Real-time linear response for gravitational perturbations

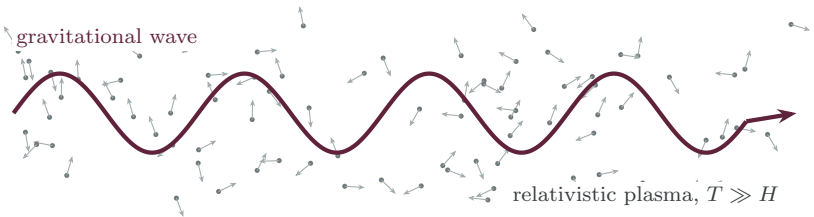
Atsuhisa Ota

Chongqing U.
Based on AO (2025); AO, H.-Y. Zhu, Y. Zhu (2026);
H. Lai, AO (2026)

September 17, 2026

A wave in a medium

Question: What happens to a gravitational wave traveling through a medium?



- The early universe *was* a medium: a hot radiation plasma, then free-streaming neutrinos
- Every primordial GW we hope to detect (CMB B-modes, PTA, LISA, ET) crossed it

Roadmap



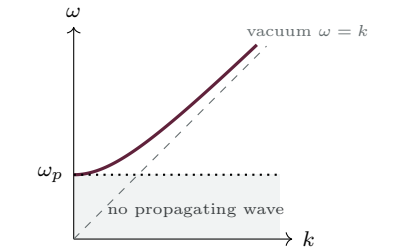
1. Waves in media: what plasma physics teaches us
2. One framework: real-time linear response of the medium
3. Medium 1: thermal radiation — a graviton “plasmon”
4. Medium 2: radiation with spin — does the plasma leave fingerprints?
5. Medium 3: a resonant, non-thermal medium — the UV structure
6. The answer and outlook



Waves in media

Plasma physics has a clear answer for light. What about gravity?

Light in a plasma



1. A gap

$$\omega^2 = k^2 + \omega_p^2, \qquad \omega_p \sim eT$$

- Below ω_p: no propagation — this is why metals reflect light
- Gauge invariant: no mass in the Lagrangian, only in the dispersion

2. Landau damping

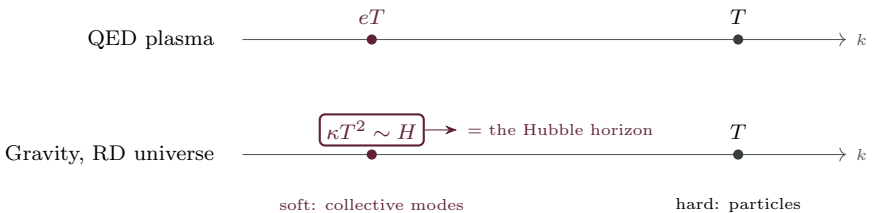
- Particles surfing with the wave exchange energy
- Damping without collisions

The plasma dresses the wave at the soft scale, where collective modes live.

[Braaten, Pisarski (1990); Le Bellac, *Thermal Field Theory*]

The gravitational version

Soft scale = coupling × temperature:



Friedmann: $H^2 = \kappa^2\rho/12 \sim \kappa^2 T^4 \;\Rightarrow\; \kappa T^2 \sim H$ ($\kappa^2 = 32\pi G$).

The graviton "plasmon gap" sits exactly at the Hubble horizon.

[Rebhan (1991); Brandt, Frenkel (1993)]

Taken literally, this is a paradox

- **Plasma intuition:** superhorizon GWs ($k < H$) lie below the gap
  $\Rightarrow$ they should oscillate and decay?

Taken literally, this is a paradox

- **Plasma intuition:** superhorizon GWs ($k < H$) lie below the gap
  $\Rightarrow$ they should oscillate and decay?
- **Standard cosmology:** superhorizon tensor modes are frozen and conserved
  $\Rightarrow$ the basis of every inflationary prediction

[Weinberg (2003)]

Taken literally, this is a paradox

- **Plasma intuition:** superhorizon GWs ($k < H$) lie below the gap
  $\Rightarrow$ they should oscillate and decay?
- **Standard cosmology:** superhorizon tensor modes are frozen and conserved
  $\Rightarrow$ the basis of every inflationary prediction

[Weinberg (2003)]

