BLACK HOLE APPARENT HORIZONS BECOME COMOVING WITH THE UNIVERSE

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- Cosmological coupling (CC) is a proposal in GR that a global FLRW cosmology and local regions of relativistic stress within it are strongly coupled.
- The idea is not new: McVittie published an exact solution almost a century ago, which interpolates between a local black hole and large-scale expanding FLRW metric. His solution is, however, pathological.
- The idea strongly re-emerged after some recent observational indications that some populations of BH masses grow as a^k (D. Farrah et al.). Eventually, if $k = 3$ they dilute as a^{-3}, leading to an emergent DE without fine tuning (see Croker et al.)
- Proposed interpolating models beyond McVittie are generally non-singular (see e.g. Cadoni et al.)

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- The complexity of the equations describing solutions that interpolate between compact objects and FLRW universes can be circumvented by studying the general behaviour of event and apparent horizons (MR and V. Faraoni, gr-qc: 2407.14549 and gr-qc: 2608.21222)
- CC can have profound impacts on the evolution of primordial black holes (MR et al astro-ph 2607.13857).
- CC affecting inspiralling SMBH can leave an imprint in the data of Pulsar Timing Arrays (MR et al. gr-qc: 2409.01801)

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1. Static horizons cannot exist in FLRW
2. How does a horizon evolve?
3. CC and primordial black holes
4. CC and Pulsar Timing Array (PTA)
5. Conclusions

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Static event horizons become null singularities

- The most general spherically symmetric and time-dependent metric can be written as

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

The areal radius is defined as $R(t,r) = r\,a(t,r)$. The apparent horizon (AH) position is fixed by

$$\boxed{\nabla_\alpha R \nabla^\alpha R = 0}\,.$$

- Null radial geodesics have tangent $u^\alpha = (u^0, u^1, 0, 0)$ that satisfy the geodesic equation

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- Combining with normalisation conditions

$$u^\alpha u^\beta g_{\alpha\beta} = 0 \quad \Rightarrow \quad \frac{u^1}{u^0} = \pm \frac{T}{a}$$

we find

$$\boxed{\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) = \frac{\dot{T}}{T} \pm \frac{2T'}{a} + \frac{\dot{a}}{a}}, \quad \boxed{\frac{d}{d\lambda}\left(\frac{1}{u^1}\right) = \frac{T'}{T} \pm \frac{2\dot{a}}{T} + \frac{a'}{a}}$$

- In the near-horizon region the Schwarzschild metric can be written as the Rindler metric plus small corrections

$$\boxed{ds^2 = ds^2_{\text{Rindler}} + \ldots = -\frac{x^2}{4}dt^2 + 4M^2\left[dx^2 + d\Omega^2\right] + \ldots}$$

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- We compare

$$ds^2 = -\frac{x^2}{4}dt^2 + 4M^2\left[dx^2 + d\Omega^2\right]$$

with

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

in the near-horizon region:

- we obtain the identifications

$$T = x/2, \quad a = 2M, \quad \dot{T} = \dot{a} = 0, \quad T' = 1/M.$$

- the equations for u^0, u^1 simplify to

$$\frac{d}{d\lambda}\left(\frac{1}{u^0}\right) \simeq \pm\frac{2}{a_{\rm H}}, \quad \frac{u^1}{u^0} = \pm\frac{x}{4M}$$

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- The equations of motion for u_0 can then be solved

$$u^0 = \frac{C}{\lambda - \lambda_0} > 0\,,$$

the positivity condition being required to maintain the vector u future-directed. Thus we have two possibilities:

- $C > 0$, $\lambda > \lambda_0$, which implies

$$\lim_{\lambda \to \infty} u^0 = 0\,,$$

that is "time stops" while $u^1 \to 0$ and also velocity vanishes.

- $C < 0$, $\lambda < \lambda_0$, which implies

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null geodesics stop propagating at a finite λ_0:
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Curvature invariants

- We found that null geodesics end (cannot originate) on (from) the horizon (the same happens for timelike geodesics).
  This is a clear case of geodesic incompleteness, which usually leads to a singularity.
- A legitimate suspicion is that these results are artefacts of a bad coordinate choice. To clarify the issue, we compute the Ricci scalar in the near-horizon limit

$$\mathcal{R} \equiv g^{\mu\nu} R_{\mu\nu} = -\frac{4a''}{a^3} + \frac{6\ddot{a}}{aT^2} - \frac{2a'T'}{a^3T} - \frac{6\dot{a}\dot{T}}{aT^3} + \frac{6\dot{a}^2}{a^2T^2} - \frac{8a'}{a^3r}$$

$$- \frac{4T'}{a^2rT} - \frac{2T''}{a^2T} + \frac{2a'^2}{a^4}$$

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We compare again

$$ds^2 = -\frac{x^2}{4}dt^2 + 4M^2\left[dx^2 + d\Omega^2\right]$$

with

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

in the near-horizon region so that we obtain the identifications

$$T = x/2, \quad a = 2M, \quad \dot{T} = \dot{a} = 0, \quad T' = 1/M.$$

We find that

$$\mathcal{R} \simeq \frac{1}{x}$$

which blows up for $x \to 0$. Thus, forcing a horizon to be static in an expanding universe turns it into a naked singularity.

