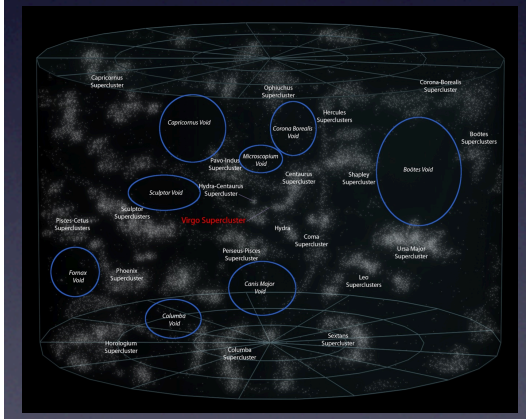
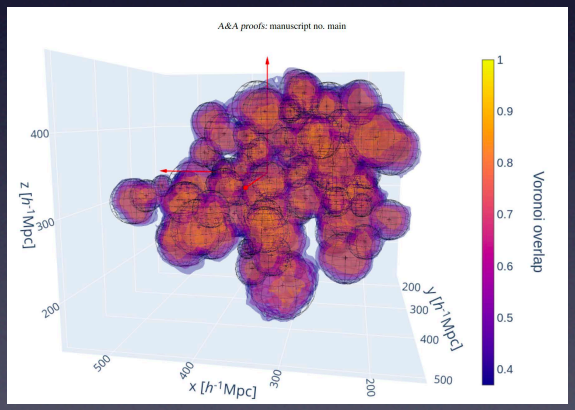
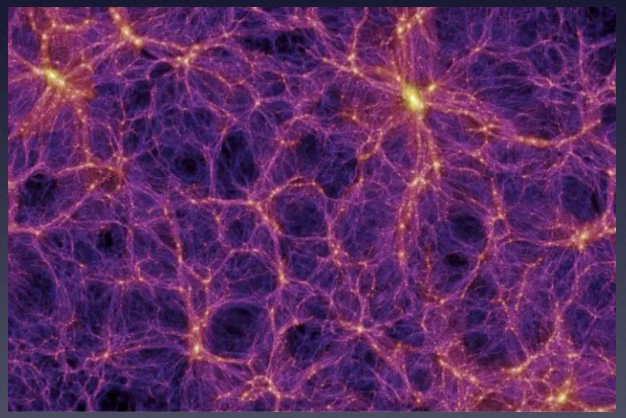
Modeling Voids as Cosmological Structures

Roland Triay, CPT - Aix Marseille University

Hot topics in Modern Cosmology
Spontaneous Workshop XVIII
September 13rd – 19th, 2026, IESC Cargese, France







* The travelling wavelets approach to gravitational instability theory:one-dimensional wavelets,*N. Benhamidouche, B. Torresani and R. Triay* - MNRAS, 302, 807 (1999)
* Lambda effect in the cosmological expansion of voids.*H-H Fliche, R.Triay* - JCAP 11:022 (2010)
* Dynamics ofVoid and its Shape in Redshift Space.*Kei-ichi Maeda, Nobuyuki Sakai, RolandTriay* - JCAP 08:026 (2011)
* Isotropization of voids

Einstein equations

In the GR theory of gravitation, the sources of the gravitational field, which are characterized by a *vanishing divergence* stress-energy tensor T in the field equations,

$$T = \mathcal{A}(g) \tag{1}$$

account for a metric tensor g with signature $(+,-,-,-)$ on the space-time manifold. $\mathcal{A}$ reads as a series of covariant tensors written in term of g and its derivatives

$$\mathcal{A}(g)_{\mu\nu} = -\mathcal{A}_0 F^{(0)}_{\mu\nu} + \mathcal{A}_1 F^{(1)}_{\mu\nu} + \mathcal{A}_2 F^{(2)}_{\mu\nu} + \dots \tag{2}$$

where $F^{(n)}$ denotes a $2n$ degree tensor. The $\mathcal{A}_n$ stand for *coupling constants*, their values have to be determined from observations.

T_{00} : energy density : mass , kinetic, thermal, electromagnetic, energy of other physical fields.
T_{0i} : momentum fluxes (frame dragging, e.g. Kerr black hole.)
T_{ij}: stress tensor : pressure, shear stress

The laws of physics are universal and invariance under diffeomorphisms acting on spacetime

For $n \leq 1$ these tensors are unique

$$F^{(0)}_{\mu\nu} = g_{\mu\nu}, \quad F^{(1)}_{\mu\nu} = S_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci tensor and R the scalar curvature. The determination of consecutive terms $n > 1$ of the expansion in eq. (2) requires one to specify the $F^{(n)}_{\mu\nu}$ by means of additional principles.