- **Earlier one-loop (in-in) results:** a *tachyonic* mass and secular growth

$$m_{\rm eff}^2 = -\tfrac{2}{5}H^2, \qquad P_h^{\text{1-loop}} \sim P_h^{\rm tree}\ln(\tau/\tau_R)$$

[AO, Sasaki, Wang (2023); Fröb, Glavan, Meda, Sawicki (2025)]

Taken literally, this is a paradox

- **Plasma intuition:** superhorizon GWs ($k < H$) lie below the gap
  $\Rightarrow$ they should oscillate and decay?
- **Standard cosmology:** superhorizon tensor modes are frozen and conserved
  $\Rightarrow$ the basis of every inflationary prediction

[Weinberg (2003)]

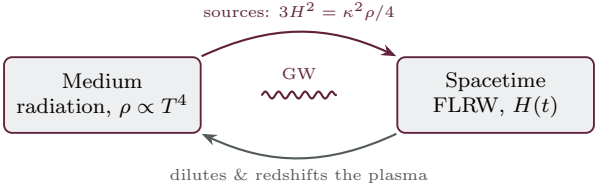
- **Earlier one-loop (in-in) results:** a *tachyonic* mass and secular growth

$$m_{\rm eff}^2 = -\tfrac{2}{5}H^2, \qquad P_h^{\text{1-loop}} \sim P_h^{\rm tree}\ln(\tau/\tau_R)$$

[AO, Sasaki, Wang (2023); Fröb, Glavan, Meda, Sawicki (2025)]

Gap, tachyon, or nothing? We need to know how the medium really responds.

For gravity, the medium makes the spacetime



- Light in a lab: the plasma sits in a fixed Minkowski space
- GWs in cosmology: the same plasma drives the expansion
- Thermal Minkowski is *off shell*: $G_{\mu\nu} = 0$ but $T_{\mu\nu} \neq 0$
- Setting $a \to 1$ kills the effect we want to study

We must compute the response on a dynamical (FLRW) background.

[AO (2025)]

One framework: linear response in FLRW

The same equation for every medium — only the response changes.

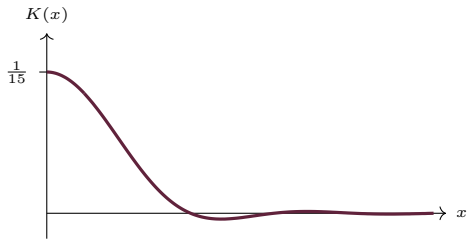
The benchmark: Weinberg's damping equation

Collisionless kinetic theory (free-streaming radiation):

$$h_{ij}'' + 2\mathcal{H}h_{ij}' + k^2 h_{ij} = -24\,\mathcal{H}^2 \int_{\tau_0}^{\tau} d\bar{\tau}\; K\big(k(\tau-\bar{\tau})\big)\,\partial_{\bar{\tau}} h_{ij}(\bar{\tau})$$

- $K(x) = j_2(x)/x^2$: a memory kernel
- Depends on the plasma only through $\mathcal{H} = aH$ (Friedmann eq.)
- Damps tensor modes at horizon entry

Here the whole radiation background free-streams; with a free-streaming fraction f the coefficient scales accordingly.



Goal: derive this from QFT on FLRW — and explain the one-loop puzzles.

[Weinberg (2004); Rebhan, Schwarz (1994)]

How would we reproduce it in QFT?

Linear response: the induced stress is a retarded commutator of stress tensors,



$$\equiv\ T^R_{\mu\nu}(x)$$

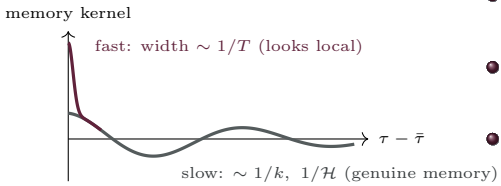
$$= i \int dy^0\, \Theta(x^0 - y^0)\, \big\langle\, \big[\ \underbrace{H_{\rm int}(y^0)}_{\frac{\kappa}{2}\int d^3y\, a^2\, h^{\mu\nu} T^I_{\mu\nu}}\ , T^I_{\mu\nu}(x)\,\big]\,\big\rangle .$$

- One diagram: two stress-tensor vertices, hard radiation in the loop
- The hard particles free-stream, exactly as in the kinetic calculation

Natural expectation: compute this one retarded diagram, and Weinberg's equation should come out.

