Negative refraction and gravitational metamaterials

From optical geometry to dark matter

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First part of the talk

1 Coordinate-dependent and coordinate-independent refractive index definitions.

2 Applications to black holes, regular black holes and naked singularities.

Second part of the talk

3 Disformal dark sector as localized soliton-like gravitational metamaterial.

4 Observational signatures of negative refraction soliton-like objects.

Main result

Dark matter $\rightharpoonup$ soliton-like solution $\Rightarrow$ modifying lensing through negative refraction response.

Talk based on:

1. O. Luongo, Class. and Quant. Grav., **42**, 225005, (2025).
2. O. Luongo, under revision, (2026).
3. O. Luongo, E. Ventola, Int. J. Mod. Phys. A, **41**, 24, 2646015, (2026).

Naive idea

Geometry → effective medium for light.

A prototype: The Gordon metric

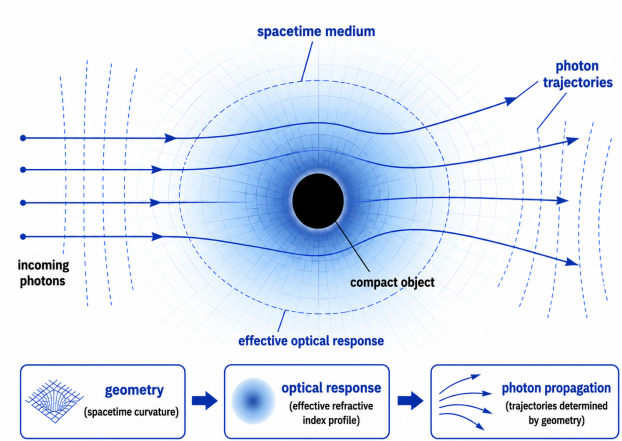
$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \left(1 - \frac{1}{n^2}\right) u_\mu u_\nu$$

with $g^{\mu\nu}_{\text{opt}} k_\mu k_\nu = 0$

Here we reverse the logic.

Goal?

From geometry to exotic refraction?



Two ways of assigning an optical response

Direct strategy

One computes the optical path from a given spacetime metric and interprets light propagation as propagation in an effective medium.

Indirect strategy

One rewrites the metric in an optical form and identifies a refractive parametrization inside the spacetime line element.

Problem

Both strategies are valid. None of them automatically defines an invariant refraction, neither its sign.

Focus on a static spherical geometry:

$$ds^2 = \beta^2 dr^2 + \gamma^2 r^2 d\Omega^2 - c^2\alpha^2 dt^2.$$

Coordinate-dependent index

$$n_D = \frac{\beta}{\alpha}\left[1 - \sin^2\sigma\left(1 - \frac{\gamma^2}{\beta^2}\right)\right]^{1/2}.$$

Direct construction

The angle σ only parametrizes the local spatial direction in a chosen metric representation.

Optical rewriting

$$ds^2 = f^2 n^2 (d\hat{r}^2 + \hat{r}^2 d\Omega^2) - c^2 f^2 dt^2.$$

First hint:

Is the refractive index sign fixed by its definition?
A negative coordinate value of n_D is not, by itself, physical negative refraction. The physical sign must be defined in the observer rest space.

Can we classify compact geometries through their optical response?

Optical message

Different metrics become different spacetime media!

Black holes

horizon plus singularity

Optical behavior

n diverges close to the horizon.

Regular black holes

horizon without singularity

Optical behavior

n remains regular near the core.

Naked singularities

no horizon, singular core

Optical behavior

n evolves smoothly with $|n| < 1$.

- Direct and indirect techniques are equivalent.
- A Snell's law can be formulated.
- Can the refractive index give hints on the optical response?

