

Emergent Time and Big-Bang

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arXiv: 1301.1361 with Jean-Philippe Uzan

arxiv: 1303.1409

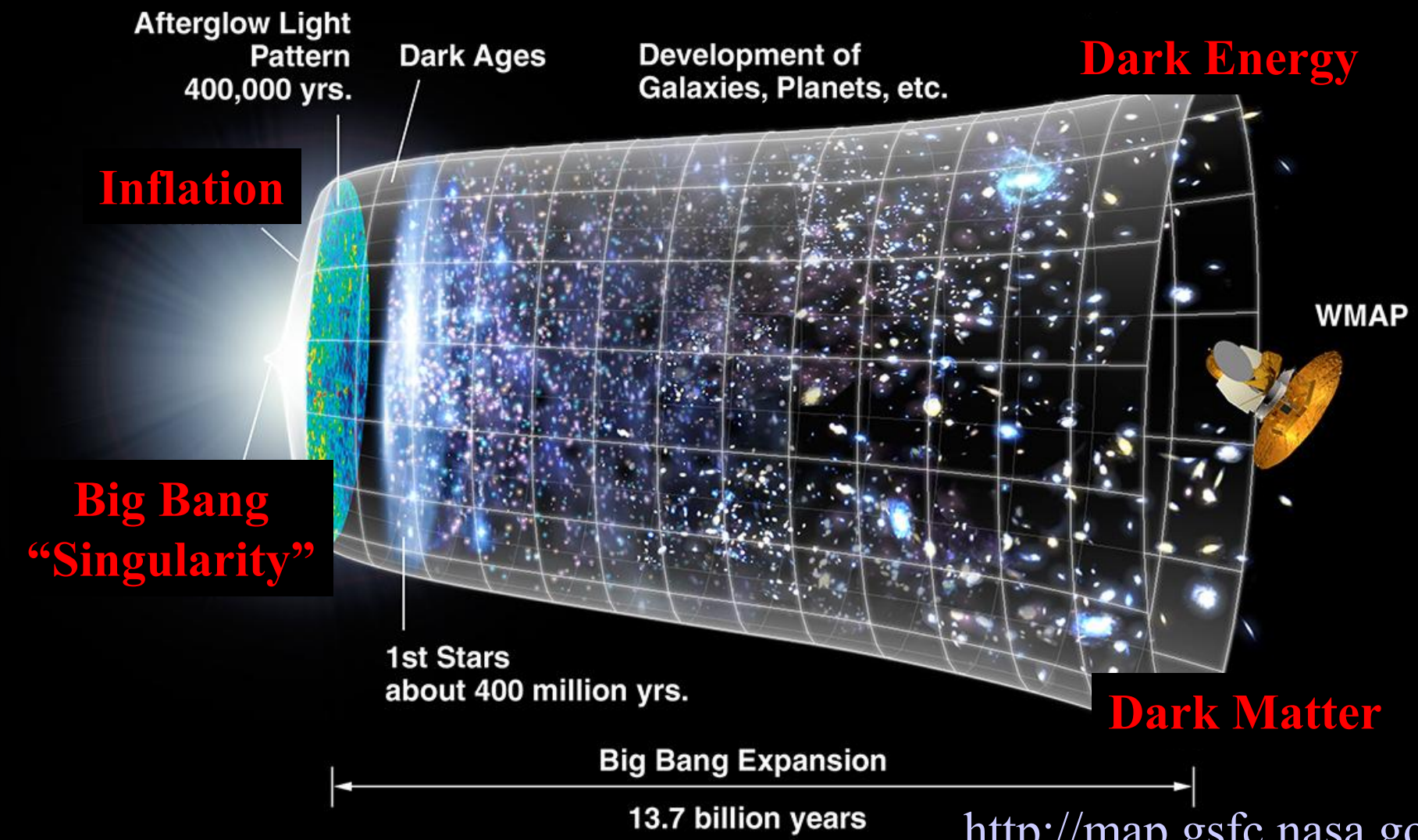
arXiv: 1403.0580 with John Kehayias & Jean-Philippe Uzan

arXiv: 2310.17266 & 2505.00112 with Justin Feng & Sante Carloni

arXiv: 2602.02646 with Justin Feng & Jean-Philippe Uzan

INTRODUCTION

History of the universe



http://map.gsfc.nasa.gov/

History of the universe

- History = dynamics = sequence of configurations parameterized by time
- Beginning of the hot universe @ reheating
- Geometrical description of the universe breaks down @ initial singularity
- Space may be emergent
- How about time? Can time be emergent?

Time and dynamics

- In any diffeo-invariant theories of gravity,
  $H = \Sigma$ constraints $= 0$ (up to boundary terms)
  ➔ no evolution of quantum state
- Dynamics should be encoded as correlations among various fields
  ➔ one of the fields plays the role of time
  e.g. inflaton during inflation
- In this sense, concept of time and dynamics may be emergent

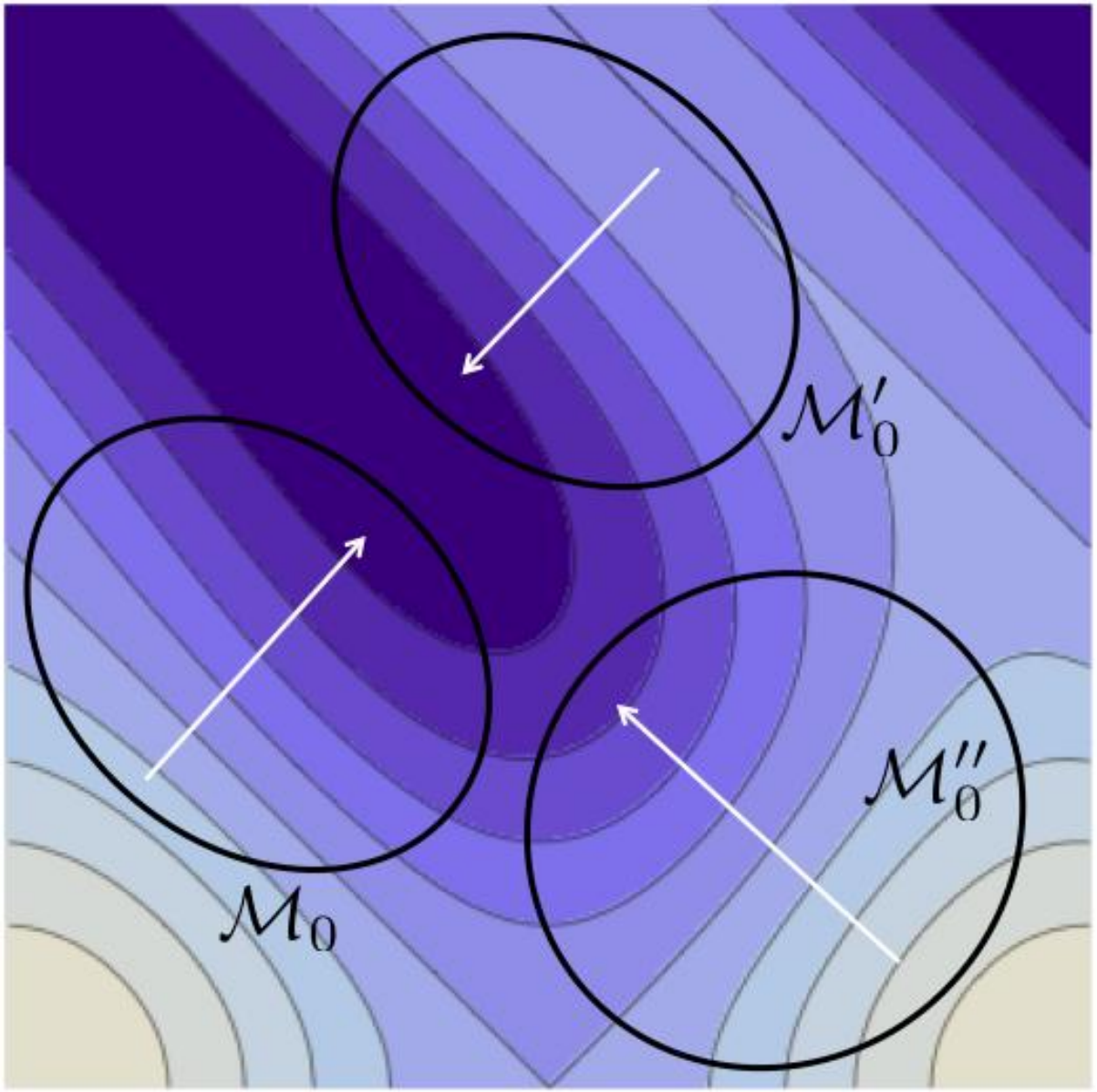
BASIC IDEA

arXiv: 1301.1361 with Jean-Philippe Uzan

Clock field

- Clock field = field playing the role of time
- It must carry at least one number
  → simplest: scalar field ϕ
- Time translational symmetry requires
  shift symmetry: $\phi \rightarrow \phi + c$
- Time reflection symmetry requires
  Z_2 symmetry: $\phi \rightarrow -\phi$