[Kubo (1957); AO (2025)]

Two time scales in retarded response



- **Fast** ($\beta = 1/T$): the plasma reacts almost instantly — looks like a local mass term
- **Slow** ($1/k$, $1/\mathcal{H}$): the genuine memory of the medium
- The slow part does reproduce K — but a local piece $-2\mathcal{H}^2 h_{ij}$ survives
- Its IR limit is $m_{\rm eff}^2 = -\frac{2}{5}H^2$: the secular growth

Is this an extra term that QFT requires — or did we do something wrong?

[AO, Zhu, Zhu (2026)]

The response has three pieces, not one

$$h_{ij}'' + 2\mathcal{H}h_{ij}' + k^2 h_{ij} = \underbrace{-\frac{\kappa}{2}\, T_{ij}^{\rm lin}[h]}_{\text{medium response}}\ , \qquad \kappa = \sqrt{32\pi G}.$$



T^R: retarded response — history of the plasma (nonlocal)



T^C: contact response — explicit metric dependence of $T_{\mu\nu}$



$T^{\rm os}$: on-shell projection — the background equations of motion

Drop any one of the three, and diffeomorphism invariance is lost.

[AO (2025); AO, Zhu, Zhu (2026)]



Medium 1: thermal radiation

Build the response on FLRW, and let symmetry referee it.

What symmetry demands of the response



QED: the photon self-energy must be transverse, $k_\mu \Pi^{\mu\nu} = 0$



Gravity on a *flat* background, $\bar{T}_{\mu\nu} = 0$: $\partial^\mu \bar{\Pi}_{\mu\nu\rho\sigma} = 0$

- Both follow from one statement: $\delta_\xi \Gamma = 0$ for the medium action
- With $\bar{T}_{\mu\nu} \neq 0$ the transversality is modified by the background stress — known since the first graviton HTL analyses

On FLRW there is more: the correction is tied to the Weyl (trace) identity.

[Rebhan (1991); Brandt, Frenkel (1993); AO (2025)]

On FLRW, diffeomorphism and Weyl are entangled

Schematically (indices suppressed):

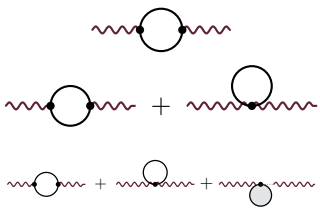
$$\partial \cdot (a^4 \bar{\Pi}) \;=\; \underbrace{\partial\,(a^2 \bar{T})}_{\text{background stress}} \;-2\,(\partial \ln a)\; \underbrace{\left[\bar{T} - \tfrac{1}{2}\,\mathrm{tr}\,\bar{\Pi}\right]}_{\text{the Weyl identity}}$$

- The bracket vanishes for conformal matter — it *is* the Weyl identity
- So a response that violates Weyl invariance violates the diffeomorphism identity as well
- On an expanding background the two symmetries become one test

That entanglement is what makes the check sharp — and sensitive to the state.

[AO (2025)]

Verdict: which construction passes?



✗ retarded only: non-covariant Schwinger terms survive

✓ **this work**: Weyl and diffeomorphism identities both hold

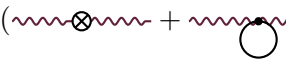
✗ perturbed ensemble: $\mathrm{tr}\,\bar{\Pi}^S \neq 0$ breaks the entangled identity

Retarded + contact in the canonical formalism is diffeomorphism consistent.

Weinberg's equation from QFT

TT projection of the full response in a radiation universe:

$$a^2\kappa^2\,\bar{\Pi}^{\mathrm{TT}}_{ijkl}h^{kl} = \mathcal{H}^2\Big(\underbrace{-2h_{ij}}_{\text{fast}} + \underbrace{24K(k\tau)\,\sigma_{ij}}_{\text{boundary}} + \underbrace{24\int_{\tau_0}^{\tau} d\bar{\tau}\,K\big(k(\tau-\bar{\tau})\big)\,\partial_{\bar{\tau}}h_{ij}}_{\text{slow memory}}\Big)$$

- Fast part cancels:  + Fast = 0.
- σ_{ij}: homogeneous solution; $\sigma_{ij} = 0$ so that superhorizon modes are not sourced (large diffeomorphism)

The background pressure eats the fast part — what is left is exactly kinetic theory.