We have,

$$\alpha = \left(1 - \frac{2M}{r}\right)^{1/2}, \qquad \beta = \alpha^{-1}, \qquad \gamma = 1.$$

The refractive index reads

$$n_S = \frac{r}{r - 2M}\left(1 - \frac{2M}{r}\sin^2\sigma\right)^{1/2}.$$

Near the horizon

$$r \to 2M \quad \Longrightarrow \quad n_S \to \infty.$$

Far from the source

$$r \gg 2M \quad \Longrightarrow \quad n_S \to 1.$$

Two simple deformations of the Schwarzschild optical profile are

$$\alpha_{SdS} = \left(1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}\right)^{1/2}, \qquad \alpha_{RN} = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{1/2}.$$

The relative departures from the Schwarzschild index, for small σ, are

$$\delta n_{SdS} = \frac{r^3\Lambda}{3(r-2M)} + \mathcal{O}(\sigma^2), \qquad \delta n_{RN} = -\frac{Q^2}{r(r-2M)} + \mathcal{O}(\sigma^2).$$

de Sitter contribution

$\Lambda > 0$ increases the optical index.

Electric charge

Q decreases the optical index.

Regular black holes

Regular black holes replace the singular core by a regular interior while preserving a horizon structure.

Bardeen	**Hayward**	**Dymnikova**	**Fan-Wang**
topological charge q	de Sitter core	length scale l_D	length scale l_{FW}

Near the regular core, the index has the universal structure

$$n(r \simeq 0) \simeq 1 + \mathcal{C}\,(1 + \cos^2\sigma) r^2,$$

where $\mathcal{C}$ depends on the specific regular solution.

Optical conclusion

The central optical divergence of Schwarzschild is replaced by a finite regular behavior.

No horizon solutions

Solutions without horizons provide a qualitatively distinct optical class.

Negative-mass Schwarzschild	**Reissner-Nordström**	**AdS-like**
repulsive solution?	$Q > M$	no horizon

For the AdS-type case,

$$\alpha = \left(1 + \frac{\Lambda r^2}{3}\right)^{1/2}, \qquad \beta = \alpha^{-1}, \qquad \gamma = 1,$$

and

$$n_{AdS} = \frac{\left(9 + 3r^2\Lambda \sin^2\sigma\right)^{1/2}}{3 + r^2\Lambda}.$$

Distinctive feature

No-horizon solutions can exhibit smooth optical evolution and regions with $|n| < 1$.

Optical distinction among the three classes

Black holes

The refractive index diverges at the horizon and the interior singularity remains hidden.

Regular black holes

The core is nonsingular and the optical profile remains finite in the inner region.

Optical limitation

Exterior data do not easily distinguish a singular black hole from a regular black hole.

What must be probed

The discriminating information is strongest close to the would-be horizon or inside it.

Black holes **Regular black holes** **No-horizon objects**

Near horizon	Near core	Toward center
n grows sharply.	n remains finite.	n evolves smoothly.

To distinguish no horizon objects from black holes one must probe rays that sample the strong field region, where the optical profile does not mimic a horizon divergence.

Gravitational Snell's law

Once spacetime is treated as a medium, the refractive index controls the change of the ray direction.

Radial gravity approximation

$$\frac{d\sigma}{d\varphi} = -1 - \frac{r}{n}\frac{dn}{dr}.$$

Geometric role

dn/dr measures the optical inhomogeneity induced by the gravitational source.

Second hint

The equation is invariant under $n \to -n$.

Physical meaning: *A sign flip of the refractive index does not by itself modify the angular bending equation. The physical sign must be interpreted through the energy flow.*

Weak-field test: the Sun

For the Sun,

$$\frac{r_{H,\odot}}{r_\odot} \simeq 4.24 \times 10^{-6} \ll 1.$$

The radial approximation then gives

$$\Delta\sigma_\odot \simeq \int_{-\pi/2}^{\pi/2} \frac{r_{H,\odot}}{r_\odot} \cos\varphi \, d\varphi \simeq \frac{2r_{H,\odot}}{r_\odot}.$$

Numerically,

$$\Delta\sigma_\odot \simeq (1.7489 \pm 0.0007) \text{ arcsec}.$$

Consistency check

This reproduces the standard weak-lensing result

$$\theta_{\text{def}} = \frac{4M}{b}.$$

From weak to strong optical regimes

Weak-field optics provides an important consistency check, but it is not the natural regime in which exotic spacetime media can be efficiently distinguished.