- Clock field does not have to be the same everywhere; multi-clock models also possible


$\mathcal{M}_0'$
$\mathcal{M}_0''$
$\mathcal{M}_0$

Effective metric

- Lorentz symmetry may be emergent (Chadha & Nielsen 1983)
- How about Lorentz signature & time itself?
- Let's suppose that there is no concept of time @ fundamental level and start with 4D Riemannian (i.e. locally Euclidean) metric with (++++) signature.
- Physical fields couple to effective metric.
- Can effective metric have signature (-+++) ?

SIMPLE EXAMPLES

arXiv: 1301.1361 with Jean-Philippe Uzan

Scalar field χ in flat space

- Suppose that $\boxed{\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0}$
- Choose one of coordinates t so that $t \equiv \dfrac{\phi}{M^2}$
- Consider the Euclidean action

$$S_\chi = \int d^4x \left[-\frac{1}{2}\delta^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) + \frac{\alpha}{M^4}\left(\delta^{\mu\nu}\partial_\mu\phi\partial_\nu\chi\right)^2 \right]$$

potential

Euclidean kinetic term

coupling to clock field

- This can be rewritten as

$$\boxed{S_\chi = \int dt d^3x \left[-\frac{1}{2} g_{eff}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) \right]}$$

$$g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if $\alpha > 1/2$!

Vector field A_μ in flat space

- Suppose that $\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0$
- Choose one of coordinates t so that $t \equiv \frac{\phi}{M^2}$
- Consider the Euclidean action

$$S_\chi = \int d^4x \left[-\frac{1}{4}\delta^{\mu\rho}\delta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} + \frac{\alpha}{M^4}\delta^{\mu\rho}\delta^{\nu\sigma}\delta^{\alpha\beta}F_{\rho\alpha}F_{\sigma\beta}\partial_\mu\phi\partial_\nu\phi \right]$$

Euclidean kinetic term — coupling to clock field

- This can be rewritten as

$$S_\chi = \int dt d^3x \left[-\frac{1}{4} g_{eff}^{\mu\rho} g_{eff}^{\nu\sigma} F_{\mu\nu}F_{\rho\sigma} \right]$$

$$g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if $\alpha > 1/2$!

Abelian Higgs field ω in flat space

- Suppose that $\partial_\mu \phi = \text{const.} \neq 0 \text{ in } \mathcal{M}_0$
- Choose one of coordinates t so that $t \equiv \frac{\phi}{M^2}$
- Consider the Euclidean action

$$D_\mu \equiv \partial_\mu - iqA_\mu$$

$$S_\chi = \int d^4x \left[-\frac{1}{2}\delta^{\mu\nu}\left(D_\mu\omega\right)^*\left(D_\nu\omega\right) - U(|\omega|^2) + \frac{\alpha}{M^4}\left|\delta^{\mu\nu}\partial_\mu\phi D_\nu\omega\right|^2 \right]$$

potential

Euclidean kinetic term

coupling to clock field

- This can be rewritten as

$$S_\chi = \int dt d^3x \left[-\frac{1}{2} g_{eff}^{\mu\nu}\left(D_\mu\omega\right)^*\left(D_\nu\omega\right) - U(|\omega|^2) \right] \qquad g_{eff}^{\mu\nu} = \begin{pmatrix} 1-2\alpha & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

- Lorentz signature emerges if $\alpha > 1/2$!

Lorentz violation

- Lorentz-invariance is broken @ UV
- **Lorentz-invariant IR fixed-point** is generic [Chadha & Nielsen 1983] (and may apply to GW as well; cf. $|c_{gw}^2 - c_\gamma^2| < 10^{-15}$ from GW170817) but running is slow (logarithmic)
- SUSY (w/o boost) or/and strong dynamics may speed-up the RG running towards Lorentz-invariant IR fixed-point
- Eg. SUSY (w/o boost) + SM gauge symmetry forbid dim-3 and -4 LV operators [Groot Nibbelink & Pospelov 2005]. Dim-5 LV operator can be forbidden by CPT.

GRAVITY @ LOW MOMENTA

arXiv: 1301.1361 with Jean-Philippe Uzan

Model of clock field and gravity

- Riemannian diffo invariance
- Shift symmetry: $\phi \rightarrow \phi + c$
- Z_2 symmetry: $\phi \rightarrow -\phi$
- Minimal # of d.o.f. → 2nd-order EOM → Riemannian shift- & Z_2-symmetric Horndeski

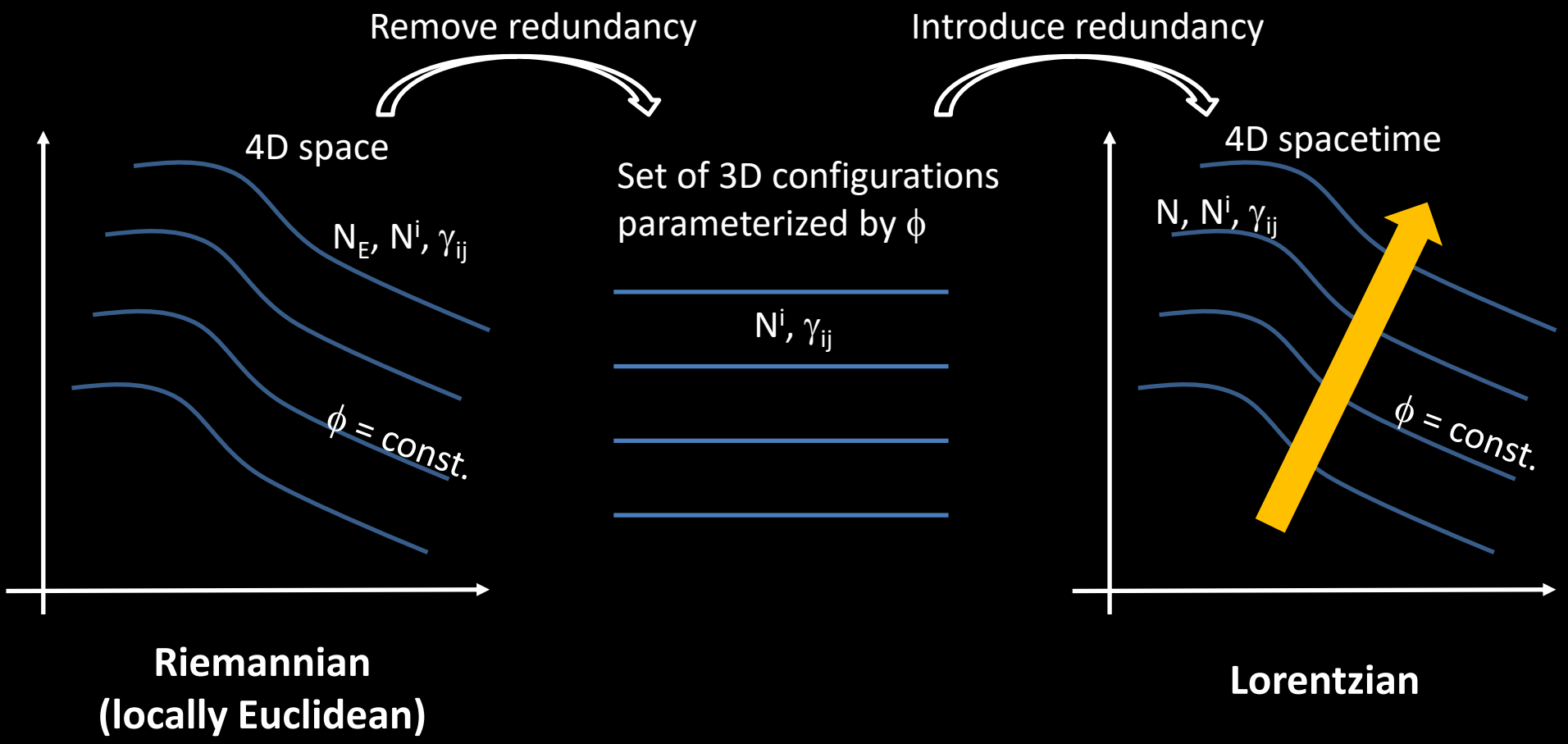
$$S_g = \int \mathrm{d}x^4\sqrt{g_{\mathrm{E}}}\Big\{G_4(X_{\mathrm{E}})R_{\mathrm{E}} - g_5 G_{\mathrm{E}}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}})\left[(\nabla_{\mathrm{E}}^2\phi)^2 - (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2\right]\Big\}$$