Test Klein-Gordon field

- We consider a massless scalar field in the s-mode, which satisfies the Klein-Gordon equation $\Box\phi(t,r)=0$, that is:

$$\ddot{\phi}+\left(\frac{3\dot{a}}{a}+\frac{\dot{T}}{T}\right)\dot{\phi}-\frac{T^2}{a^2}\phi''-\frac{T}{a^2}\left(T'+\frac{Ta'}{a}+\frac{2T}{r}\right)\phi'=0\,.$$

  We recall that arbitrarily close to the horizon $\dot{a}=\dot{T}=0$ and $T=x/2\to 0$. The above equation boils down to

$$\ddot{\phi}\simeq 0 \quad\Rightarrow\quad \phi(t)=\phi_0+\phi_1 t\to\infty$$

- From the trace of the stress tensor fluid $\Pi_{\alpha\beta}$ we have

$$\Pi\equiv 3P-\rho=(\dot{\phi}/T)^2\to\infty \quad \text{as} \quad T\simeq\frac{x}{2}\to 0$$

  However $\rho=\dot{\phi}^2/2=\text{const}$ so the pressure diverges. This also means that perturbations grow unbounded.

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- We consider again the metric

$$ds^2 = -T^2(t,r)dt^2 + a^2(t,r)\left(dr^2 + r^2 d\Omega^2\right)$$

- If the AH exists, its areal radius satisfies

$$\nabla^c R(t,r) \nabla_c R(t,r)\Big|_{\rm AH} = 0$$

so that

$$R_{\rm AH}(t) = a\left(t, r_{\rm AH}(t)\right) r_{\rm AH}(t)$$

- We find that the rate of change:

$$\frac{1}{R_{\rm AH}}\frac{dR_{\rm AH}}{dt} = \frac{1}{M}\frac{dM}{dt} = H_{\rm AH}\left(1 - \frac{a_{\rm AH}\dot{r}_{\rm AH}}{T_{\rm AH}}\right)$$

where M is the Misner-Sharp-Hernandez mass.

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Let us see the various possibilities

- $R_{\rm AH} = \text{const}$ leads to a static, null event horizon, so it is unphysical. It is obtained when $\dot{r}_{\rm AH} = T_{\rm AH}/a_{\rm AH}$.
- $\dot{r}_{\rm AH} = 0$ leads to an AH that is exactly comoving: then

$$\frac{1}{M}\frac{dM}{dt} = \frac{1}{R_{\rm AH}}\frac{dR_{\rm AH}}{dt} = H_{\rm AH} \quad \Rightarrow \quad M(t) = M_0 a(t)$$

  and the MSH mass shows a linear cosmological coupling.
- When the AH forms, it can also be slower or faster than comoving, depending on whether, at its formation,

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Possible evolutions of slower than comoving AH:

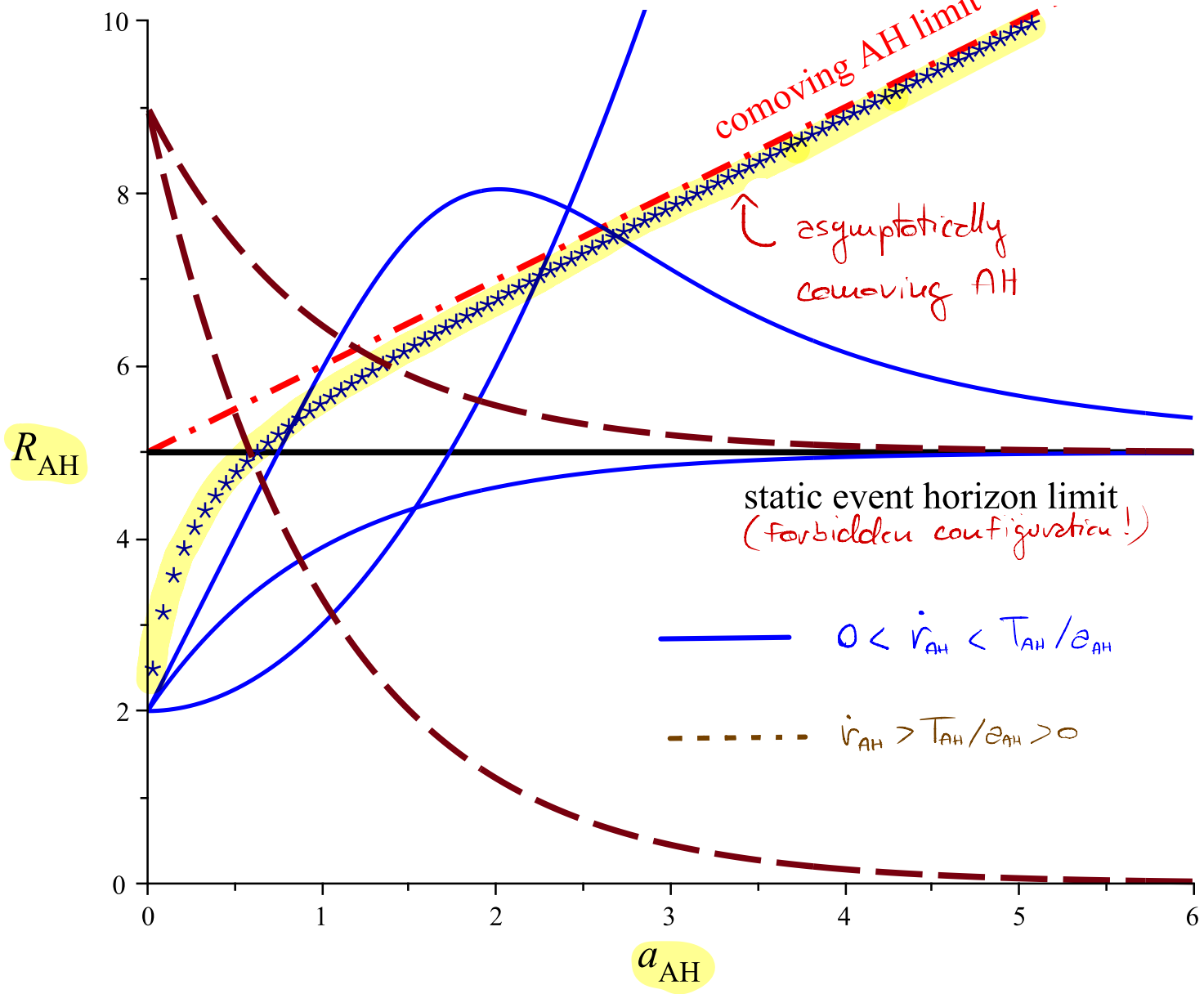


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Observational signatures

Let's assume that black holes are cosmologically coupled: is there any observational evidence?

- Observations suggest that SMBHs must have grown much faster at redshifts $z > 6$ than standard astrophysical channels [Volonteri et al. Nat. Rev. Phys. 3, 732 (2021)].
- Recent reports show plausible evidence for cosmological coupling between the masses of SMBH and the scale factor, obtained from data of black hole populations in red elliptical galaxies in the redshift range $0 < z \leq 2.5$ [Farrah et al. AJL 944, L31 (2023)].
- Virgo/LIGO reveal the presence of much heavier stellar mass black holes than predicted from well-established models of stellar evolution [Abbott et al. PRL 125, 101102 (2020)].
- What are other possible effects?

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Effects on Hawking evaporation and PBH as dark matter

Hawking evaporation and growth due to CC are competing processes.

- Assuming that CC switched on at some $z_{\rm CC}$ so that

$$M(z) = M\theta(z - z_{\rm CC}) + M\left(\frac{1+z_{\rm CC}}{1+z}\right)^k \theta(z_{\rm CC} - z)$$

that is $M \propto a^k$ after some $z_{\rm CC}$ and assuming adiabaticity

- we can model the two effects as (MR et al astro-ph 2607.13857)

$$\frac{dM(t)}{dt} = \Gamma_{\rm HR}[M(t)] + \Gamma_{\rm CC}[M(t), t]$$

where

$$\Gamma_{\rm HR} = -\frac{\mathcal{C}_{\rm HR}}{M(t)^2}, \qquad \Gamma_{\rm CC}[M(t), t] = kH(t)M(t)$$