Weak-field regime

Solar-System optics

Strong-field regime

Compact-object optics

Suppressed corrections

$$Q, \qquad \Lambda, \qquad q, \qquad l_D, \qquad l_{FW}$$

give negligible or extremely small optical effects.

Enhanced sensitivity

Neutron stars and white dwarfs provide stronger compactness, enhancing the response of $n(r)$ and the corresponding deflection.

Role

The weak-field limit is not sufficiently sensitive to discriminate among exotic spacetime media.

Observable trend

Deviations become appreciable only for sufficiently compact sources or for configurations close to their maximal parameter range.

Flat space as an optical test

Flatness does not automatically imply a trivial optical response. In standard spherical Minkowski coordinates one has

$$ds^2 = -d\eta^2 + d\rho^2 + \rho^2 d\Omega^2, \qquad n = 1.$$

However, after introducing spherical Rindler coordinates,

$$r^2 = (\rho - b)^2 - \eta^2,$$
$$At = \tanh^{-1}\left(\frac{\eta}{\rho - b}\right),$$

the same flat construction leads to

$$ds^2 = -A^2 r^2 dt^2 + dr^2 + r^2\left(\frac{b}{r} + \cosh(At)\right)^2 d\Omega^2.$$

Optical lesson

The optical index can become nontrivial even in a flat-spacetime construction. Thus, the sign of n is not by itself a curvature invariant.

Rindler refractive index and Unruh temperature

From the direct prescription, the spherical Rindler refractive index is

$$n_R = \frac{1}{Ar}\left[1 - \sin^2\sigma\left(1 - \left(\frac{b}{r} + \cosh(At)\right)^2\right)\right]^{1/2}.$$

Optical sign

$n_R > 0 \quad \text{if} \quad A > 0, \qquad n_R < 0 \quad \text{if} \quad A < 0.$

Unruh temperature

$$T_U = \frac{|A|}{2\pi k_B}.$$

Third hint:

The sign of the acceleration parameter controls the sign assigned to the optical index. Nevertheless, the thermal response depends on the modulus of A, not on its sign.

Physical interpretation: *A negative n_R does not imply a negative Unruh temperature. It signals an optical sign choice, while the observable thermal scale remains positive.*

Metamaterials!

Veselago's medium is characterized by

$$\varepsilon < 0, \qquad \mu < 0.$$

Although

$$n^2 = \varepsilon\mu > 0,$$

the physical branch is left-handed,

$$n = -\sqrt{\varepsilon\mu}.$$

Physical content

The essential feature is the reversal between phase propagation and energy transport. Maxwell theory gives

$$\mathrm{k} \cdot \mathrm{S} < 0.$$

Why the sign of n is not enough? Reason 1: coordinate-dependence

A coordinate-dependent refractive index can become negative because of the parametrization of the metric functions. **This does not imply negative refraction in the Veselago sense**.

Coordinate independent refractive index:

For a stationary spacetime, define the conserved Killing energy and the frequency measured by a local observer

$$E = -\xi^\mu k_\mu, \qquad k^\nu \nabla_\nu E = 0, \qquad \omega = -u^\mu k_\mu.$$

The scalar refractive amplitude is

$$n_{\rm abs} = \frac{-u^\mu k_\mu}{-\xi^\mu k_\mu} = \frac{\omega}{E} > 0.$$

Key point

Negative refraction cannot be the sign of ω/E. Should we write $\pm\frac{\omega}{E}$?