$$X_{\mathrm{E}} \equiv g_{\mathrm{E}}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \qquad (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2 \equiv g_{\mathrm{E}}^{\nu\rho}g_{\mathrm{E}}^{\sigma\mu}(\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)(\nabla_\rho^{\mathrm{E}}\nabla_\sigma^{\mathrm{E}}\phi)$$

- Redefinition of $G_4(X_E)$ → $g_5 = 0$

$$S_g = \int \mathrm{d}x^4\sqrt{g_{\mathrm{E}}}\Big\{G_4(X_{\mathrm{E}})R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}})\left[(\nabla_{\mathrm{E}}^2\phi)^2 - (\nabla_\mu^{\mathrm{E}}\nabla_\nu^{\mathrm{E}}\phi)^2\right]\Big\}$$

Riemannian diffeo. vs Lorentzian diffeo.



Correspondence

$$S_g = \int \mathrm{d}x^4 \sqrt{g_{\mathrm{E}}} \left\{ G_4(X_{\mathrm{E}}) R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G_4'(X_{\mathrm{E}}) \left[(\nabla_{\mathrm{E}}^2 \phi)^2 - (\nabla_\mu^{\mathrm{E}} \nabla_\nu^{\mathrm{E}} \phi)^2 \right] \right\}$$

$$g_{\mu\nu} = g^{\mathrm{E}}_{\mu\nu} - \frac{\partial_\mu \phi \partial_\nu \phi}{X_c}$$

$$g^{\mu\nu} = g_{\mathrm{E}}^{\mu\nu} + \frac{g_{\mathrm{E}}^{\mu\rho} g_{\mathrm{E}}^{\nu\sigma} \partial_\rho \phi \partial_\sigma \phi}{X_c - X_{\mathrm{E}}}$$

$$\frac{1}{X} = \frac{1}{X_c} - \frac{1}{X_{\mathrm{E}}} \qquad X_c = \frac{M^4}{N_c^2}$$

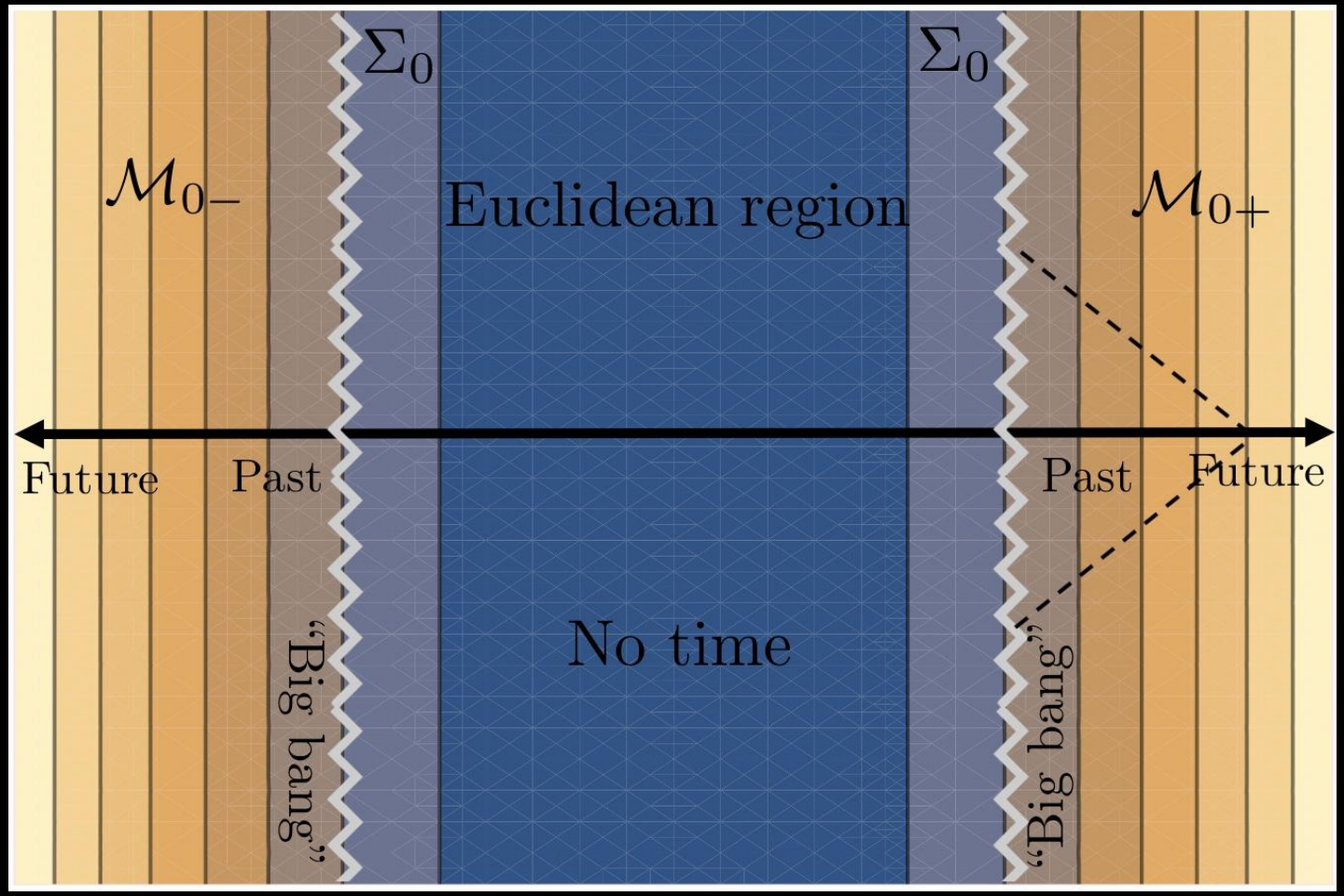
$$\frac{f(X)}{\sqrt{X}} = \frac{G_4(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}} \qquad \frac{P(X)}{\sqrt{X}} = \frac{\mathcal{K}(X_{\mathrm{E}})}{\sqrt{X_{\mathrm{E}}}}$$

$$S_g = \int \mathrm{d}x^4 \sqrt{-g} \left\{ f(X) R + 2 f'(X) \left[(\nabla^2 \phi)^2 - (\nabla^\mu \nabla^\nu \phi)(\nabla_\mu \nabla_\nu \phi) \right] + P(X) \right\}$$

