Classification Of $f(R)$ Theories And Resummations In Higher-Curvature Modifications Of Starobinsky Inflation

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Renormalization-Group Analogy in Inflation

QFT		**Inflation**
RG flow	$\longleftrightarrow$	Slow roll
Couplings $\alpha, \lambda, \ldots$	$\longleftrightarrow$	Slow-roll parameters $\varepsilon,\ \eta_H, \ldots$
Beta functions	$\longleftrightarrow$	Background equations for $\varepsilon, \eta_H, \ldots$
Sliding scale μ	$\longleftrightarrow$	Conformal time τ
Correlation functions	$\longleftrightarrow$	Perturbation spectra
Callan-Symanzik equation and RG	$\longleftrightarrow$	Conservation of spectra at superhorizon

D. Anselmi, JCAP 01 (2021) 048
$R + R^2 \qquad R + R^2 + C^2$

D. Anselmi, F. Fruzza, **MP**, CQG 38 (2021) 22, 225011
Classification of single-field with $V(\phi)$ w/ and w/o C^2

D. Anselmi, JCAP 07 (2021) 037
B. Grzadkowski, **MP**, JCAP 07 (2025) 030
Double-field inflation

MP, PRD 113 (2026) 2, 023502
Classification of $f(R)$ (**this talk**)

$f(R)$ Theories

Action

$$S(g) = -\frac{1}{2\kappa^2}\int \sqrt{-g}\, f(R), \qquad \kappa^2 = 8\pi G,$$

Equations of motion

$$FR_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f + (g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu)F = 0, \qquad F \equiv \frac{\partial f}{\partial R}$$

On FLRW background

$$g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = \mathrm{d}t^2 - a(t)^2\mathrm{d}x^i\mathrm{d}x^i \quad \Rightarrow$$

$$H\dot{F} - F\dot{H} - FH^2 - \frac{f}{6} = 0$$

$$\ddot{F} + 2\dot{H}F - H\dot{F} = 0$$

Slow-Roll Expansion

Convenient slow-roll parameter

$$\alpha \equiv -\frac{\dot{F}}{2HF},$$

Other useful quantities

(All power series in α)

$$\varepsilon \equiv -\frac{\dot{H}}{H^2}, \qquad \nu \equiv -\frac{1}{Ha\tau}, \qquad \beta_\alpha \equiv \frac{\mathrm{d}\alpha}{\mathrm{d}\ln|\tau|}$$

EOMs

$$-\nu\beta_\alpha = \alpha(1+2\alpha) - (1-\alpha^2)\varepsilon, \qquad \varepsilon = 1 + 2\alpha + \frac{f}{6FH^2}$$

Slow-Roll Conditions

Typically we assume

$$\varepsilon, |\eta_H| \ll 1, \qquad \eta_H \equiv \frac{\dot{\varepsilon}}{\varepsilon H}$$

From the EoMs

$$\varepsilon = \frac{\alpha + 2\alpha^2 + v\beta_\alpha}{1-\alpha}, \qquad \eta_H = -\frac{v\beta_\varepsilon}{\varepsilon}.$$

$$\Downarrow$$

$$\alpha \ll 1, \qquad \beta_\alpha = b_0\alpha^2 + \mathcal{O}(\alpha^3) \qquad \Longleftrightarrow \qquad \varepsilon = \alpha + \mathcal{O}(\alpha^2), \qquad \eta_H = \mathcal{O}(\alpha)$$

Classification

Class I $\beta_\alpha = b_0\alpha^2 + \dots$ Class II $\beta_\alpha = b_1\alpha^3 + \dots$ Class III $\beta_\alpha = b_2\alpha^4 + \dots$

etc...