Gravity + Electromagnetism

Start from ordinary electromagnetism minimally coupled to the gravitational metric:

$$S_{\mathrm{EM}} = -\frac{1}{4}\int d^4x\sqrt{-g}\, F_{\mu\nu}F^{\mu\nu}.$$

In the geometric optics limit,

$$F_{\mu\nu} = \mathrm{Re}\left[f_{\mu\nu}e^{i\Theta/\eta}\right], \qquad k_\mu = \nabla_\mu\Theta.$$

At leading order, $k_\mu k^\mu = 0$, $k^\nu\nabla_\nu k_\mu = 0$. **NO SOLUTION!** The stress tensor is null dust,

$$T^{\mu\nu}_{\mathrm{EM}} = \rho k^\mu k^\nu, \qquad \rho > 0.$$

Consequence

The photon cone is the metric null cone and the energy flux is locked to the phase direction.

What do we need then? A covariant negative refraction criterion

Let s^μ be a unit spatial vector orthogonal to u^μ and decompose

$$k_\mu = -\omega u_\mu + p_\mu, \qquad u^\mu p_\mu = 0, \qquad p_\perp = s^\mu k_\mu.$$

For the stress tensor transporting electromagnetic energy, define

$$J^\mu = -T^{\mu\nu} u_\nu, \qquad J_\perp = s_\mu J^\mu.$$

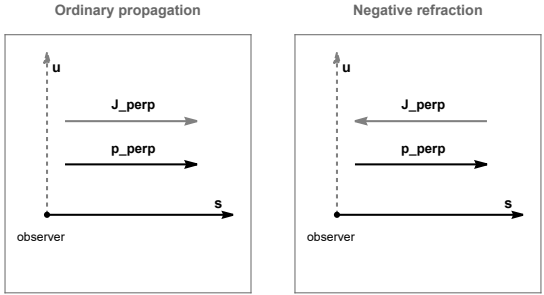
Invariant criterion

$$p_\perp J_\perp < 0.$$

The signed refractive index is therefore

$$n_{\text{eff}} = \text{sgn}(p_\perp J_\perp) n_{\text{abs}}.$$

Intuitive physical meaning



Consequence

The photon cone is the metric null cone and the energy flux is NOT locked to the phase direction.

Can invisibility be obtained?

Yes, but not necessarily! $\longrightarrow$ Realistically: **exotic refraction**!

Three optical quantities, one physical sign

n_D

Coordinate-dependent optical diagnostic.

n_D is not a sign criterion.

n_{abs}

Invariant scalar magnitude:

$$n_{\mathrm{abs}} = \frac{\omega}{E} > 0.$$

It cannot encode negative refraction.

n_{eff}

A signed new refractive index:

$$n_{\mathrm{eff}} = \mathrm{sign}(p_\perp J_\perp) n_{\mathrm{abs}}.$$

Physical prescription

Negative refraction means phase momentum and electromagnetic energy flux are oppositely oriented.

What a gravitational metamaterial must do

A gravitational metamaterial is not a coordinate redefinition of n. It is a sector in which the photon cone and the energy flux acquire a nontrivial optical response.

Minimal GR plus Maxwell

Metric photon cone. Null-dust flux. Phase and energy transport are locked.

Metamaterial sector

Modified photon cone. Nontrivial optical mixing. Energy flux can reverse relative to phase propagation.

WARNING:

1. What can realize the negative refraction?
2. Do we need localization or generalization of the exotic optical property?

What do we need?

Non-linear electrodynamics?

Realizing a gravitational metamaterial?

The minimal route to gravitational negative refraction is to give photons an optical metric distinct from the gravitational metric:

$$\tilde{g}_{\mu\nu} = C(\phi, X) g_{\mu\nu} + D(\phi, X)\phi_\mu\phi_\nu, \qquad X = g^{\mu\nu}\phi_\mu\phi_\nu.$$

The inverse metric is

$$\tilde{g}^{\mu\nu} = \frac{1}{C}\left(g^{\mu\nu} - \eta\phi^\mu\phi^\nu\right), \qquad \eta = \frac{D}{C + DX}.$$