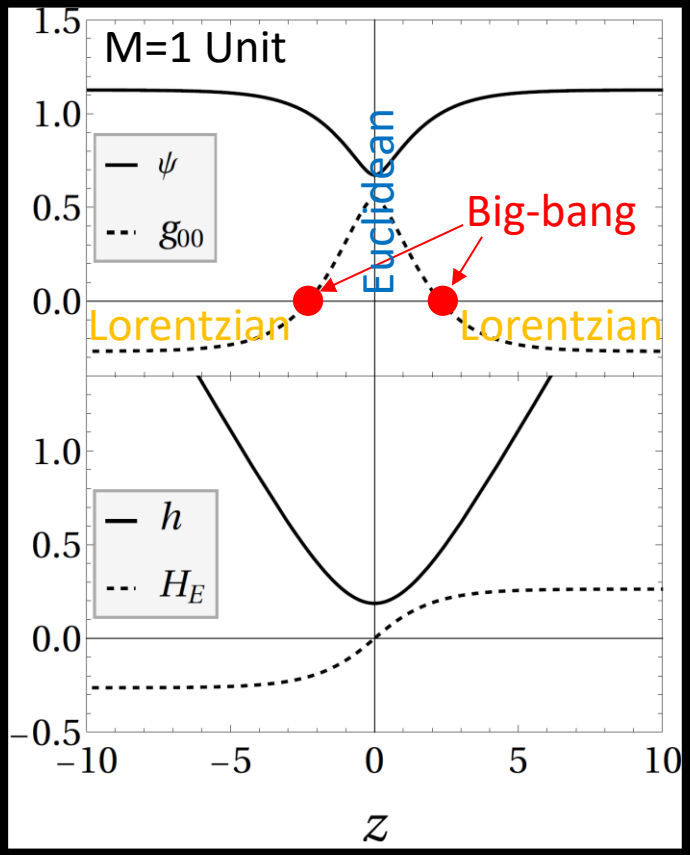
EMERGENT BIG-BANG

arXiv: 2602.02646 with Justin Feng & Jean-Philippe Uzan

Mirror Universe from Emergent Big-Bang



Mirror Universe from Emergent Big-Bang



Euclidean metric

$$g^{\mathrm{E}}_{\mu\nu}\mathrm{d}x^{\mu}\mathrm{d}x^{\nu} = N^2_{\mathrm{E}}(z)\mathrm{d}z^2 + \mathrm{e}^{2h(z)}\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

Effective metric

$$g_{\mu\nu} = g^{\mathrm{E}}_{\mu\nu} - \frac{\partial_\mu\phi\partial_\nu\phi}{M^4} \qquad g_{00} = \left(\frac{\psi^2}{M^4} - 1\right)$$

Definitions

$$H_{\mathrm{E}} \equiv \dot{h}/N_{\mathrm{E}} \qquad \psi(z) = \dot{\phi}/N_{\mathrm{E}}$$

Hubble expansion rate

$$H = \left(\frac{\psi^2}{M^4} - 1\right)^{-1/2} H_{\mathrm{E}}$$

diverges @ $g_{00} = 0$

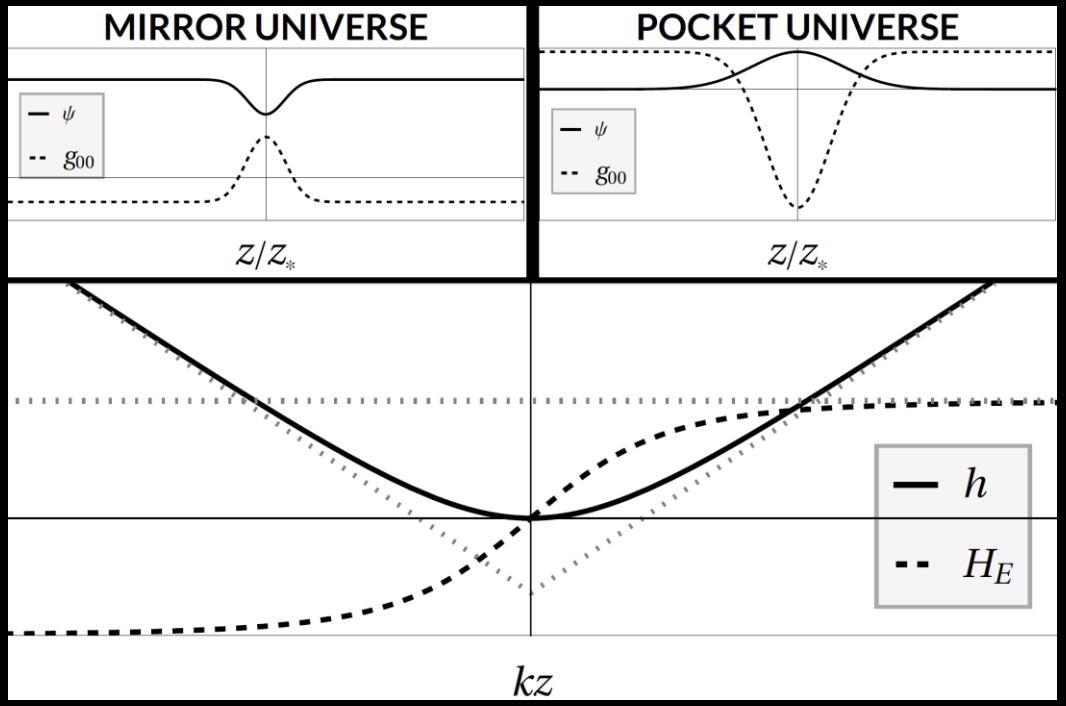
Concrete example

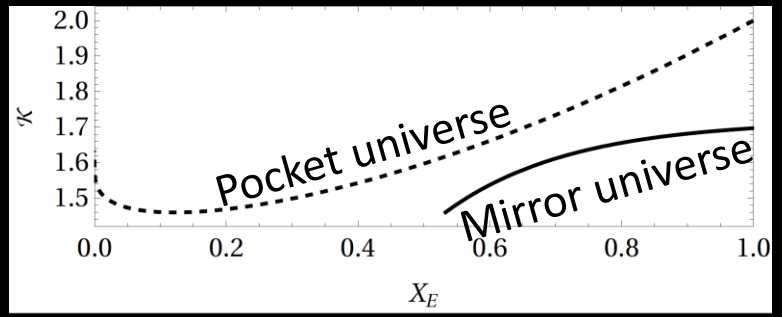
$$S_g = \int \mathrm{d}x^4\sqrt{g_{\mathrm{E}}}\left\{G_4(X_{\mathrm{E}})R_{\mathrm{E}} + \mathcal{K}(X_{\mathrm{E}}) - 2G'_4(X_{\mathrm{E}})\left[(\nabla^2_{\mathrm{E}}\phi)^2 - (\nabla^{\mathrm{E}}_\mu\nabla^{\mathrm{E}}_\nu\phi)^2\right]\right\}$$

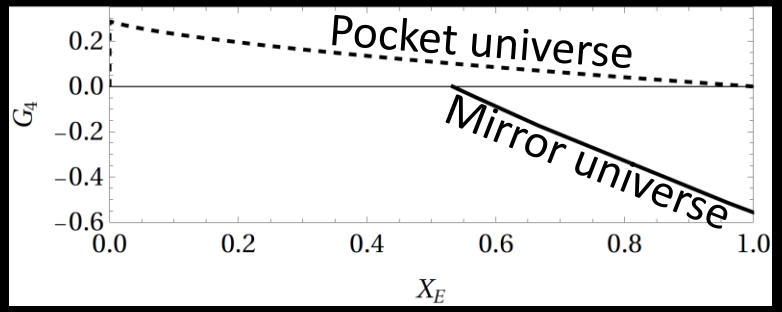
$$G_4 = g_0 + g_1 X_{\mathrm{E}}, \quad \mathcal{K} = k_0 + \tfrac{1}{2}k_2(X_0 - X_{\mathrm{E}})^2$$