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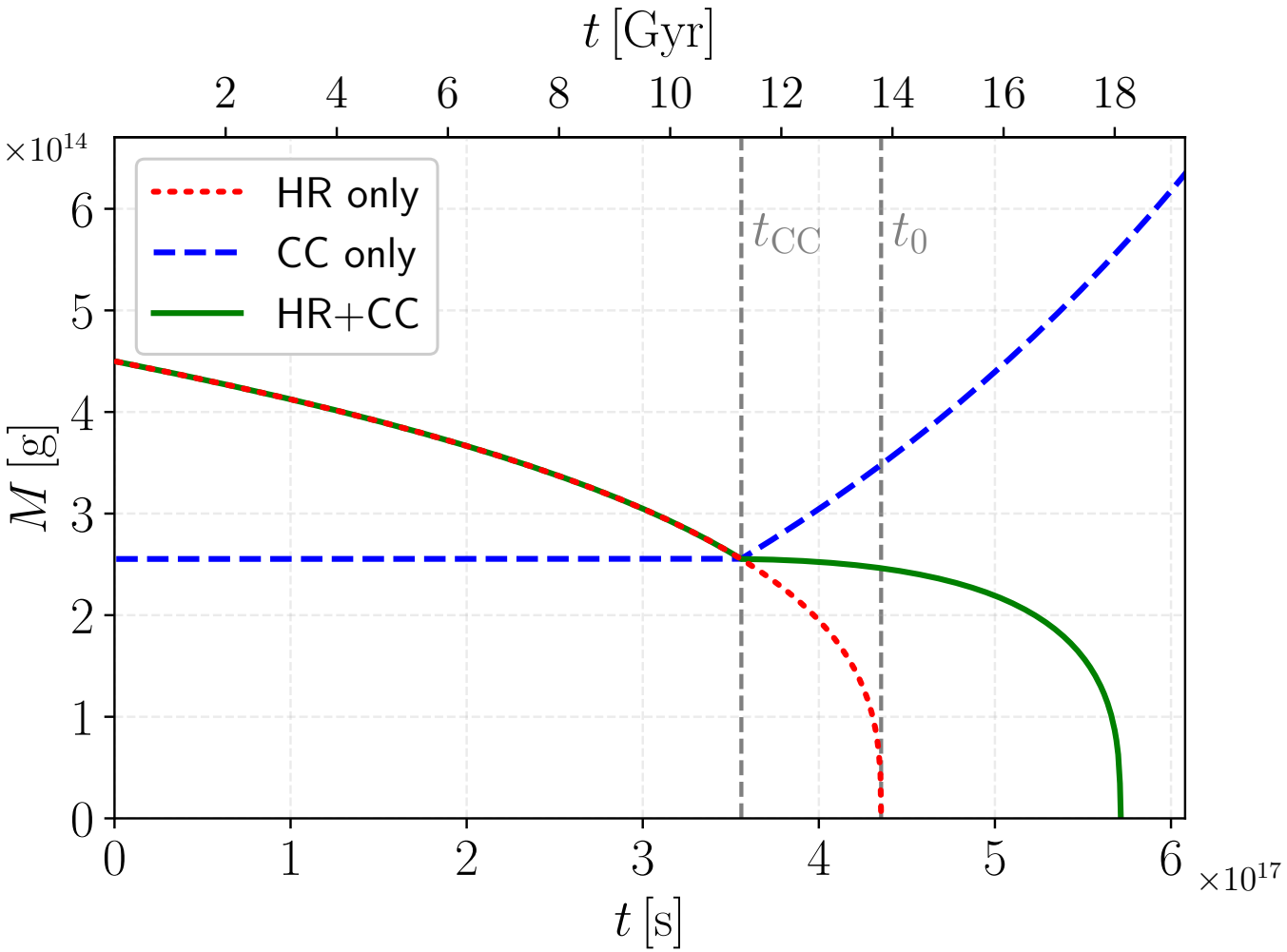
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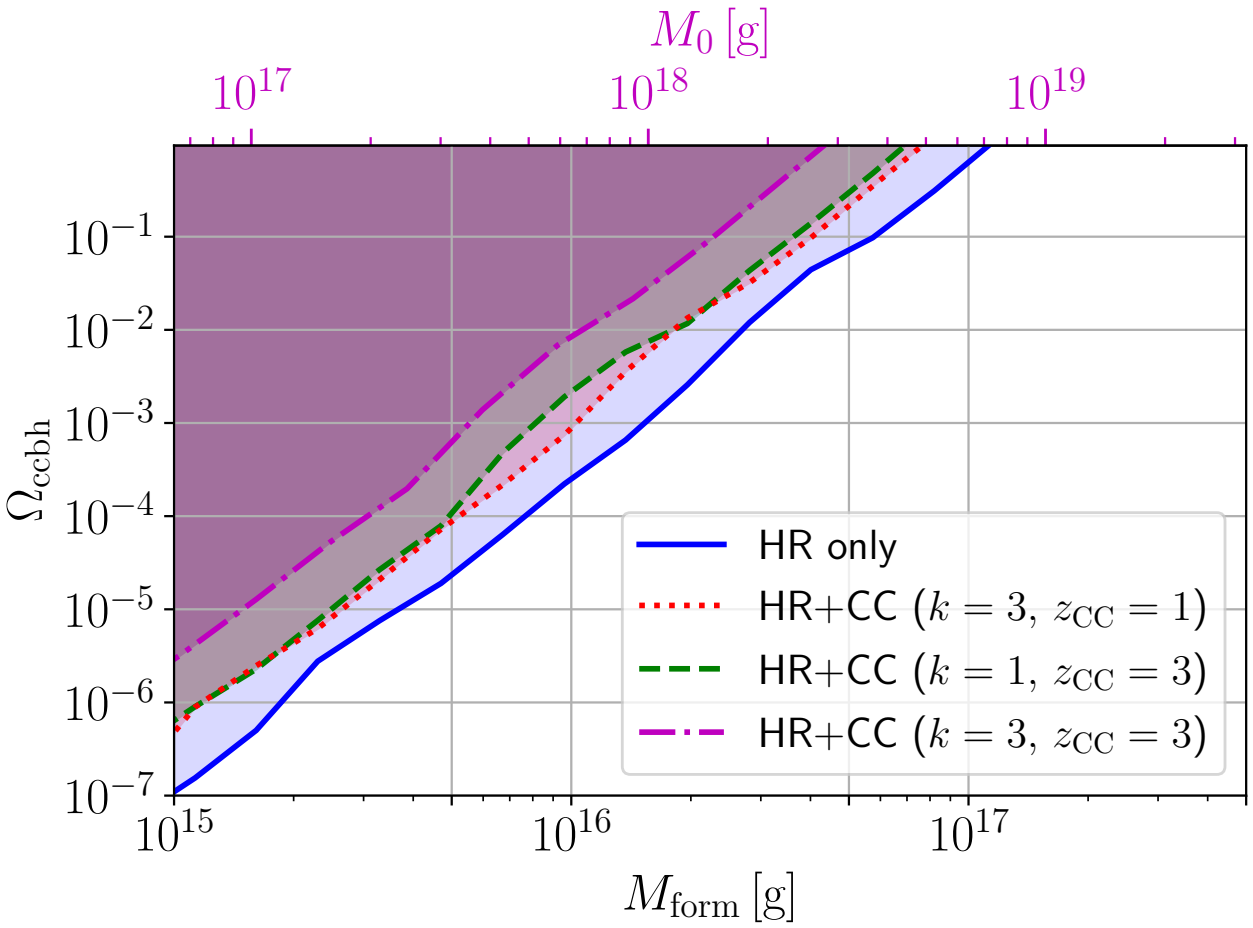
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Evolution of a primordial BH with initial mass 4.5×10^{14} g:



Effects on Hawking evaporation and PBH as dark matter



If dark matter is made of primordial black holes, CC alters the mass window.

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1 Static horizons cannot exist in FLRW

2 How does a horizon evolve?

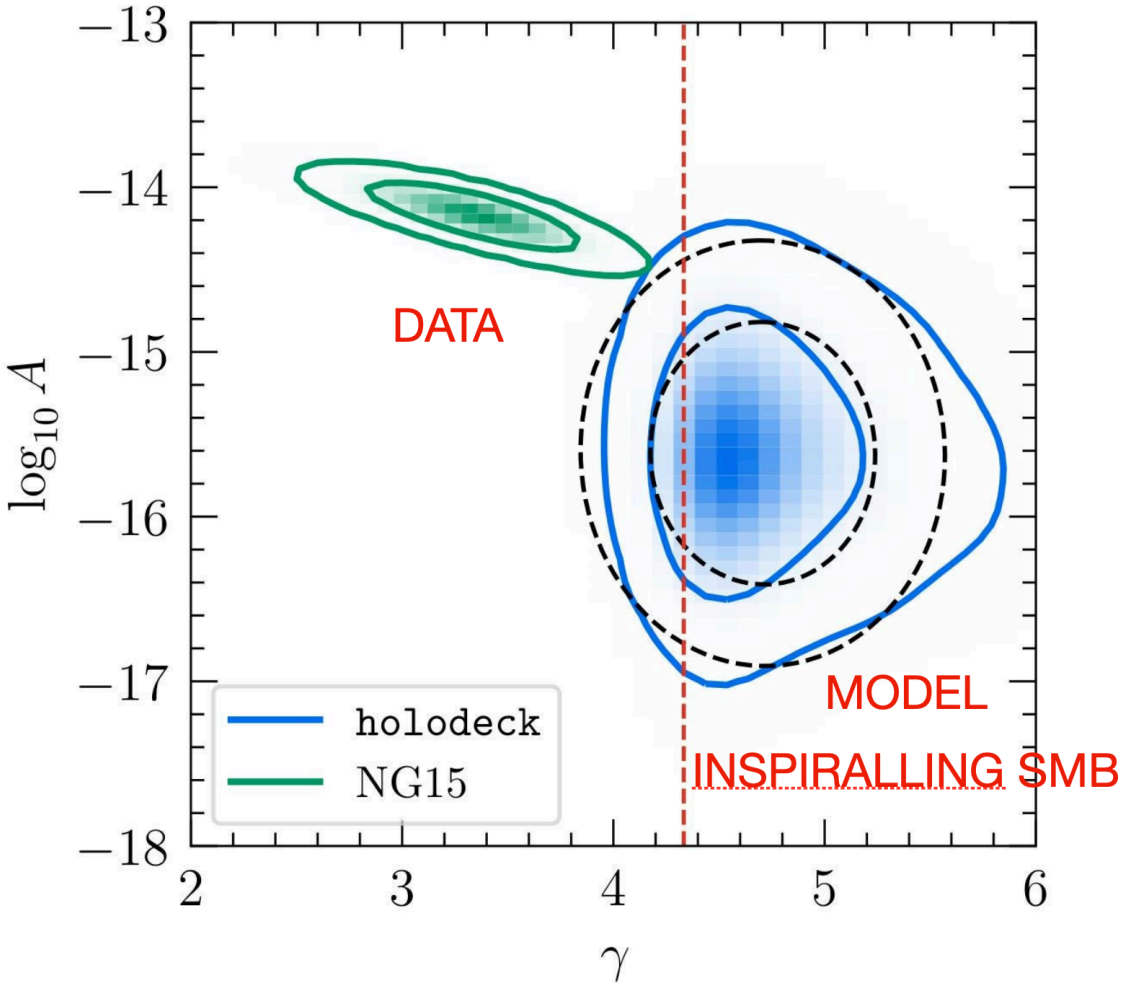
3 CC and primordial black holes

4 CC and Pulsar Timing Array (PTA)

5 Conclusions

Effects on Pulsar Timing Array

Pulsar Timing Array (PTA) experiments like NanoGrav are designed to measure the stochastic background of gravitational waves by means of pulsar signal distortions. If the background is generated by merging supermassive black holes there is a problem with data.



Effects on Pulsar Timing Array