[AO (2025)]

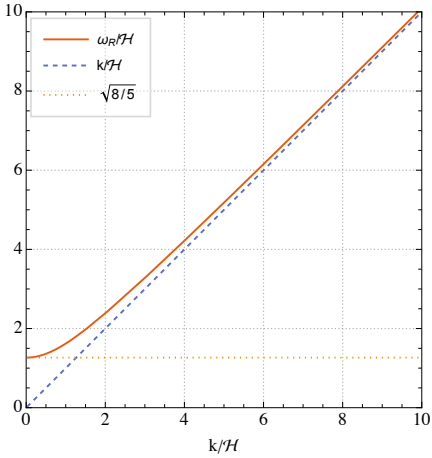
A graviton plasmon

Adiabatic approximation ($\mathcal{H} \approx$ const, $v = ah$):

$$\omega^2 = k^2 + \mathcal{H}^2\, \mathcal{I}\left(\frac{\omega - i\mathcal{H}}{k}\right)$$

- $k \ll \mathcal{H}$: a gap, $\omega_p^2 = \frac{8}{5}\,\mathcal{H}^2$
- $k \gg \mathcal{H}$: $\omega^2 \simeq k^2 + 2\mathcal{H}^2$

Just like QED: a gap in $\omega(k)$, not a mass in $\mathcal{L}$.



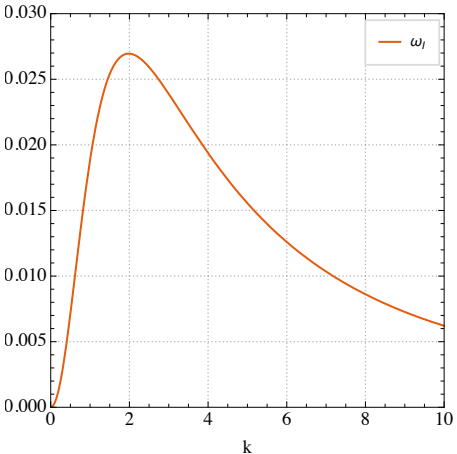
[AO (2025)]

Landau damping of gravitational waves

Subhorizon expansion of the complex pole:

$$\omega = k + \frac{\mathcal{H}^2}{k} + i\frac{\mathcal{H}^3}{k^2} + \left(\ln\frac{\mathcal{H}}{2k} + 5 - \frac{3\pi i}{2}\right)\frac{\mathcal{H}^4}{k^3} + \cdots$$

- The branch cut gives $i\pi$: genuine Landau damping at $\mathcal{O}(\mathcal{H}^4/k^3)$
- $\mathcal{H}$ plays the Debye mass; $(\mathcal{H}/k)^3$ is a volume factor, as in plasma physics
- Suppressed deep inside the horizon



Im ω peaks near horizon crossing

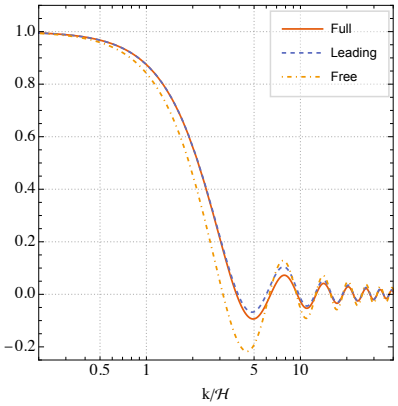
[AO (2025)]

. . . and yet superhorizon modes stay frozen

Real-time equation, $k \ll \mathcal{H}$:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' + \frac{8}{5}\mathcal{H}^2\big(h_{ij} - h_{ij}(0)\big) = 0$$

- The gap is there, but the initial-time boundary of the retarded kernel supplies a source
- $h_{ij} = h_{ij}(0)$: frozen until horizon entry
- The medium acts mainly *around* horizon crossing: a few tens of percent in amplitude
- Deep inside: extra damping = Landau damping



orange: full; blue: leading gradient; yellow: free

[AO (2025)]

The gap is real — but the universe remembers its initial conditions.