$$(g_0, g_1, k_0, k_2, X_0) = (1, -1, 1, -1.55, 1)$$

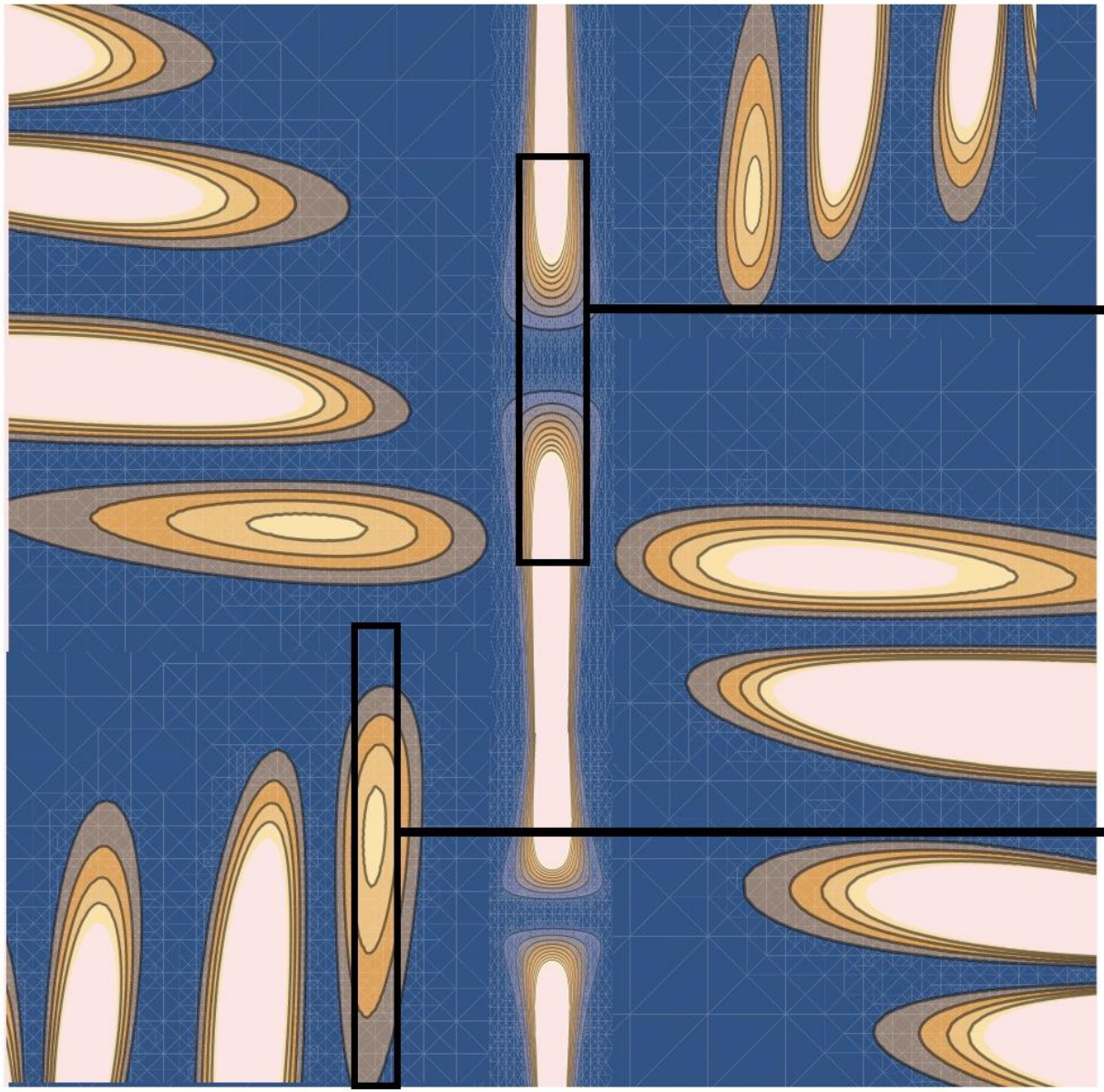
Reconstruction of $G_4(X_E)$ and $K(X_E)$



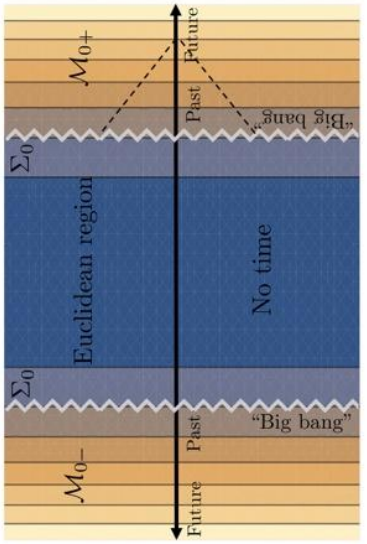


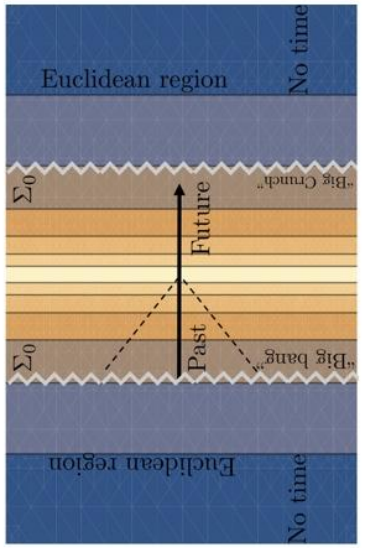


Possible Global Picture



Mirror universes





Pocket universe

CANDIDATE UV THEORY

arXiv: 1303.1409

Possible UV completion: scalar-tensor version of Stelle's gravity

arxiv: 1303.1409

- Riemannian (i.e. locally Euclidean) theory
- Shift symmetry: $\phi \rightarrow \phi + c$
- Z_2 symmetry: $\phi \rightarrow -\phi$
- 4D parity invariance: $x^\mu \rightarrow -x^\mu$
- Power-counting renormalizable
  → include all possible terms up to dim-4
- Building blocks

$$g^E_{\mu\nu} \quad \nabla^{\mathrm{E}}_\mu \quad R^{\mu\nu\rho\sigma}_{\mathrm{E}} \quad \phi \quad \partial_\mu\phi \quad \nabla^{\mathrm{E}}_\mu\nabla^{\mathrm{E}}_\nu\phi$$

UV action (4th derivatives)

$$X_{\rm E} \equiv g_{\rm E}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$I_4 = \int dx^4\sqrt{g_{\rm E}}\left[c_1 R_{\rm E}^2 + c_2 R_{\rm E}^{\mu\nu}R^{\rm E}_{\mu\nu} + c_3 R_{\rm E}^{\mu\nu\rho\sigma}R^{\rm E}_{\mu\nu\rho\sigma} + c_4 X_{\rm E} R_{\rm E}\right.$$
$$\left. + c_5 R_{\rm E}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c_6 X_{\rm E}^2 + c_7(\nabla^2_{\rm E}\phi)^2 + c_8(\nabla^{\rm E}_\mu\nabla^{\rm E}_\nu\phi)^2\right]$$

Relevant deformations (2nd derivatives & 0th derivatives)

$$I_2 = \int dx^4\sqrt{g_{\rm E}}\left[c_9 R_{\rm E} + c_{10} X_{\rm E}\right] \qquad I_0 = c_{11}\int dx^4\sqrt{g_{\rm E}}$$

Integration by parts (+ redef of $c_{7,8}$) & rescaling of ϕ

$$\Rightarrow \quad c_5 = 0 \qquad c_6 = 1$$

Total action

$$I = \int dx^4\sqrt{g_{\rm E}}\left[2Z\Lambda_{\rm E} - ZR_{\rm E} + \frac{1}{2\lambda}C^2_{\rm E} - \frac{\omega}{3\lambda}R^2_{\rm E} + \frac{\theta}{\lambda}E_{\rm E}\right.$$
$$\left. + X^2_{\rm E} - 2X_\star X_{\rm E} + \alpha(\nabla^2_{\rm E}\phi)^2 + \beta(\nabla^{\rm E}_\mu\nabla^{\rm E}_\nu\phi)^2 + \gamma X_{\rm E}R_{\rm E}\right]$$

$$C^2_{\rm E} \equiv R_{\rm E}^{\mu\nu\rho\sigma}R^{\rm E}_{\mu\nu\rho\sigma} - 2R_{\rm E}^{\mu\nu}R^{\rm E}_{\mu\nu} + R^2_{\rm E}/3$$