- Cosmological coupling can leave an imprint on PTA data on the stochastic gravitational wave background (SGWB), when attributed to the merger of supermassive black holes. The standard amplitude is:

$$A^2 = \frac{4(G\mathcal{M})^{5/3}}{3\pi^{1/3}H_0}\int_{z_{\rm min}}^{z_{\rm max}}\frac{\dot{n}(z)dz}{(1+z)^{4/3}E(z)}$$

- Because the inspiral phase is very long, the mass growth can significantly alter the amplitude of SGWB through an increasing chirp mass $\mathcal{M}$. Assume Croker's growth model $(-3 < k < 3)$

$$M(z) = M_i\Theta(z - z_i) + M_i\left(\frac{1+z_i}{1+z}\right)^k\Theta(z_i - z)$$



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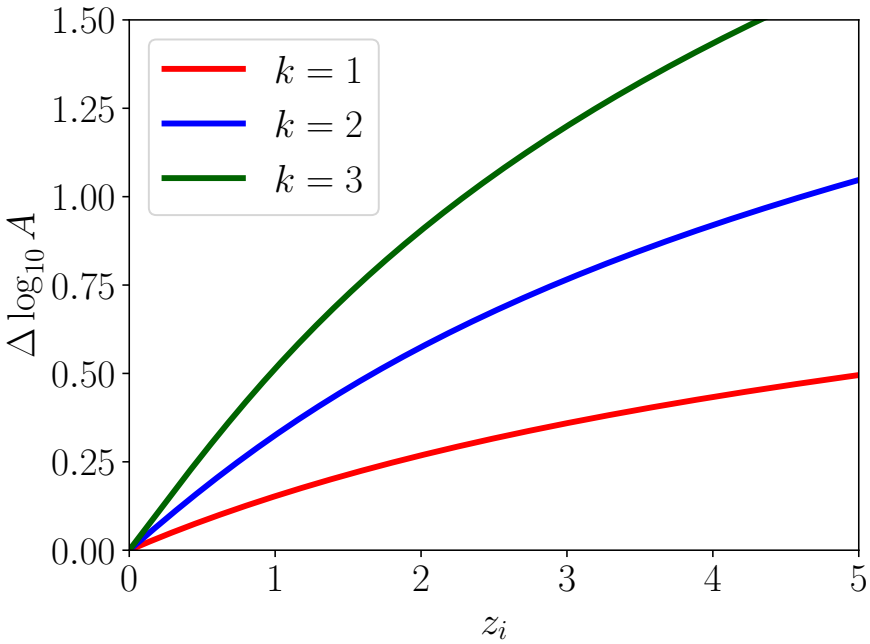
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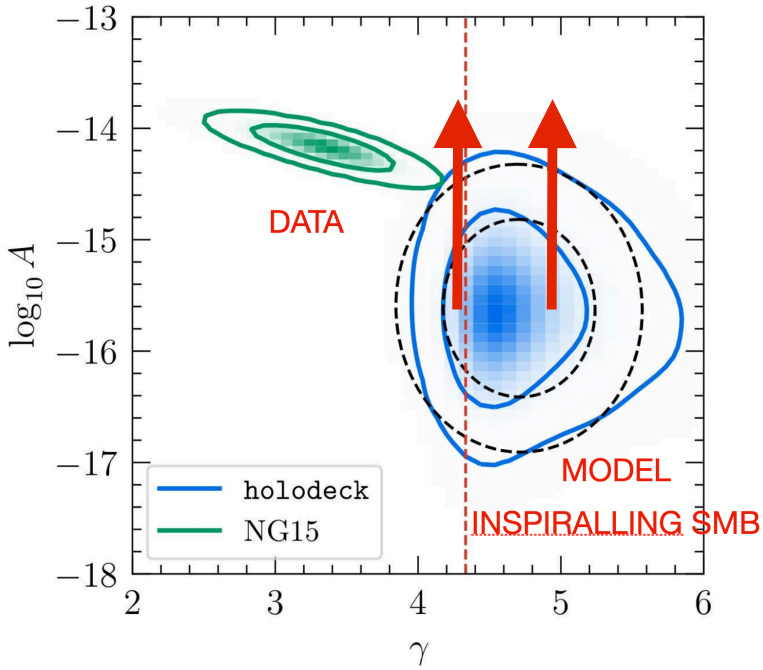