Hyperbolicity requirements

$$C > 0, \qquad C + DX > 0.$$

Decompose the scalar gradient in the local observer frame:

$$\phi^\mu = \dot{\phi} u^\mu + q s^\mu, \qquad \dot{\phi} = -u_\mu \phi^\mu, \qquad q = s_\mu \phi^\mu.$$

In the reduced (u, s) sector,

$$\tilde{g}^{ab} = \begin{pmatrix} -A & M \\ M & B \end{pmatrix},$$

with

$$A = \frac{1 + \eta \dot{\phi}^2}{C}, \qquad B = \frac{1 - \eta q^2}{C}, \qquad M = -\frac{\eta \dot{\phi} q}{C}.$$

Metamaterial interpretation

M is the gravitational analogue of magneto-electric mixing in a bianisotropic metamaterial.

Key point

A tilted cone is a **necessary condition**. The flux criterion is a **sufficient condition**.

Metamaterial localization?

Let the phase covector in the reduced sector be

$$k_a = \begin{pmatrix} \omega \\ p_\perp \end{pmatrix}.$$

The optical null condition gives $Bp_\perp^2 + 2M\omega p_\perp - A\omega^2 = 0$ and therefore

$$\frac{p_\perp}{\omega} = \frac{-M \pm \sqrt{M^2 + AB}}{B}.$$

The ray vector is the Hamiltonian flow, $\ell^a = \tilde{g}^{ab} k_b$, $\ell_\perp = M\omega + Bp_\perp$.

Energy flux in the disformal sector

Let photons propagate on the disformal optical metric $S_\gamma = -\frac{1}{4}\int d^4x\sqrt{-\tilde{g}}\,\tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$.

Geometric optics on the photon cone

$$\tilde{T}^{\mu\nu} = \rho\,\ell^\mu\ell^\nu, \qquad \rho > 0. \qquad \Longrightarrow \qquad \tilde{g}_{\mu\nu}\ell^\mu\ell^\nu = 0.$$

Varying the photon action with respect to the gravitational metric gives

$$T^{\mu\nu} = \Omega\rho\left[C\ell^\mu\ell^\nu - \Lambda(\phi_\alpha\ell^\alpha)^2\phi^\mu\phi^\nu\right].$$

Physical meaning of the disformal coupling

$$\Lambda \equiv D_X - \frac{D}{C}C_X.$$

Λ controls the nonstandard flux that can compete with the ordinary positive ray term.

Exact criterion and necessary ingredients

The observer-measured flux is

$$J^\mu = \Omega\rho\left[C\omega\ell^\mu - \Lambda(\phi\cdot\ell)^2\dot{\phi}\phi^\mu\right].$$

Projecting along s^μ gives

$$J_\perp = \Omega\rho\left[C\omega\ell_\perp - \Lambda(-\dot{\phi}\omega + q\ell_\perp)^2\dot{\phi}q\right].$$

Exact negative-refraction condition

$$p_\perp\left[C\omega\ell_\perp - \Lambda(-\dot{\phi}\omega + q\ell_\perp)^2\dot{\phi}q\right] < 0.$$

The necessary ingredients

$$\dot{\phi} \neq 0, \qquad q \neq 0, \qquad \Lambda \neq 0.$$

Localization of the negative refraction shell

The condition $p_\perp J_\perp < 0$ does not need to hold everywhere.

Near the center

Regularity suppresses the spatial gradient,

$$q = s_\mu \phi^\mu \to 0.$$

Far from the source

Localization suppresses both gradients,

$$\dot{\phi} \to 0, \qquad q \to 0.$$

Finite active shell

Negative refraction naturally appears only in a bounded spacetime region $\longrightarrow$ **LOCALIZATION**.

Gravitational metamaterials

A gravitational metamaterial is a regular optical object satisfying

$$\boxed{p_\perp J_\perp < 0}, \qquad \text{plus :}$$

1. **regularity**: The field that produces negative refraction might not diverge;
2. **localization**: The field that produces negative refraction might be localized;
3. **no horizon**: We are not dealing with black hole solutions;
4. **hyperbolic photon cone**: No superluminal contribution is allowed;
5. **finite active shell**: Negative refraction is only in a finite domain.