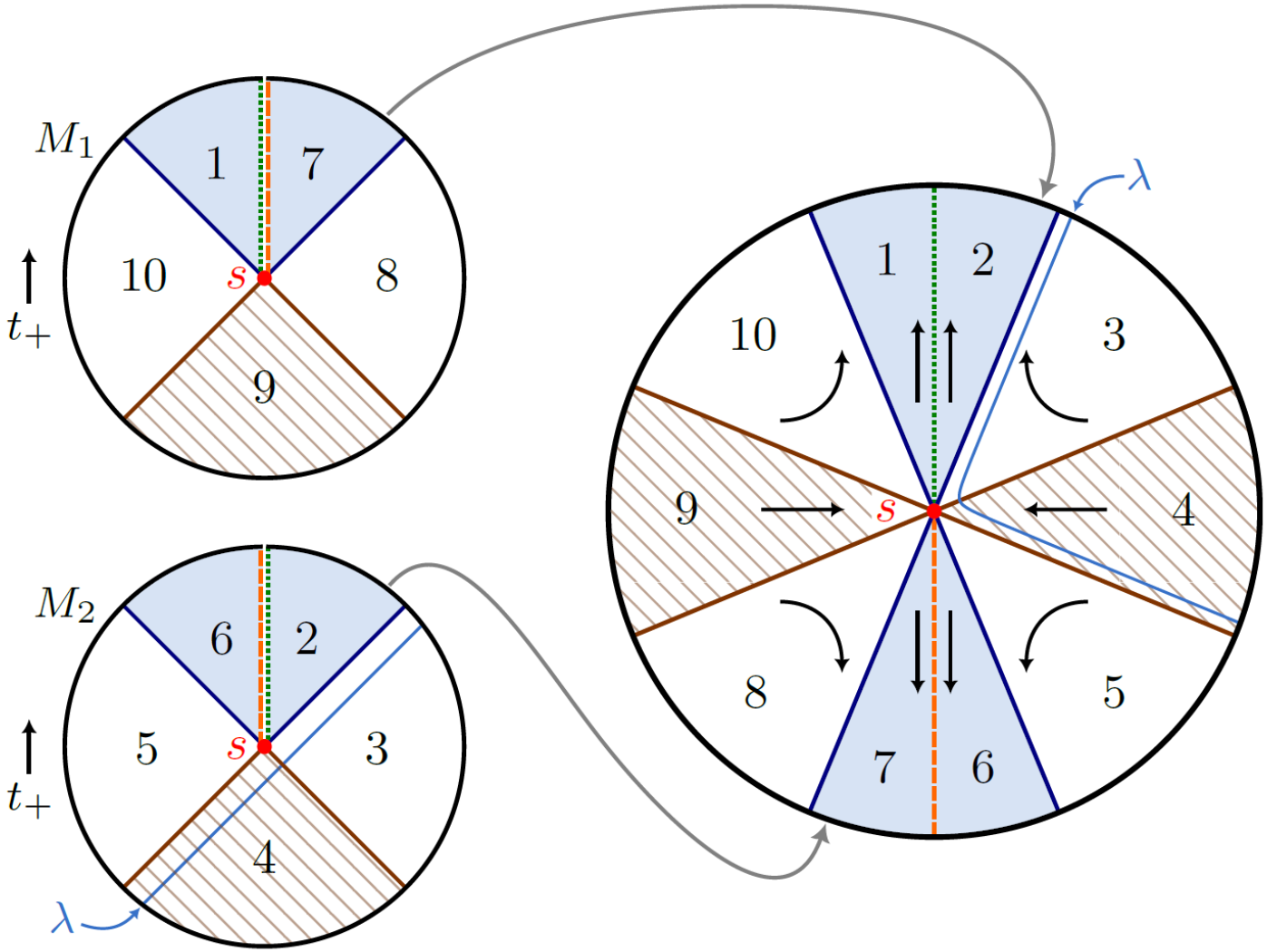
$$E_{\rm E} \equiv R_{\rm E}^{\mu\nu\rho\sigma}R^{\rm E}_{\mu\nu\rho\sigma} - 4R_{\rm E}^{\mu\nu}R^{\rm E}_{\mu\nu} + R^2_{\rm E}$$

- This theory is indeed renormalizable (Muneyuki & Ohta 2013).
- RG flow has not yet been studied.
- Lorentzian signature is recovered in IR under a certain condition [arXiv: 2505.00112 with Justin Feng & Sante Carloni].
- In UV, higher derivative terms become important and physics is purely Euclidean.
- Matter action also needs to be UV completed by Euclidean higher derivative terms.
- Euclidean quantization needs to be developed.
- …