Effects on Pulsar Timing Array

The amplitude changes according to

$$A^2 = \frac{4(G\mathcal{M})^{5/3}}{3\pi^{1/3}H_0}(z_i+1)^{5k/3}\int_{z_{\min}}^{z_{\max}} dz\,\frac{\dot{n}(z)}{(1+z)^{(5k+4)/3}E(z)}$$





M. Calzà, F. Gianesello, M.R:, S. Vagnozzi, *Sci.Rep. 14 (2024) 31296*

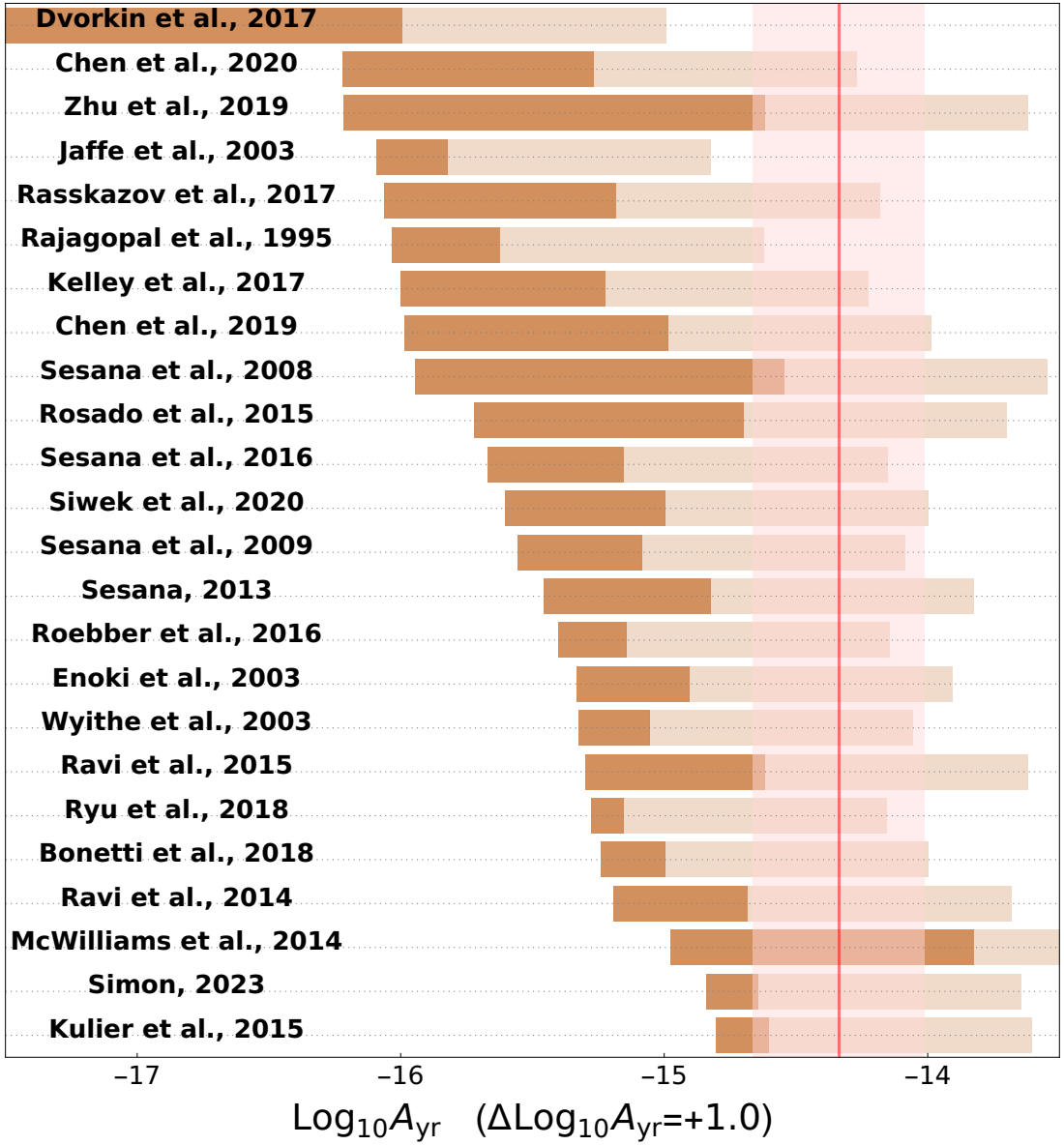


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- An exactly static black hole event horizon embedded in the time-dependent geometry generates a naked null singularity located at a finite areal radius. All radial null and timelike geodesics are future-inextensible at this static horizon leading to geodesic incompleteness, which define spacetime singularities (Hawking-Penrose theorems). This is confirmed by a divergent Ricci scalar.
- Scale separation is not sufficient: static horizon in expanding universes cause catastrophic effects no regardless of the magnitude of the expansion rate!
- The evolution of apparent horizons seems to be generic: in the long term they become comoving.
- CC has an impact in modelling DM as primordial black holes. It also affects PTA signals.