Reinterpreting dark matter

Is it a localized configuration that modifies the photon sector covariantly?

A solitonic optical profile

A simple localized profile is

$$\phi(t,r) = A\frac{1+\varepsilon\tanh(\Omega t)}{1+r^2/L^2}, \qquad 0<\varepsilon<1.$$

Its temporal and radial gradients are

$$\dot{\phi} = A\varepsilon\Omega\,\mathrm{sech}^2(\Omega t)\frac{1}{1+r^2/L^2}, \qquad q = -\frac{2A[1+\varepsilon\tanh(\Omega t)]r/L^2}{(1+r^2/L^2)^2}.$$

For standard $t>0$ and $r>0$,

$$\dot{\phi}q < 0.$$

Key point

The profile is regular at the origin, localized at infinity and optically active only at intermediate radii.

Finite mass and horizon-free regime

A dark configuration must define a finite gravitational source:

$$m(r) = 4\pi \int_0^r \rho(r') r'^2 dr', \qquad \frac{2m(r)}{r} < 1.$$

The second inequality prevents a horizon in the lensing region.

For a separable profile $\phi(t, r) = f(t)\Phi(r)$, finite mass requires

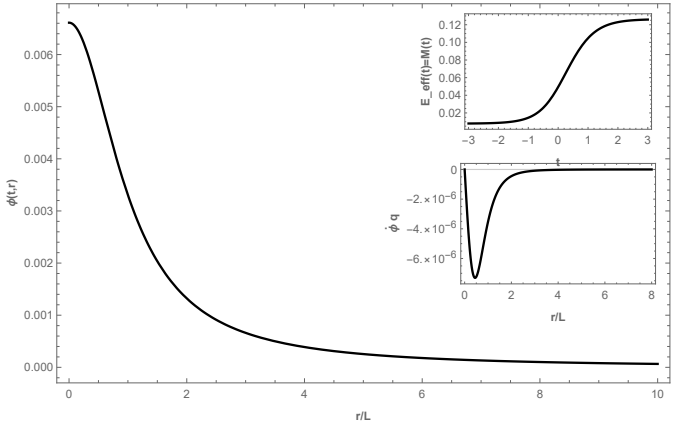
$$\int_0^\infty r^2 \Phi^2 dr < \infty, \qquad \int_0^\infty r^2 (\Phi')^2 dr < \infty.$$

Not black hole optics

The phenomenology is not shadow physics. The object must be seen by lensing rays as a horizon-free optical medium.

Shell nature

At $r = 0$, $q = 0$ and at large r, both $\dot{\phi}$ and q decay.



A scalar potential can support a long-lived weak-pressure regime:

$$U(\phi) = \frac{1}{2}\mu^2\phi^2 + \frac{\lambda_\phi}{4}\phi^4 + \cdots, \qquad \mu^2 > 0.$$

A dust condition is

$$|\Omega^2 - \mu^2| \ll \mu^2, \qquad 0 < (\Phi')^2 \ll \mu^2\Phi^2.$$

Dynamical role

Weak pressure allows the configuration to cluster approximately as cold dark matter.

Optical role

The disformal photon cone remains nontrivial and can affect lensing.

A minimal weak-field disformal branch in which the photon response depends on the scalar invariant X:

$$C = 1, \qquad D = D_0 + \lambda X \quad \rightarrow \quad \Lambda = D_X - \frac{D}{C} C_X = \lambda.$$

Localized negative refraction

The parameter λ controls the genuinely disformal flux term:

$$p_\perp \left[\omega_\ell \ell_\perp - \lambda (q \ell_\perp - \dot{\phi} \omega_\ell)^2 \dot{\phi} q \right] < 0.$$

Negative refraction occurs only where this term reverses the projected energy flow.