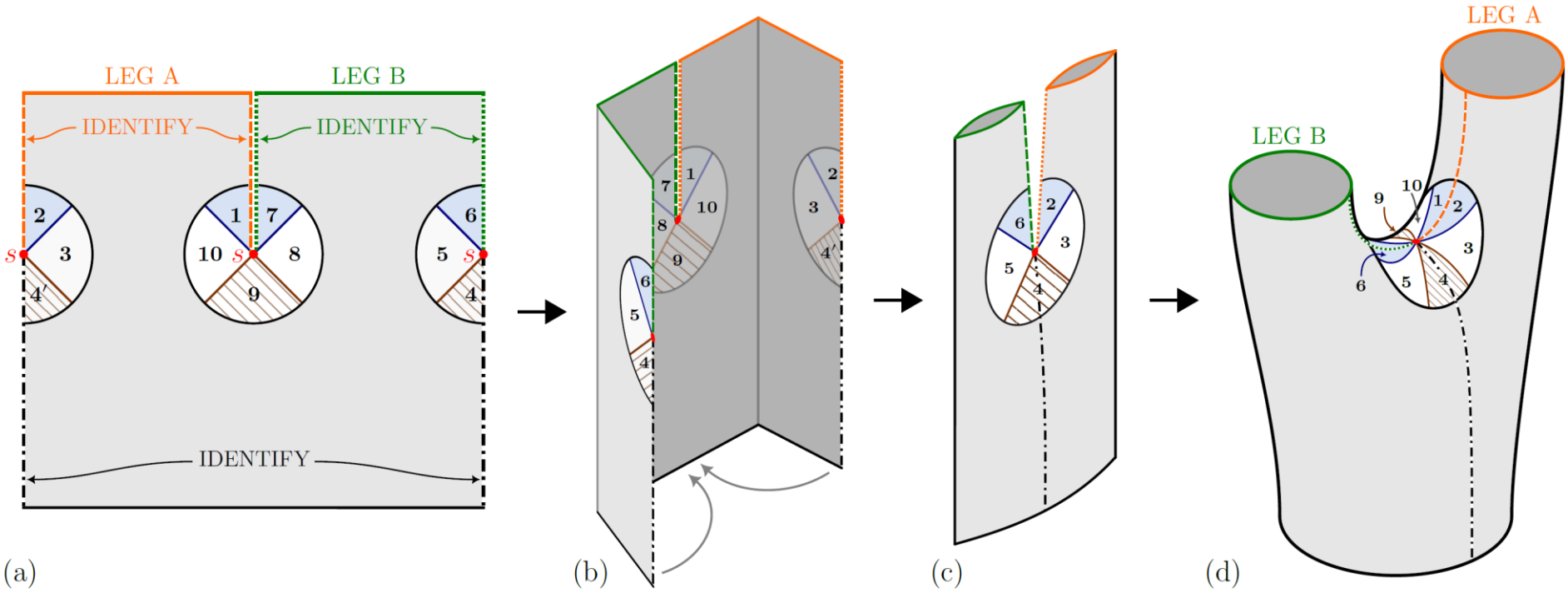
END OF BH EVAPORATION

arXiv: 2310.17266 with Justin Feng & Sante Carloni

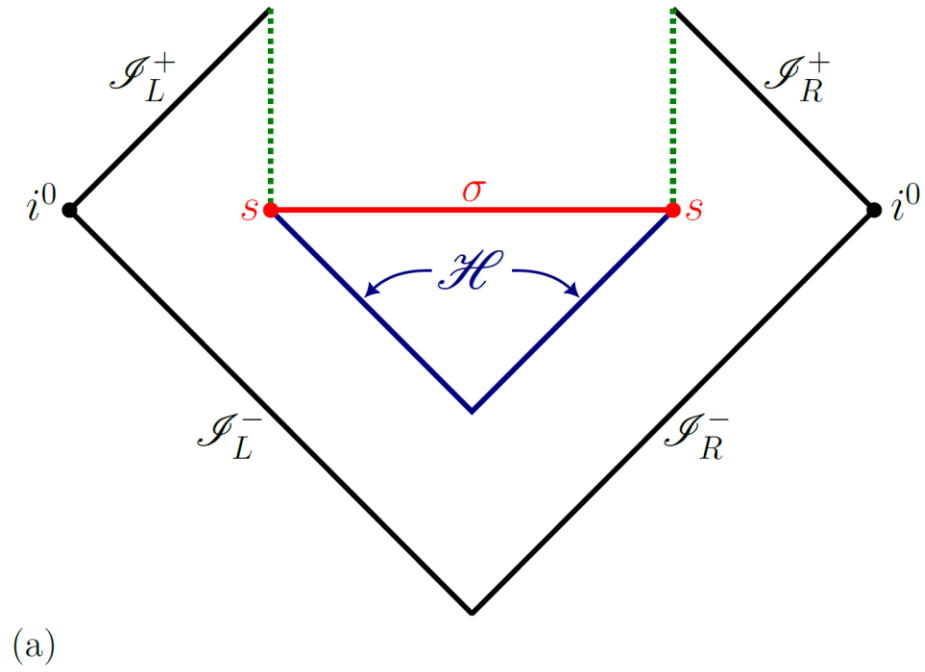
Saddle-like causally discontinuous singularity