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THANK YOU FOR YOUR ATTENTION!



Backup slides



Isotropic and Rindler coordinate

Schwarzschild metric in spherical and isotropic coordinates

$$ds^2 = -\left(1 - 2M/\tilde{r}\right) dt^2 + \left(1 - 2M/\tilde{r}\right)^{-1} d\tilde{r}^2 + \tilde{r}^2 d\Omega^2$$

$$\tilde{r} = r\left(1 + \frac{M}{2r}\right)^2 \Rightarrow \; ds^2 = -\left(\frac{2r - M}{2r + M}\right)^2 dt^2 + \left(1 + \frac{M}{2r}\right)^4 \left[dr^2 + r^2 d\Omega^2\right]$$

Note that the horizon now is located at $r = M/2$! Set

$$\frac{x}{2} = \frac{2r - M}{2r + M} \quad \Leftrightarrow \quad r = \frac{M(x+2)}{2(2-x)}, \quad dr = \frac{2M}{(x-2)^2} dx$$

The metric becomes

$$ds^2 = -\frac{x^2}{4} dt^2 + \frac{1024M^2}{(x+2)^4(x-2)^4} dx^2 + \frac{64M^2}{(x+2)^4(x-2)^4} d\Omega^2$$

In the limit $x \to 0$ we have the near-horizon Rindler metric

$$ds^2 = -\frac{x^2}{4} dt^2 + 4M^2 \left[dx^2 + d\Omega^2\right] \quad \text{with} \quad \mathcal{R} = \frac{1}{2M^2}.$$



Null geodesics in isotropic coordinates

Schwarzschild solution in isotropic coordinates:

$$ds^2 = -\left(\frac{2r-M}{2r+M}\right)^2 dt^2 + \left(1+\frac{M}{2r}\right)^4 \left[dr^2 + r^2 d\Omega^2\right]$$

Null radial ray with tangent vector $(u^0, u^1, 0, 0)$ with normalisation condition

$$u^\alpha u_\alpha = 0 \quad \Rightarrow \quad \frac{u^1}{u^0} = \pm \frac{4r^2(M-2r)}{(M+2r)^3}$$

In the near-horizon region $r \to M/2$ we have :

$$\frac{x}{2} \to \frac{1}{M}\left(r - \frac{M}{2}\right) \quad \text{and} \quad \frac{u^1}{u^0} \simeq \mp \frac{1}{4M}\left(r-\frac{M}{2}\right) \quad \Rightarrow \quad \frac{u^1}{u^0} \to \mp \frac{x}{8}$$

The geodesic equation for u^0 is

$$\frac{du^0}{d\lambda} + 2\Gamma^0_{01} u^0 u^1 = 0 \quad \Rightarrow \quad \frac{du^0}{d\lambda} - \frac{8M u^0 u^1}{(M-2r)(M+2r)} = 0$$



In the near-horizon region

$$\frac{du^0}{d\lambda} \mp \frac{1}{2M}(u^0)^2 = 0$$

The solution has the form

$$u^0 = \mp \frac{C}{\lambda - \lambda_0}$$

with C and λ_0 arbitrary constant.



- Can we interpolate FLRW and Schwarzschild?
- After the discovery of the Schwarzschild solution, McVittie proposed in 1933 a metric that interpolates it with FLRW at large scale.
- But why bother? The size of a black is negligible compared to the Universe (separation of scales)
- Until 1929 the Universe was though to be static (Einstein found a solution of his equations provided a cosmological constant Λ was introduced in its equations)
- In 1929 everything changed when Hubble discovered the eponymous expansion law (Λ became the "biggest blunder"...) and it became sensible to look for an interpolating solution.



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- The metric of the McVittie solution in isotropic coordinates is

$$ds^2 = -\left[1 - \frac{m_0}{2ra(t)}\right]^2 \left[1 + \frac{m_0}{2ra(t)}\right]^{-2} dt^2$$
$$+a(t)^2 \left[1 + \frac{m_0}{2ra(t)}\right]^4 \left(dr^2 + r^2 d\Omega^2\right)$$

- For $a = 1$ it becomes Schwarzschild, for $a(t) \to \infty$ it becomes FLRW.
- The source is a fluid with G^1_0 (unphysical no-accretion condition) and

$$8\pi P = -3H^2 - 2\dot{H}\left(1 + \frac{m_0}{2ra(t)}\right)\left(1 - \frac{m_0}{2ra(t)}\right)^{-1}$$

- The solution shows a singularity at the horizon located at $r = m(t)/2$, where pressure diverges and energy conditions are violated.



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