Background expansions

We can derive v, H and F as expansions in α

$$\frac{\mathrm{d}}{\mathrm{d}t} = -vH\beta_\alpha \frac{\mathrm{d}}{\mathrm{d}\alpha}$$

$$\beta_\alpha \frac{\partial v}{\partial \alpha} = 1 - v - \varepsilon \qquad \text{with} \qquad v(\alpha) = \sum_{n=0}^{\infty} v_n \alpha^n$$

Same for H and F (up to overall functions of α)

$$v\beta_\alpha \frac{\partial F}{\partial \alpha} = 2\alpha F, \qquad v\beta_\alpha \frac{\partial H}{\partial \alpha} = H\varepsilon \qquad F = F_0 \Phi(\alpha) \sum_{n=0}^{\infty} f_n \alpha^n, \qquad H = H_0 h(\alpha) \sum_{n=0}^{\infty} h_n \alpha^n,$$

“Running” slow-roll parameter

Introduce the rescaled conformal time and its log $\eta \equiv -k\tau$ $\ell \equiv \ln \eta\,.$

$$\frac{\mathrm{d}\alpha(\ell)}{\mathrm{d}\ell} = \beta_\alpha(\alpha) = b_n \alpha^{n+2} + \mathcal{O}(\alpha^{n+3}), \qquad \alpha(\ell = 0) = \alpha_k$$

Since k is introduced by hand we have $\dfrac{\mathrm{d}\alpha}{\mathrm{d}\ln k} = 0, \quad \Rightarrow \quad \dfrac{\mathrm{d}\alpha_k}{\mathrm{d}\ln k} = -\beta_\alpha(\alpha_k)\,.$

Class I: $\alpha^I(\ell) = \dfrac{\alpha_k}{1 - b_0 \alpha_k \ell} \qquad \alpha_k = \dfrac{\alpha_{k_*}}{1 + b_0 \alpha_{k_*} \ln k/k_*}$

Improved Power Spectra: $\mathscr{P}(\alpha, \alpha_k) \underset{\eta\to 0}{\longrightarrow} \mathscr{P}(\alpha_k) \qquad \dfrac{\mathrm{d}\mathscr{P}(\alpha_k)}{\mathrm{d}\ln k} = -\beta_\alpha(\alpha_k)\dfrac{\mathrm{d}\mathscr{P}(\alpha_k)}{\mathrm{d}\alpha_k}$

Perturbations

$$S_2(w) = \frac{1}{2}\int \mathrm{d}\eta \left[w'^2 - w^2 + \frac{2+\sigma}{\eta^2} w^2 \right] \qquad \Rightarrow \qquad w'' + w - \frac{2+\sigma}{\eta^2} w = 0, \qquad \sigma = \sum_{n=1}^{\infty} s_n \alpha^n$$

$$w(\eta) = \sum_{n=0}^{\infty} w_n(\eta) \alpha_k^n \qquad\qquad w_n'' + w_n - \frac{2}{\eta^2} w_n = \frac{g_n(\eta)}{\eta^2},$$

$$w_n(\eta) = \int_{\eta}^{\infty} \mathrm{d}\eta' \frac{g_n(\eta')}{\eta\eta'^3} \left[(\eta - \eta')\cos(\eta - \eta') - (1 + \eta\eta')\sin(\eta - \eta') \right]$$

Power Spectra

$$\mathscr{P}_{\rm t} = \frac{4\kappa^2 v(\alpha_k)^2 H(\alpha_k)^2 \left| Q_{0{\rm t}}(\alpha_k) \right|^2}{\pi^2 F(\alpha_k)}, \qquad \mathscr{P}_{\rm s} = \frac{\kappa^2 v(\alpha_k)^2 H(\alpha_k)^2 (1-\alpha_k)^2 \left| Q_{0{\rm s}}(\alpha_k) \right|^2}{12\pi^2 \alpha_k^2 F(\alpha_k)\,.}$$

$$r = 48 \left(\frac{\alpha_k}{1-\alpha_k} \right)^2 \left| \frac{Q_{0{\rm t}}(\alpha_k)}{Q_{0{\rm s}}(\alpha_k)} \right|^2$$

where

$$\eta w(\eta) = Q(\ln \eta) + Y(\eta), \qquad Y(\eta) \underset{\eta \to 0}{\longrightarrow} 0$$

$$Q_0(\alpha_k) \equiv Q(0)$$

Spectral Indices

$$n_{\rm s}-1=2\left\{1-\frac{1-\alpha_k}{v(\alpha_k)}+\beta_\alpha(\alpha_k)\left[\frac{1}{\alpha_k(1-\alpha_k)}-\frac{{\rm d}\ln|Q_{0{\rm s}}(\alpha_k)|}{{\rm d}\alpha_k}\right]\right\}$$

$$n_{\rm s}=2\left[1-\frac{1-\alpha_k}{v(\alpha_k)}-\beta_\alpha(\alpha_k)\frac{{\rm d}\ln|Q_{0{\rm t}}(\alpha_k)|}{{\rm d}\alpha_k}\right],$$

