_{ij}^T - k^2h_{ij}^Th_{ij}^T\right)$$

Constrained gravity

Coupling to matter

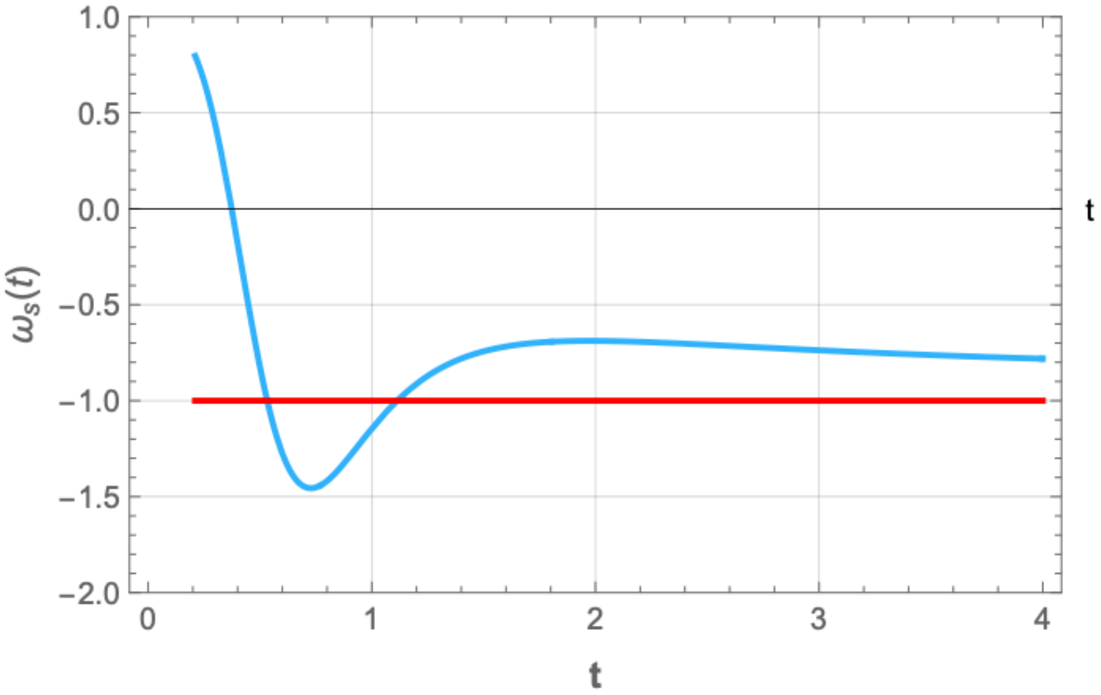
If we couple the matter minimally to gravity, it will not affect the evolution of the cosmological background

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}(R-2\Lambda) - \frac{m^2}{2}\sigma^2 + \frac{\xi}{2}R\sigma^2\right] + S_m$$

We can couple it non-minimally to the constrained scalar:

$$S_{CS} + \int d^4\sqrt{-g}\sigma^2\mathscr{L}_m$$

Evaluated in the minimal frame.



This is an example of a deep dynamical mechanism, which leads to rich phenomenology.

Branches with vector fields

The minimal version

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}(R-2\Lambda) - m^2 A_\mu A^\mu + \frac{1}{2}\xi R A_\mu A^\mu - \xi R^{\mu\nu}A_\mu A_\nu\right]$$

Constraint: $A_0\left(m^2 - \frac{6\xi\dot{a}^2}{a^2}\right) = 0$

Only tensor modes propagate in for cosmological spacetimes.

Two de Sitter solutions:

1) $A_0 \neq 0$ $\qquad H^2_{dS_1} = \frac{m^2}{6\xi}$ $\qquad c_T^2 = \frac{(M_P^2 - \xi A_0^2)}{(M_P^2 + \xi A_0^2)}$

2) $A_0 = 0$ $\qquad H^2_{dS_2} = \frac{\Lambda}{3}.$ $\qquad c_T^2 = 1$

Based on the work with T. Daniel, E-print: 2603.18147

Branches with vector fields

Stealth black holes

Ansatz:

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

$$A_\mu = (A_0(r), A_1(r), 0, 0)$$

Equations:

$$\begin{aligned}
0 =& \mathrm{e}^{-2\alpha} A_0^2 \xi + \mathrm{e}^{-4\beta}\xi\left(-6\beta' r A_1^2 + 4A_1' A_1 r + A_1^2\right) + M_\mathrm{P}^2 + \mathrm{e}^{-2\beta}\left(2M_\mathrm{P}^2\beta' r + A_1^2\xi - M_\mathrm{P}^2\right) \\
&+ \mathrm{e}^{-2\beta-2\alpha} A_0^2\xi\left(2\beta' r - 1\right), \\
0 =& \mathrm{e}^{-2\alpha} A_0^2\xi - 3\,\mathrm{e}^{-4\beta} A_1^2\xi\left(2\alpha' r + 1\right) - M_\mathrm{P}^2 - \mathrm{e}^{-2\beta}\left(-2\alpha' M_\mathrm{P}^2 r - A_1^2\xi - M_\mathrm{P}^2\right) \\
&- \mathrm{e}^{-2\beta-2\alpha}\xi\left(-2\alpha' r A_0^2 + 4A_0' A_0 r + A_0^2\right), \\
0 =& -2\xi A_0\left(2\beta' r + \mathrm{e}^{2\beta} - 1\right), \\
0 =& -A_1^2\xi^2\left(2\alpha' r - \mathrm{e}^{2\beta} + 1\right),
\end{aligned}$$

Solution:

$$ds^2 = -\left(1 - \frac{r_g}{r}\right)dt^2 + \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2$$

$$A_1 = \frac{c_1\sqrt{r}}{r r_g - 1}, \qquad \text{and} \qquad A_0^2 = \frac{r_g^3 c_2 r - c_2 r_g^2 + c_1^2}{r\, r_g^2}$$

Based on the work with T. Daniel, E-print: 2603.18147

Summary & Outlook

Degrees of freedom are the building blocks of physical theories, which encode their main behavior in both linear and non-linear regimes.

Their number changes depending on the background.

➡ Crucial to know the stability, and non-linear behavior

Possible to find through LLMs with in context learning

➡ A working framework whenever we have an algorithmic problem in physics

Constrained gravity — framework in which gravity is modified through non-propagating external fields.

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}(R-2\Lambda) - \frac{m^2}{2}g(\sigma) + \frac{\xi}{2}f(R)g(\sigma)\right] + S_m$$

Holds hints for the cosmological constant, and admits phantom-like DE solutions in a consistent manner.

Thank you!

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