Weak-field optical branch

Use $\alpha^2 = f = \beta^{-2}$ and $\gamma = 1$, with

$$f(r) = 1 - \frac{2M}{r} + \cdots .$$

For equatorial photons, $\mathcal{H} = \frac{1}{2} \tilde{g}^{\mu\nu} k_\mu k_\nu = 0$, $E = -k_t$, $b = \frac{L_z}{E}$.

Averaging in time

The observational lens is treated as adiabatically stationary:

$$\langle Y \rangle_T = \frac{1}{T} \int_{-T/2}^{T/2} Y(t)\, dt, \qquad \tau = \frac{\Omega T}{2}, \qquad y = \tanh \tau.$$

The useful temporal moments are

$$K_T = \frac{\epsilon^2 \Omega^2}{\tau} \left(y - \frac{y^3}{3} \right), \quad Q_T = 1 + \epsilon^2 \left(1 - \frac{y}{\tau} \right), \quad \mathcal{M}_T = \frac{\epsilon^2 \Omega^2}{\tau} \left[y - \frac{y^3}{3} + \epsilon^2 \left(\frac{y^3}{3} - \frac{y^5}{5} \right) \right].$$

The averaged radial potential becomes

$$\bar{V}_{\rm eff} = -\frac{1}{f} + \frac{b^2}{r^2} - \eta \frac{\mathcal{A}^2 K_T P^2}{f^2} - \frac{\eta^2 \mathcal{A}^4 \mathcal{M}_T P^2 (P')^2}{f \bar{B}_{\rm opt}}.$$

with $\bar{B}_{\rm opt}(r) = 1 - \eta(r) f(r) \mathcal{A}^2 Q_T \left[P'(r) \right]^2$.

The bending angle is

$$\hat{\alpha}(b) = 2\int_{r_0}^{\infty} \frac{b\,dr}{r^2\sqrt{-f(r)\bar{B}_{\mathrm{opt}}(r)\bar{V}_{\mathrm{eff}}(r;b)}} - \pi.$$

Observable split

At leading weak-field order, the ray is evaluated on $r^2 = b^2 + z^2$. The optical correction naturally separates as

$$\delta\hat{\alpha}_{\mathrm{opt}}^{\mathrm{Born}} = \delta\hat{\alpha}_{\Delta W}^{\mathrm{Born}} + \delta\hat{\alpha}_{\bar{B}}^{\mathrm{Born}}.$$

Hamiltonian deformation

$$\delta\hat{\alpha}_{\Delta W}^{\mathrm{Born}}$$

comes from the time component and the time-radial mixing of the optical metric.

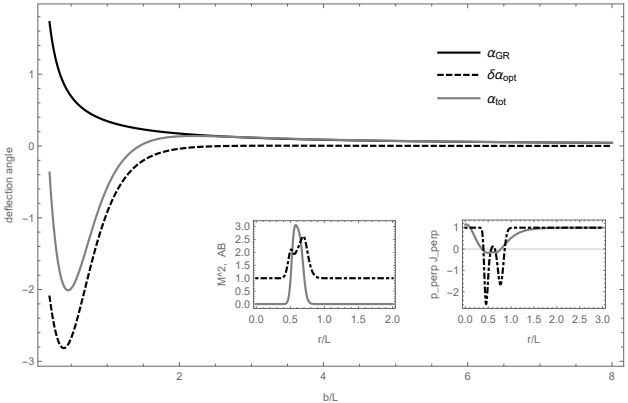
Radial cone factor

$$\delta\hat{\alpha}_{\bar{B}}^{\mathrm{Born}}$$

comes from the static radial deformation of the photon cone.

Reading the plot

The deflection residual is localized in impact parameter. The strong-mixing interval marks where the optical cone is sufficiently tilted, while the region with $p_\perp J_\perp < 0$ identifies the finite negative-refraction shell.