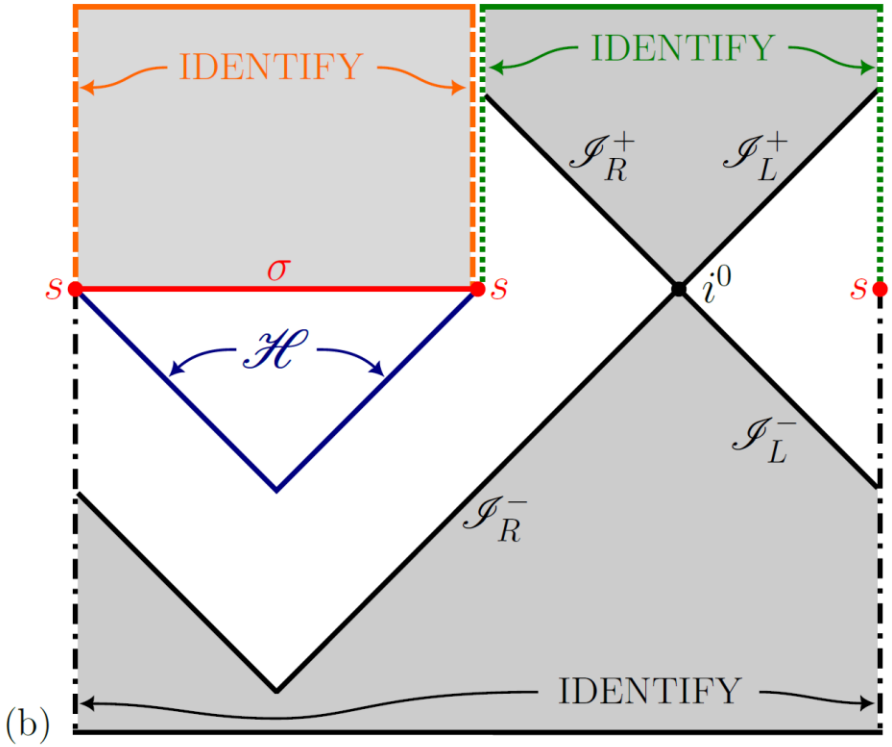


Trousers spacetime

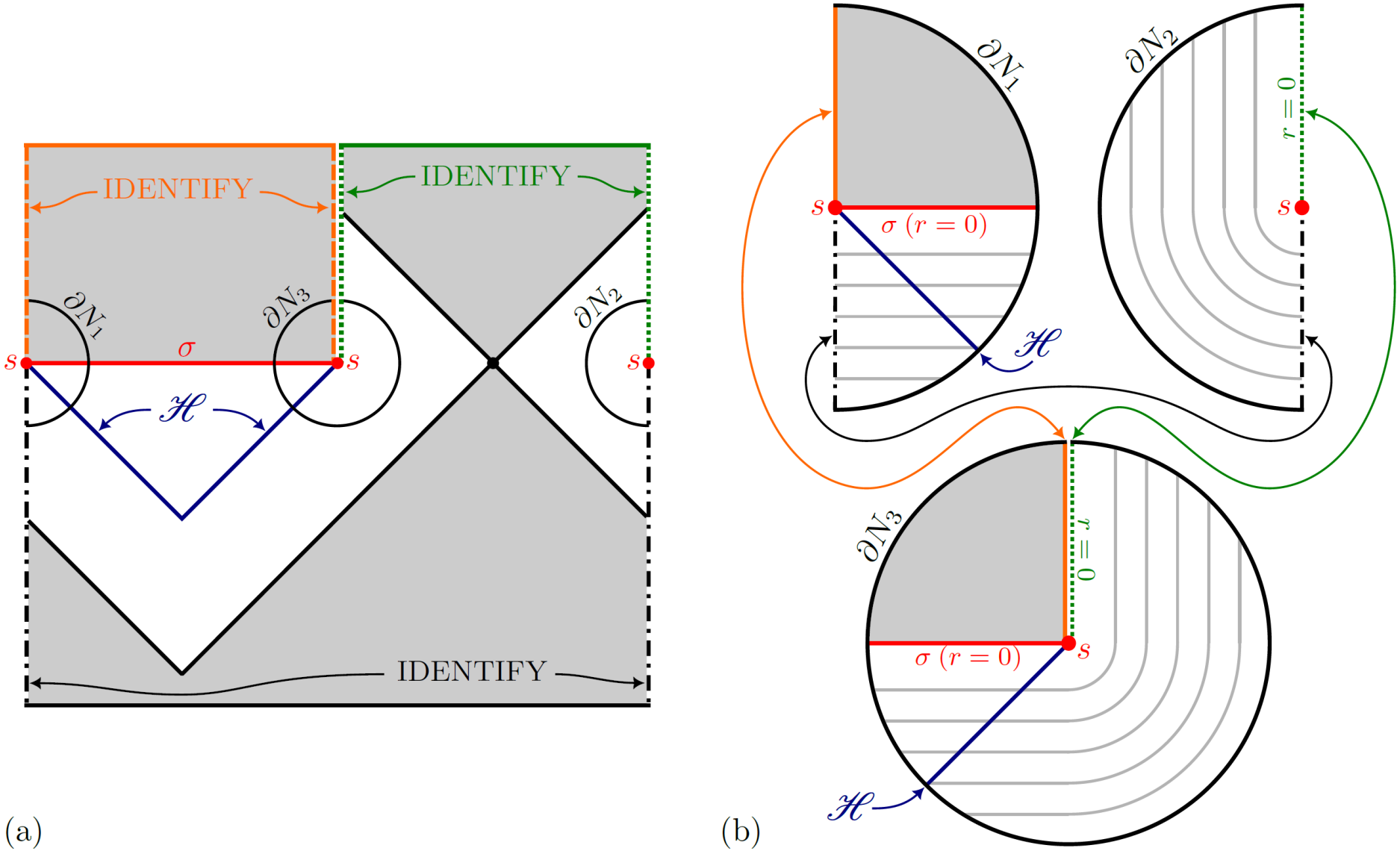


BH evaporation in 2D

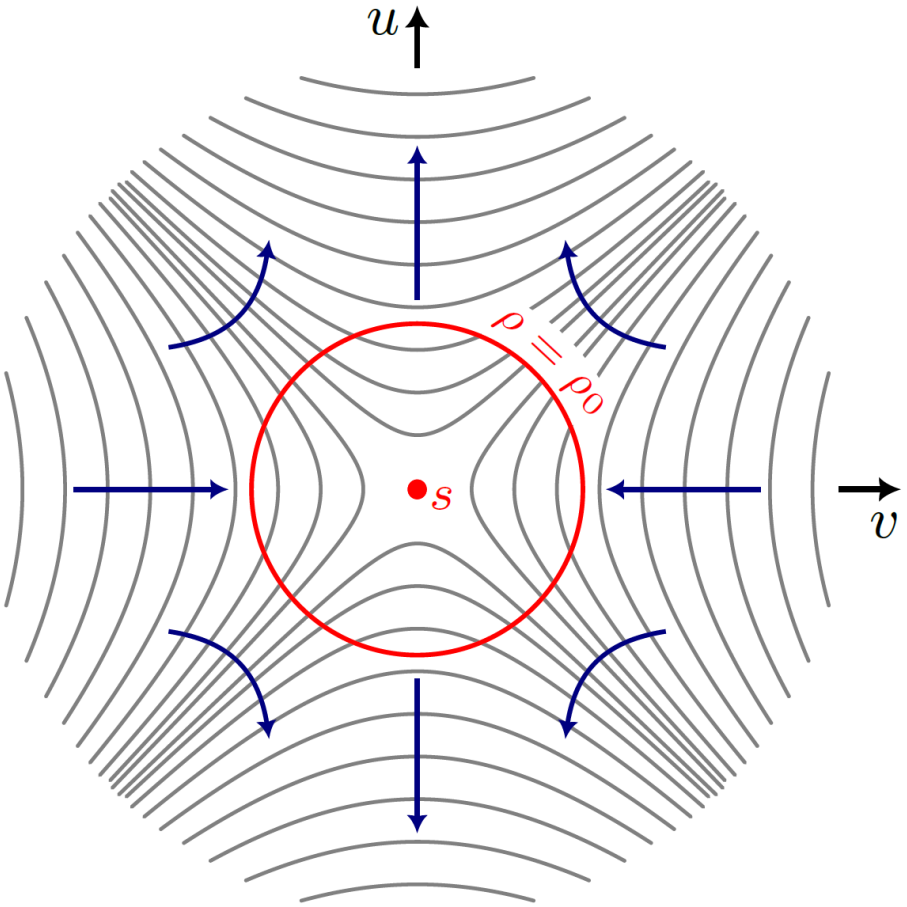




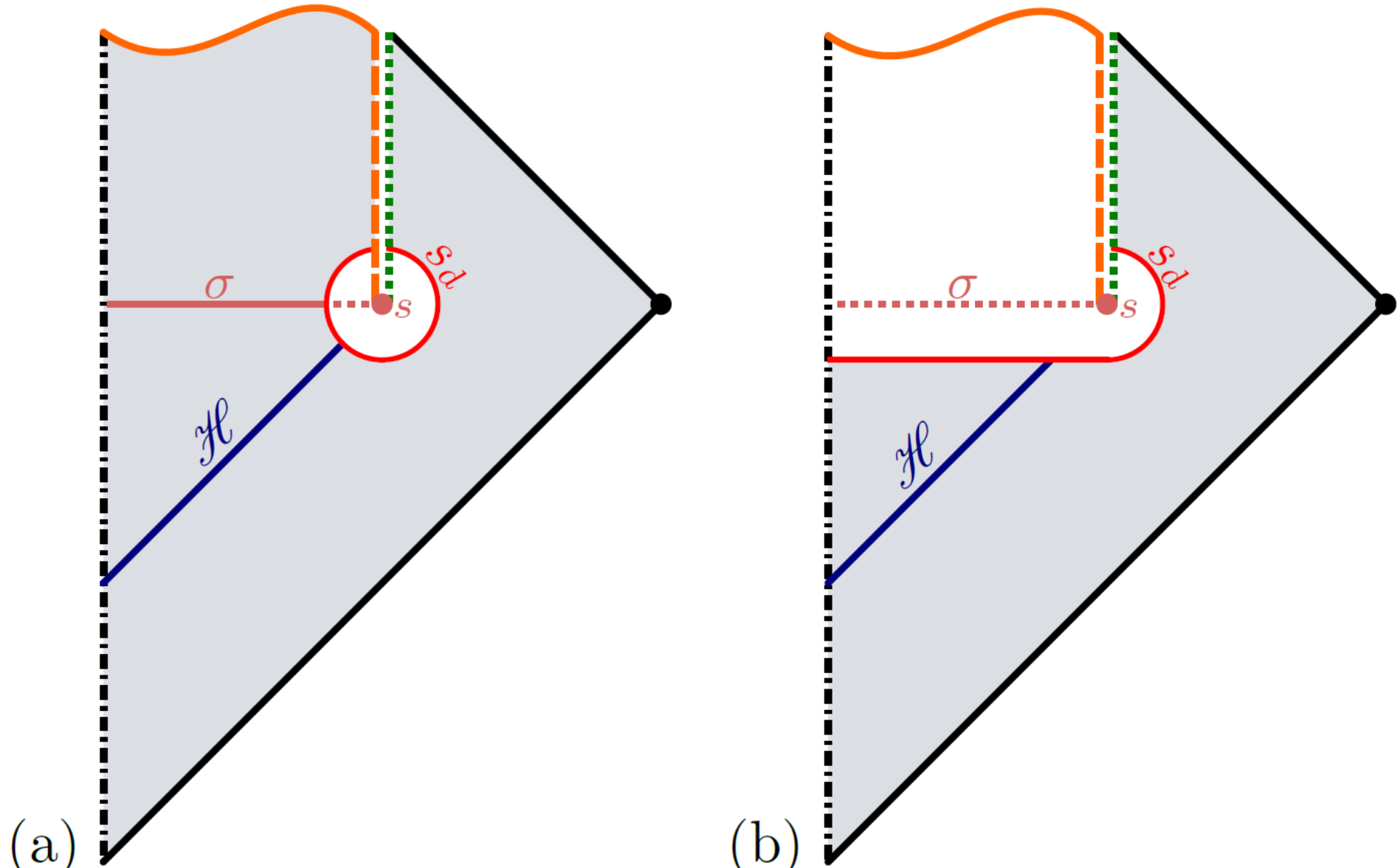
BH evaporation in (d+1)-D



Possible UV completion



Possible global picture



SUMMARY & DISCUSSIONS

Summary

- Lorentzian dynamics can emerge as an effective property of a fundamentally Riemannian theory.
- This requires introduction of a field playing the role of time, a clock field.
- This idea was applied to scalar, vector, (Dirac, Weyl, Majorana) spinor fields and gravity as explicit examples.
- In a simple realization, the clock field/gravity sector is described by the Riemannian version of a shift- and Z_2-symmetric Horndeski theory.
- We found a FLRW solution and analyzed stability of scalar- and tensor- perturbations.
- Big-bang may be replaced by Euclidean geometry.
- Endpoint of BH evaporation may be Euclidean geometry.
- We proposed a renormalizable Riemanian theory as a candidate UV theory.

Future works

- Emergent big-bang in the renormalizable Riemannian theory?
- Cosmological constant problem as boundary value problem?
- Emergent black holes?
- Lorentzian dS/CFT from Euclidean AdS/CFT?
- Emergence of Lorentz symmetry at low energy? [Chadha & Nielsen 1983; Groot Nibbelink & Pospelov 2005]
- Development of quantum theory?
- …

Gravity 2027

RESCEU International Symposium on Quantum Aspects of Gravity

- **Dates: February 15-19, 2027**
- **Venue: Hongo campus, The University of Tokyo, Japan.**
- **Main topics: quantum aspects of gravity, including string/quantum gravity, quantum cosmology, quantum field theory in curved spacetime, inflationary quantum fluctuations, quantum effects on gravity experiments, and so on.**

https://www.resceu.s.u-tokyo.ac.jp/symposium/gravity2027/index.html