At the leading order with $\beta_\alpha=b_n\alpha^{2+n}+\mathcal{O}(\alpha^{3+n})$

$$n_{\rm t}\simeq-6\alpha_k^2,\qquad\forall n\geq 0$$

$$r\simeq 48\alpha_k^2,\qquad\forall n\geq 0,$$

$$n_{\rm s}-1\simeq\begin{cases}2\alpha_k b_0 & \text{if } n=0,\\ 2\alpha_k^2(b_1-3) & \text{if } n=1,\\ -6\alpha_k^2 & \text{if } n\geq 2.\end{cases}$$

Predictions

$$r_* \simeq \begin{cases} \dfrac{12(1-n_{s*})^2}{b_0^2} & \text{if } n=0, \\ \dfrac{24(1-n_{s*})}{3-b_1} & \text{if } n=1, \\ 8(1-n_{s*}) & \text{if } n \geq 2. \end{cases}$$

$r_* < 0.035$

$n_{s*} = 0.974 \pm 0.003$ (P-ACT) ACT Collaboration, JCAP 11 (2025) 062

$n_{s*} = 0.9682 \pm 0.0032$ (Planck+SPT+ACT+Bicep/Keck)

$n_{s*} = 0.9728 \pm 0.0029$ (Planck+SPT+ACT+Bicep/Keck+DESI BAO)
Balkenhol et al., OJA Vol. 9 2026

Every class larger than II is excluded because they give $r_* \simeq 0.2 - 0.3$

Only class I and class II survive with

$b_0 \lesssim -0.48 \quad -0.59 \quad -0.50$ $\qquad$ $b_1 \lesssim -14.8 \quad -18.8 \quad -15.6$

Number of e-folds

$$N_k = \int_{t_k}^{t_f} \mathrm{d}t\; H(t) = -\int_{\alpha_k}^{\alpha_f} \frac{\mathrm{d}\alpha'}{v(\alpha')\beta_\alpha(\alpha')} = -\frac{1}{b_n(n+1)\alpha_k^{1+n}} + \mathcal{O}(\alpha_k^0)\,.$$

$$N_* \simeq \begin{cases} \dfrac{2}{1-n_{\mathrm{s}*}} & \text{if } n=0, \\ \dfrac{1}{1-n_{\mathrm{s}*}}\left(1-\dfrac{3}{b_1}\right) & \text{if } n=1. \end{cases}$$

Only class I and class II survive with

$$N_*^{n=0} \simeq \begin{cases} 77 \pm 9, \\ 63 \pm 7, \\ 74 \pm 8. \end{cases} \qquad N_*^{n=1} \simeq \begin{cases} 47 \pm 7, \\ 37 \pm 4, \\ 42 \pm 6. \end{cases}$$

$N_* > 60$ might be compatible with non-instantaneous reheating

Zharov, Sobol, Vilchinskii, PRD 112 (2025) 2, 023544 Drees, Xu, PLB 867 (2025) 139612

Reconstructing $f(R)$

We know already $\varepsilon(\alpha)$, $H(\alpha)$ and $F(\alpha)$ then we also have

$$\hat{f}(\alpha) = 6FH^2\,(\varepsilon - 1 - 2\alpha) \qquad R(\alpha) = -\,6H^2(2-\varepsilon)$$

Inverting $R(\alpha)$ we get $\alpha(R)$ and plug it back into $\hat{f}(\alpha)$ $\qquad f(R) = \hat{f}(\alpha(R))$

Class I $$f(R) = C_0\tilde{R}^2\left[1 + \frac{c_0}{\tilde{R}^{\gamma}} + \frac{c_1}{\tilde{R}^{2\gamma}} + \mathcal{O}(\tilde{R}^{-3\gamma})\right] \qquad \gamma \equiv -\,2b_0$$