Dynamical mass versus lensing mass

Thus the clean target is not a rotation-curve potential, but a mismatch:

$$M_{\mathrm{lens}}(b) = \frac{b}{4}\hat{\alpha}(b) = M_{\mathrm{dyn}}(b) + \delta M_{\mathrm{meta}}(b).$$

Interpretation

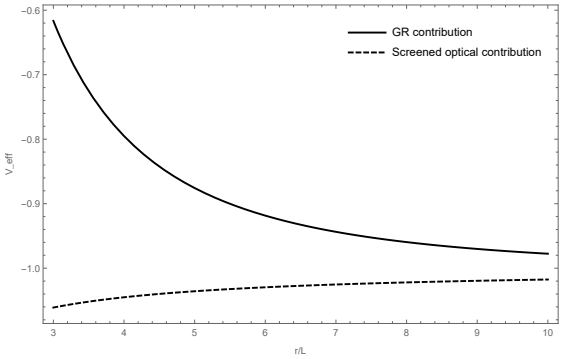
δM_{meta} is not extra material mass.

The Einstein's equivalence principle?

Matter and photons probe different geometrical structures:

$$\text{Baryons} \longrightarrow g_{\mu\nu}, \qquad \text{Photons} \longrightarrow \widetilde{g}_{\mu\nu}.$$

- Addressing Solar System constraints.
- Addressing cosmological bounds.



Photon-metric departure

$$h^{(\gamma)}_{\mu\nu} = \frac{\widetilde{g}_{\mu\nu}}{C} - g_{\mu\nu} = \frac{D}{C}\phi_\mu\phi_\nu.$$

$$\epsilon_t = \left|\frac{D}{C}\dot{\phi}^2\right|, \quad \epsilon_s = \left|\frac{D}{C}q^2\right|, \quad \epsilon_{ts} = \left|\frac{D}{C}\dot{\phi}q\right|.$$

X-window realization

$$D(\phi, X) = D_0 + \lambda_\star X W(X), \; W(X) = \exp\left[-\frac{(X - X_\star)^2}{\Delta_X^2}\right]$$

$X \simeq X_\star \Rightarrow$ active shell, $\quad X \not\simeq X_\star \Rightarrow$ screened.

Screening hierarchy

$$\epsilon^{\rm loc}_{t;s;ts} \lesssim 10^{-15}, \qquad \epsilon_{\rm shell} \not\simeq 0.$$

$$C > 0, \qquad C + DX > 0.$$

Local effect is suppressed, while the shell response remains active.

Fiducial numerical bounds

$\mathcal{A} = 5.0\times10^{-3}$, $L = 1$, $\mu = 20$, $\varepsilon = 0.6$, $\Omega = 1$, $t_0 = 0.6$.

$C_0 = 1$, $\quad D_0 = 0$, $\quad \lambda_\star = -3.2\times10^{9}$, $\quad \lambda_\phi = 0$, $\quad \omega = 1$.

Physical message

Screening protects local and cosmological propagation, while a finite shell can still generate an optical lensing residual.

What would falsify the scenario?

Dynamical versus optical observables

Nonrelativistic tracers follow $g_{\mu\nu}$ and measure the dynamical mass. Photons follow $\widetilde{g}_{\mu\nu}$ and measure the disformal optical potential.

Observable hints

- A negative refraction shell can produce impact parameter dependent corrections.
- The shell may enhance bending in one region and reduce it in another.
- Distortions can be observed.
- Newton potential can change *inside general relativity*.

Astrophysical target

- The main test is the correct mismatch between lensing maps and dynamical tracers.
- Even if the localized solution does not lead to any dark sector, the negative refraction phenomenon may be directly observed in analogy to solid state physics.

Conclusions

1. The refractive index studied in multiple cases.
2. The invariant gravitational analogue of Veselago's condition has been provided.
3. A disformal photon metric can generate a nonstandard flux contribution.
4. Metamaterials as dark matter.

Perspectives

1. Deriving the most general class of scalar profiles.
2. Studying backreaction.
3. Testing the model with data (galaxy cluster lensing, Einstein-radius shifts, shear profiles, magnification and time delays.)
4. Consequences in cosmology?

Contact

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Thanks!