Origin of the Cosmological Constant

Let us choose units of mass M and of length L. Hence, one has $[g_{\mu\nu}] = L^2$, and thus $[g^{\mu\nu}] = L^{-2}$, $[R_{\mu\nu}] = 1$ and $[R] = L^{-2}$. The specific mass density and the pressure belong to T^{μ}_{ν}, one has $[T_{\mu\nu}] = ML^{-1}$. Hence, one obtains

$$[\mathcal{A}_0] = ML^{-3}, \quad [\mathcal{A}_1] = ML^{-1}, \quad \ldots [\mathcal{A}_n] = ML^{2n-3}$$

which shows their relative contributions for describing the gravitational field with respect to scale. Namely, the larger their degree n the smaller their effective scale. Equivalently, the estimation of $\mathcal{A}_0$ demands observational data located at scale larger than the one for $\mathcal{A}_1$, *etc.*... The *transition scale* between the first two terms is defined by

$$\tau_{1/0} = \sqrt{|\mathcal{A}_1/\mathcal{A}_0|} \qquad (3)$$

A solution to the Cosmological Constant Problem. R. Triay - International Journal of Modern Physics D 14,10,1667 (2005)

Effect of the Cosmological Constant

The comparison of Schwarzschild solution with the (modified) Poisson equation

$$\mathrm{div}\vec{g} = -4\pi \mathrm{G}\rho + \Lambda$$

provides us with

$$\mathcal{A}_1 = \frac{1}{8\pi \mathrm{G}} \sim 5\,10^{26}\ \mathrm{g\,cm^{-1}}, \quad \mathcal{A}_0 = \frac{\Lambda}{8\pi \mathrm{G}}, \quad \vec{g} = \left(-\mathrm{G}\frac{m}{r^3} + \frac{\Lambda}{3}\right)\vec{r}$$

and $\tau_{1/0} \approx 1/\sqrt{\Lambda} \sim 7h^{-1}$ Gyr. If $\Lambda > 0$ then there is a critical distance

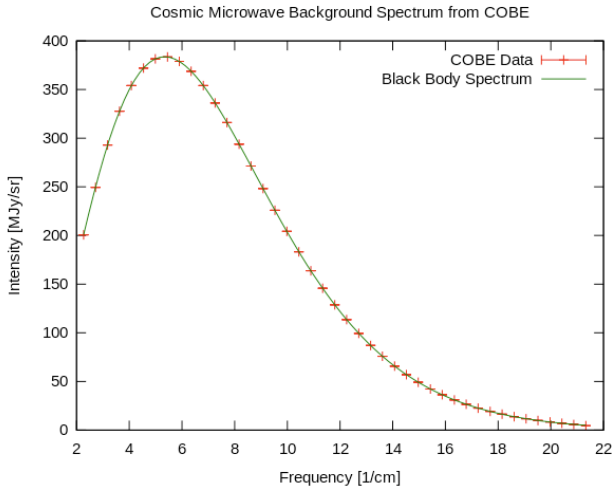
$$r_\circ = \sqrt[3]{3m\mathrm{G}/\Lambda}$$

where the gravity vanishes ; it is attractive if $r < r_\circ$ and repulsive if $r > r_\circ$.

We live in an oven in thermodynamic equilibrium at temperature of 3° K

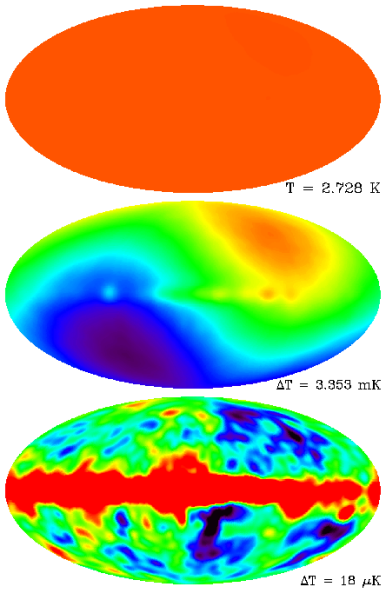
Cosmological time and space

Sky shows a black body spectrum which shows a PERFECT thermodynamics equilibrium in the Universe. Such a characteristic is compatible with the recession of galaxies (cosmological expansion) solely in GR.