Class II $$f(R) = \frac{C_0\tilde{R}^2}{(3\gamma\ln\tilde{R})^{1/\gamma}}\left[1 + \frac{d_1}{\ln\tilde{R}} + \frac{d_2}{\ln^2\tilde{R}} + \mathcal{O}(\ln^{-3}\tilde{R})\right] \qquad \gamma \equiv -\,b_1/6,\quad b_2 = 4b_1$$

$$\tilde{R} = \frac{R}{12\bar{H}^2} \qquad c_n = c_n(b_n,\, b_{n-1},\, \ldots) \qquad d_n = d_n(b_n,\, b_{n-1},\, \ldots)$$

Examples

Class I

$$f(R) = R - \frac{1}{6M^2}R^2 + 2\Lambda, \qquad f(R) = \frac{1}{6M^2}R^2 + 2M^2\,.$$

$$f(R) = R - \frac{1}{6M^2}R^2 - \frac{\lambda}{M}(-R)^{3/2}, \qquad f(R) = -\frac{R^2}{6M^2}\left(1 + \frac{3}{2}\frac{M^4}{M^4 + R^2}\right).$$

Class II

$$f(R) = -\frac{F_0 R^2}{24H_0^2\left[2 + 3\ln\left(\frac{-R}{12H_0^2}\right)\right]}$$

Where are all the other polynomials in R?

Class 0: linear β_α

If we accept a linear beta function $\bar{\beta}_\alpha = \bar{b}\alpha + \beta_\alpha, \qquad \bar{b} < 0. \qquad \dfrac{\mathrm{d}\alpha}{\mathrm{d}\ell} = \bar{b}\alpha, \qquad \Rightarrow \qquad \alpha(\eta) = \alpha_k \eta^{\bar{b}}\,.$

$$f(R) = C_0\left[1 - 2\left(1+\tilde{R}\right) + \bar{c}\left(1+\tilde{R}\right)^2 + \mathcal{O}\left(\left(1+\tilde{R}\right)^3\right)\right], \qquad \bar{c} = \bar{c}(\bar{b})\,.$$

$$\varepsilon \sim \alpha + \mathcal{O}(\alpha^2) \qquad \beta_\varepsilon \sim \varepsilon + \mathcal{O}(\varepsilon^2) \qquad \eta_H = -\frac{\nu\beta_\varepsilon}{\varepsilon} \sim \mathcal{O}(\varepsilon^0)$$

$$\varepsilon = (1+\bar{b})\alpha + \mathcal{O}(\alpha^2), \qquad \eta_H = -\bar{b} + \mathcal{O}(\alpha)$$

The condition $|\eta_H| \ll 1$ is violated unless $\bar{b}$ is small (compared to α) but $\alpha(\eta) \underset{\eta\to\infty}{\longrightarrow} 0$

$\Downarrow$

Far enough in the past there is a period of ***constant-roll*** inflation

Motohashi, Starobinsky, Yokoyama, JCAP 09 (2015) 018 Motohashi, Starobinsky, EPJC 77 (2017) 8 538

Observables in Class 0

$$n_{s*} - 1 = 2\bar{b} - \frac{6\alpha_* \left(\bar{b} - 1\right)\left(\bar{b} + b_0\right)}{3 - 2\bar{b}} + \mathcal{O}(\alpha_*^2)$$

$\bar{b}$ should be not too large.

$$\mathscr{P}_{t*} = \frac{4C_t^2}{\pi^2}\left[1 + \alpha_*^2\left(\frac{3}{\bar{b}} - 4 + \frac{10 - 8\gamma_M}{1 + 2\bar{b}} + 2\gamma_M\right) + \mathcal{O}(\alpha_*^3)\right]$$

$\bar{b}$ should be not too small

Warning!

$$f(R) = R - \frac{1}{6M^2}R^2 + \sum_{i=3}^{N}\frac{\gamma_n}{M^{2n+2}}R^n$$

$\bar{b}$ and b_i all depend on γ_n!