- A GW in thermal radiation is a plasma wave: gap $\frac{8}{5}\mathcal{H}^2$ and Landau damping
- Cosmological boundary conditions protect superhorizon modes
- The FLRW Ward identity fixes the state and removes the secular growth



Medium 2: radiation with spin

Does the gravitational wave learn what the plasma is made of?

Two conflicting expectations

Kinetic theory

- Response $\propto g_* T^4$: counts on-shell degrees of freedom
- Friedmann eq. trades $g_* T^4$ for H^2
- $\Rightarrow$ spin blind, universal

Conformal field theory

$$\langle T_{\mu\nu}(x) T_{\rho\sigma}(0) \rangle = \frac{C_T}{x^8} I_{\mu\nu\rho\sigma}$$

$$C_T = n_0 + 3n_{\frac{1}{2}} + 12n_1$$

- $\Rightarrow$ spin weighted

Is kinetic theory missing something, or does QFT conspire?

[Weinberg (2004); Osborn, Petkou (1994)]

The local IR pieces in $\mathcal{H}^2$

Spin	**Slow**	**Fast**	**Contact**	**On-shell**	**F+C+O**	**Fast+Slow**
	memory	nonlocal	$\delta T/\delta g$	background		IR limit of T^R
0	$+\frac{8}{5}$	-2	0	$+2$	**0**	$-\frac{2}{5}$
$\frac{1}{2}$	$+\frac{16}{5}$	-2	-2	$+4$	**0**	$+\frac{6}{5}$
1	$+\frac{16}{5}$	-8	$+4$	$+4$	**0**	$-\frac{24}{5}$

$-\frac{2}{5} : +\frac{6}{5} : -\frac{24}{5} = 1 : -3 : 12$ — the thermal counterpart of C_T

Mass^2 contributions in units of $\kappa^2\rho_{b/f}/12$, per relativistic degree of freedom.

The local terms cancel for every spin; what survives is the slow memory, universal once the Friedmann equation fixes the background.

[AO, Zhu, Zhu (2026)]

The old tachyon, explained

Mass^2 for superhorizon tensor modes, in units of $\mathcal{H}^2$:

fast response (time scale $1/T$)	-2	
slow response (memory, $k \to 0$)	$+\frac{8}{5}$	
one-loop IR result = fast + slow	$-\frac{2}{5}$	the reported "tachyon"

The old tachyon, explained

Mass^2 for superhorizon tensor modes, in units of $\mathcal{H}^2$:

fast response (time scale $1/T$)	-2	
slow response (memory, $k \to 0$)	$+\frac{8}{5}$	
one-loop IR result = fast + slow	$-\frac{2}{5}$	the reported "tachyon"
+ on-shell projection (background pressure)	$+2$	cancels the fast part
adiabatic dispersion relation	$+\frac{8}{5}$	the graviton plasmon

The old tachyon, explained

Mass^2 for superhorizon tensor modes, in units of $\mathcal{H}^2$:

fast response (time scale $1/T$)	-2	
slow response (memory, $k \to 0$)	$+\frac{8}{5}$	
one-loop IR result = fast + slow	$-\frac{2}{5}$	the reported "tachyon"
+ on-shell projection (background pressure)	$+2$	cancels the fast part
adiabatic dispersion relation	$+\frac{8}{5}$	the graviton plasmon
+ initial-time boundary of the kernel	$\frac{8}{5}\mathcal{H}^2\big(h_{ij} - h_{ij}(0)\big)$:	frozen

Nothing was wrong with the loop — pieces of the response were missing.

[AO (2025); AO, Zhu, Zhu (2026)]



Medium 3: a resonant, non-thermal medium

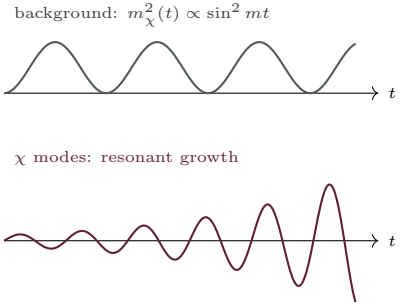
Far from equilibrium, the response is UV divergent: sort out the local terms first.