Resummations in $f(R) = R + R^2 + R^3$

MP, arXiv 09/2026 (I hope :D)

$$f(R) = R - \frac{1}{6M^2}R^2 + \frac{\gamma^2}{16M^4}R^3, \qquad [\gamma] = 0$$

EoM

$$\left[1 + \frac{27H^2\gamma^2}{2M^2}\left(1 - \frac{\varepsilon}{2}\right)\right]\frac{\dot{\varepsilon}}{H} = -3\varepsilon\left(1 - \frac{\varepsilon}{2}\right) + \frac{M^2}{2H^2} - \frac{9\gamma^2H^2}{8M^2}(2-\varepsilon)^2(1+10\varepsilon)$$

Two Possible approaches

1) Keep γ exact and expand in α (produces powers of α/γ)

2) Expand in γ first and then in α (produces powers of γ/α)

Case 1): Expand first in α then in γ

De Sitter spacetime is a solution of the EoM

$$\bar{H} = \frac{M}{\sqrt{3\gamma}}, \qquad \bar{R} = -\frac{4M^2}{\gamma}, \qquad \Rightarrow \qquad H(\alpha) = \bar{H}\sum_{n=0}^{\infty} h_n \alpha^n, \qquad R(\alpha) = \bar{R}\sum_{n=0}^{\infty} r_n \alpha^n, \quad h_0,\, r_0 = 1,$$

From $H(R)$ and $\varepsilon(R)$ we solve $\dot{\alpha}\dfrac{\partial H}{\partial \alpha} = \varepsilon H^2$ for the coefficients r_n

$$r_n(\gamma) = \frac{\tilde{r}_n(\gamma)}{\gamma^n} \qquad \tilde{r}_n(\gamma) \text{ regular for } \gamma \to 0\,. \qquad \text{This suggests an expansion in } \xi = \frac{\alpha}{\gamma}$$

The series can be rearranged as $R = \bar{R}\sum_{n=0}^{\infty} R_n(\xi)\gamma^n \qquad R_n(\xi) = \text{power series}$

e.g. $$R_0(\xi) = 1 - \xi + \frac{\xi^2}{2} - \frac{\xi^4}{8} + \frac{\xi^6}{16} - \frac{5\xi^8}{128} + \frac{7\xi^{10}}{256} + \ldots = \frac{1}{\xi + \sqrt{1+\xi^2}}$$

Resummed functions and “Improved” Slow-Roll Expansion

From the equation for R we can find $R_n(\xi)$ exactly (same for H, v, ε, β_α)

“*Improved*” slow-roll expansion

Expand in α while keeping ζ of order 1

$$R = \bar{R}\sum_{n=0}^{\infty} R_n(\xi)\gamma^n = -\frac{2M^2}{\alpha}\sum_{n=0}^{\infty}\tilde{R}_n(\zeta)\alpha^n$$

$$\tilde{R}_n(\zeta) = 2\zeta^{n-1}R_n(1/\zeta), \qquad \zeta = \gamma/\alpha = 1/\xi$$

$$\beta_\alpha = -2\alpha^2\sqrt{1+\zeta^2} + \mathcal{O}_3$$

$$R = -\frac{4M^2}{\alpha}\frac{1}{1+\sqrt{1+\zeta^2}} + \mathcal{O}_0$$

$$H \simeq \frac{M}{\sqrt{3\alpha}}\frac{1}{\sqrt{1+\sqrt{1+\zeta^2}}} + \mathcal{O}_0$$

Case 2): Expand first in γ then in α

De Sitter spacetime is **NOT** a solution of the EoM

$$R(\alpha) = -\frac{2M^2}{\alpha}\sum_{n,m=0}^{\infty} r_{nm}\alpha^n\left(\frac{\gamma}{\alpha}\right)^{2m} = -\frac{2M^2}{\alpha}\sum_{n=0}^{\infty}\hat{R}_n(\zeta)\alpha^n$$

$$\hat{R}_n(\zeta) = \sum_{m=0}^{\infty} r_{nm}\zeta^{2m}$$

They coincides with he Taylor series of $\tilde{R}_n(\zeta)$ around $\zeta = 0$

$$\hat{R}_n(\zeta) = \tilde{R}_n(\zeta), \qquad \forall n \geq 0.$$

Improved Observables

$$\mathscr{P}_{\rm s} = \frac{M^2\kappa^2}{96\pi^2\alpha_k^2}\left\{1-\alpha_k\left[2-(5-4\gamma_M)\sqrt{1+\zeta_k^2}\right]+\mathcal{O}_2\right\}$$

$$n_{\rm s}-1 = -4\alpha_k\sqrt{1+\zeta_k^2}\left[1+\alpha_k\left(\frac{-5+4\gamma_M}{2\sqrt{1+\zeta_k^2}}-\frac{31}{6}\sqrt{1+\zeta_k^2}+\frac{7+9\zeta_k^2}{1+\zeta_k^2}\right)+\mathcal{O}_2\right]$$

$$r = 48\alpha_k^2\left\{1+2\alpha_k\left[1+2\left(\gamma_M-2\right)\sqrt{1+\zeta_k^2}\right]+\mathcal{O}_2\right\}$$

Comparison with Recent Calculations

Recent attempts to reconcile ACT data by adding Higher-curvature terms

A series of ambulance-chasing papers

Modak, Phys. Rev. D 112, 115006 (2025)

Ketov, Pozdeeva, Vernov, JCAP 12 (2025) 040

Gialamas, Katsoulas, Tamvakis, JCAP 09 (2025) 060

Addazi, Aldabergenov, Ketov, Phys.Lett.B 869 (2025) 139883

Bezerra-Sobrinho, Medeiros Eur.Phys.J.C 86 (2026) 4, 416

Del Grosso, Urbano, Ziccolella arXiv:2607.09808

and more…

$$n_s = 1 - 4\alpha - \frac{2\gamma^2}{\alpha} + \mathcal{O}(\gamma^3) = 1 - 4\alpha_k\sqrt{1+\zeta_k^2}$$

Require $\gamma^2<0$ for a larger n_s

Our formulas are the same with the substitution $\gamma^2 \to -\gamma^2$

with the caveat that $\sqrt{1-\frac{\gamma^2}{\alpha^2}}$ is not always real.

Effective field theory approach

E.g. $N_k = 50-60$ they have $0.006 \lesssim \gamma \lesssim 0.009$ $\quad 0.009 \lesssim \alpha \lesssim 0.01$

$\frac{\gamma^2}{16M^4}R^3 = \frac{cR^3}{\Lambda^4}$ with $c = \mathcal{O}(1) \;\Rightarrow\; \Lambda \simeq 20M - 30M$ questionable for EFT

Renormalizable Gravity with Purely Virtual Particles

D. Anselmi, **MP,** JHEP 06 (2017) 066
D. Anselmi, JHEP 06 (2017) 086
D. Anselmi, **MP,** JHEP 11 (2018) 021

$$S(g) = -\frac{1}{\kappa^2}\int\sqrt{-g}\left(R - \frac{1}{6M^2}R^2 + \frac{1}{2m_\chi^2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma} + 2\Lambda_C\right).$$

With new prescription to remove the ghost (see Damiano's talk yesterday)
Only one tensor and one scalar perturbations survive

$$\theta_s = 1 - n_s, \qquad \rho = \frac{M^2}{m_\chi^2}$$

$$\mathscr{P}_s = \frac{4M^2G}{3\pi\theta_s^2}\left[1 + \frac{5}{12}\theta_s + \mathcal{O}(\theta_s^2)\right] \qquad \mathscr{P}_t = \frac{8M^2G}{\pi(2+\rho)}\left[1 - \frac{3\theta_s}{2(2+\rho)} + \mathcal{O}(\theta_s^2)\right]$$

$$r = \frac{6\theta_s^2}{2+\rho}\left[1 - \frac{\theta_s(28+5\rho)}{12(2+\rho)}\right] + \mathcal{O}(\theta_s^4) \qquad n_t = -\frac{9\alpha\theta_s^2}{4(6\alpha+\beta)}\left[1 - \frac{\theta_s(4-7\rho)}{12(2+\rho)}\right] + \mathcal{O}(\theta_s^4)$$

D. Anselmi, E. Bianchi, **MP,** JHEP 07 (2020) 211

Conclusions

Stop using non-renormalizable models to study inflation

(Or at least don’t cry when they do not work well)