Gravitational waves through preheating

Toy model: a scalar χ with a resonant mass, in Minkowski space

$$m_\chi^2(t) = g^2\phi(t)^2 = 4qm^2\sin^2(mt)$$

- Mathieu equation: parametric resonance
- Not GW *production* — GW *propagation* through the resonant medium
- Vacuum initial state: no Boltzmann suppression ⇒ UV divergent response



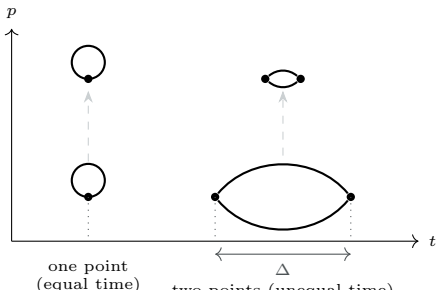
First question: which local terms appear, and how are they renormalized?

[Kofman, Linde, Starobinsky (1994, 1997); Lai, AO (2026)]

The response is built from *unequal-time* correlators

$$T_{ij}^{\rm lin}(x^0,\vec{k}) \;\sim\; \kappa\int_0^{x^0} d\Delta \int dp\, dp'\, w(k,p,p')\, G^R_{p'}(x^0, x^0-\Delta)\, G^K_p(x^0, x^0-\Delta)\, h_{ij}(x^0-\Delta)$$

- The singular part is not the large-p limit alone: it is $p\to\infty$ *with* $\Delta\to 0$, at fixed phase $p\Delta$
- So one bookkeeping parameter counts both, $\eta\sim 1/p\sim\Delta$, and the Δ^{-n} singularities are removed by dimensional regularization



[Parker, Fulling (1974); Lai, AO (2026)]

Divergences and counterterms

$$S^{\rm ct} = \int d^4x\sqrt{-g}\left(c_1 + c_2 R + c_3\, C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + \cdots\right)$$

UV order	**Divergence**	**Counterterm**	**Interpretation**
$r = 2$	$\Box^2 h_{ij}$	$c_3\, C^2$	flat-space divergence
$r = 4, 5$	$\Box h_{ij},\ \dot{h}_{ij}$	$c_2(t)\, R$	time-dependent G
$r = 6$	h_{ij}	$c_1(t)$	time-dependent Λ

$$c_2 = \frac{m^2 q \sin^2(mt)}{24\pi^2\epsilon}, \qquad c_3 = -\frac{1}{1920\pi^2\epsilon}$$

[Lai, AO (2026)]

A familiar lesson, again

- The tadpole $\langle T_{\mu\nu} \rangle$ and the response need *different* c_1:

$$c_1^{\text{lin}} - c_1^{\text{tad}} = -\frac{m^4 q \cos(2mt)}{4\pi^2 \epsilon} \qquad \text{(next-to-leading adiabatic order)}$$

- Reason: Minkowski + an external $m_\chi(t)$ is off shell
  $\Rightarrow$ the Ward identity linking tadpole and self-energy fails
- Covariant completion, e.g. $c_2 = g^2\phi^2/(96\pi^2\epsilon)$, suggests consistency once ϕ and a are on shell

Same lesson as in thermal media: put the background on shell.

[Lai, AO (2026)]

The answer and outlook

So, what happens to a gravitational wave in a medium?

The answer, medium by medium

Medium	What happens to the GW	What made it work
Thermal radiation	gap $\frac{8}{5}\mathcal{H}^2$, Landau damping, tens of percent in amplitude at horizon entry; superhorizon modes frozen	contact terms, FLRW Ward identity, time boundary
Radiation with spin	universal Weinberg damping — no spin imprint	fast + contact + on-shell $= 0$
Resonant medium	local response $\to$ time-dependent G and Λ; finite memory kernel next	adiabatic–UV expansion; on-shell background

Three lessons

1. **The medium makes the spacetime.** Work on FLRW, on shell.
2. **Keep every piece of the response.** Retarded + contact + on-shell projection.
3. **Symmetry is the referee; the time boundary matters.**

A gravitational wave in the early universe is a plasma wave whose gap is the horizon.

Outlook

- The finite memory kernel in resonant and non-thermal media (preheating, reheating)
- Helicity-dependent response from chiral plasmas
- Loop effects in ultra-slow-roll inflation and induced GWs
- Decoherence of primordial GWs in the primordial plasma
- Scalar and vector sectors
- The full Schwinger–Keldysh effective action (noise)

Target observables: CMB B-modes, PTA, LISA, ET.

Thank you for your attention.

References

- A. Ota, *Collective properties of gravitational perturbations in thermal environments on a dynamical background*, arXiv:2510.22346, to be revised.
- A. Ota, H.-Y. Zhu, Y. Zhu, *Real-time gravitational wave response in thermal spinning fields*, arXiv:2601.03631.
- H. Lai, A. Ota, *Ultraviolet structure of real-time gravitational wave linear response in a resonant scalar field*, arXiv:2606.16296.

Contact: aota@cqu.edu.cn

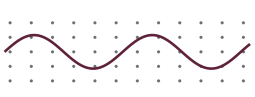
Backup: the Ward identity in full

$$\partial^\mu(a^4\bar{\Pi}_{\mu\nu\rho\sigma}) - \partial_\rho(a^2\bar{T}_{\sigma\nu}) - \partial_\sigma(a^2\bar{T}_{\rho\nu}) + 2a^2(\partial_\nu \ln a)\Big(\bar{T}_{\rho\sigma} - \tfrac{1}{2}a^2\eta^{\alpha\beta}\bar{\Pi}_{\alpha\beta\rho\sigma}\Big) = 0$$

- Weyl identities of conformal matter: $\eta^{\mu\nu}\bar{T}_{\mu\nu} = 0,\ \bar{T}_{\mu\nu} - \frac{1}{2}a^2\eta^{\rho\sigma}\bar{\Pi}_{\rho\sigma\mu\nu} = 0$
- Read against a test function ξ^ν vanishing on the boundary; surface terms are then dropped
- The form of the identity depends on the convention used to define $h_{\mu\nu}$ (e.g. $g^{\mu\nu}$, $g_{\mu\nu}$, or exponential): it must match the convention used for $\bar{\Pi}$
- Matching them is exactly what earlier analyses left ambiguous

[AO (2025); cf. Rebhan (1991)]

Backup: which ensemble, and why it matters



global Gibbs
ensemble fixed: the wave moves the particles



local Gibbs
the ensemble itself follows the metric

- **Global Gibbs:** the whole response comes from the retarded commutator
- **Local Gibbs:** the ensemble re-equilibrates in the perturbed metric — intuitive, and used in earlier work
- It leaves a bulk piece $\bar{\Pi}^S$, and the $\nu = 0$ identity then demands $\mathcal{H}\,\eta^{\mu\nu}\bar{\Pi}^S_{\mu\nu\rho\sigma} = 0$: impossible while $\mathcal{H} \neq 0$
- Local equilibrium already contains the late-time response $\Rightarrow$ double counting

[AO (2025); AO, Sasaki, Wang (2023)]

Backup: contact terms and Schwinger terms

Conformal scalar as a proxy for radiation, canonical operator formalism on FLRW:

- ∂_0 does not commute with $\Theta(x^0 - y^0)$
  $\Rightarrow$ non-covariant Schwinger terms appear in T^R
- The contact response (variation at fixed χ, π_χ) cancels them exactly:

$$\langle T^R_{\mu\nu} \rangle = \big(\text{covariant HTL kernel}\big) + \big(\text{Schwinger}\big), \qquad \langle T^C_{\mu\nu} \rangle = -\big(\text{Schwinger}\big)$$

- Causality built in: no analytic continuation needed
- Same lesson as condensed matter: dropping contact terms breaks gauge invariance (optical conductivity, Hall viscosity)

[AO (2025)]

Backup: relation to the one-loop power spectrum

Integrate out the radiation only:

$$e^{iW_{\rm th}[h]} = \oint_{\hat{d}_\chi} \mathcal{D}\chi\, e^{iS_{\rm RD}[\chi;h]}, \qquad W_{\rm th}[h] = -\tfrac{1}{2}\, h\, \Gamma\, h$$

$$W[j] = \tfrac{1}{2}\, j\, (G^{-1} - \Gamma)^{-1} j = \tfrac{1}{2}\, j\, (G + G\Gamma G + \cdots)\, j$$

- One-loop P_h correction $= G^R \Gamma G^K$: the first term of the resummation
- The single-vertex (seagull) diagram = contact + on-shell pieces
- Secular growth: wrong time boundary; non-convergence: needs resummation

[AO, Zhu, Zhu (2026)]