Velocities relative to the CMB

- The solar system barycenter $v = 368 \pm 2\,\mathrm{km/s}$ towards $(263.85° \pm 0.10°, 48.25° \pm 0.04°)$.
- The Galaxy and the Local Group of galaxies $v_{LG} = 627 \pm 22\mathrm{km/s}$ toward $(276° \pm 3°, 30° \pm 3°)$



Planck temperature

$$\Theta = \frac{U}{kT}$$

Jean-Marie Souriau : "Mécanique Statistique, Groupe de Lie et Cosmologie", Coll. Internationaux du CNRS N°– 237 Géométririe symplectique et physique mathématique (1974)

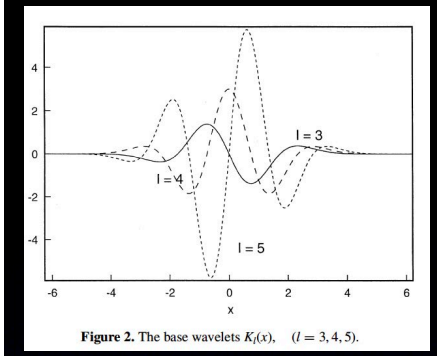


Figure 2. The base wavelets $K_l(x)$, $(l = 3, 4, 5)$.

Gravitational Instability

Poisson equations, which read (in comoving coordinates)

$$\frac{\partial \delta}{\partial t} + \frac{\partial}{\partial x}[(1+\delta)u] = 0, \tag{2}$$

$$\frac{\partial u}{\partial t} + 2H\,u + u\frac{\partial u}{\partial x} = -\frac{1}{a^2}\frac{\partial \phi}{\partial x}, \tag{3}$$

$$\frac{1}{a^2}\frac{\partial^2 \phi}{\partial x^2} = 4\pi\rho_b\delta, \tag{4}$$

where $H = \dot{a}/a = 2/3t^{-1}$ stands for the Hubble constant and the gravitational potential $\psi = \psi(r,t)$ acts through the function $\phi(x,t) = \psi - 1/2\,\ddot{a}x^2$.

- **Voids are non linear structures** — e.g., Traveling Wavelets approach to the Gravitational Instability theory. – One dimensional Wavelets . – N. Benhamidouche, B. Torresani, R. Triay. -MNRAS 302,807(1999)

- « vacuum gravitational repulsion » ?
  e.g., *Schwarzschild solution of Einstein Eq. with a Cosmological Constant*

$$\vec{g} = \left(-\mathrm{G}\frac{m}{r^3} + \frac{\Lambda}{3}\right)\vec{r}$$

$$r_\circ = \sqrt[3]{3m\mathrm{G}/\Lambda}$$

No effect in the Solar neighborhood
Small effect in the outerpart of the Galaxy
Intervenes at the edge of LSC
Homogeneity scale

$$r_\circ \sim 10^2 h^{-2/3}\ \mathrm{yr}$$
$$r_\circ \sim 5\,10^5 h^{-2/3}\ \mathrm{yr}$$
$$r_\circ \sim 4\,10^8 h^{-2/3}\ \mathrm{yr}$$
$$(\sim 100\,\mathrm{Mpc})$$

Spherical voids as Gravitational Structures

- Modeling **a "single void" embedded in a Friedmann Lemaître Universe** as the first step to a more global investigation (foam like structure)

  - - (sphere) Newton - Friedmann model

  - - (sphere) GR "FL^{in} - FL^{out}" Model - distant

  - - (ellipsoid-revolution) Newton - Friedmann model
    Embeded in the comoving space

Friedmann - Newton Model



Covariant Global solution to Euler- Poisson Eqs. System

Reference Coordinates $(t, \quad \vec{x} = \frac{\vec{r}}{a}), \qquad a > 0$

$$H = \frac{\dot{a}}{a} = \sqrt{\frac{\Lambda}{3} - \frac{K_\circ}{a^2} + \frac{8\pi G}{3}\frac{\rho_\circ}{a^3}}$$

$$K_\circ = \frac{8\pi G}{3}\rho_\circ + \frac{\Lambda}{3} - H_\circ^2 \leq \sqrt[3]{(4\pi G\rho_\circ)^2\,\Lambda}, \qquad a_\circ = 1$$

$$\vec{v}_c = \alpha\vec{x}, \qquad \vec{g}_c = \beta\vec{x}$$

$$\frac{d\alpha}{dt} + 4\alpha^2 + 2H\alpha - \frac{2\pi G}{3}\frac{\rho_c}{a^3} = 0$$

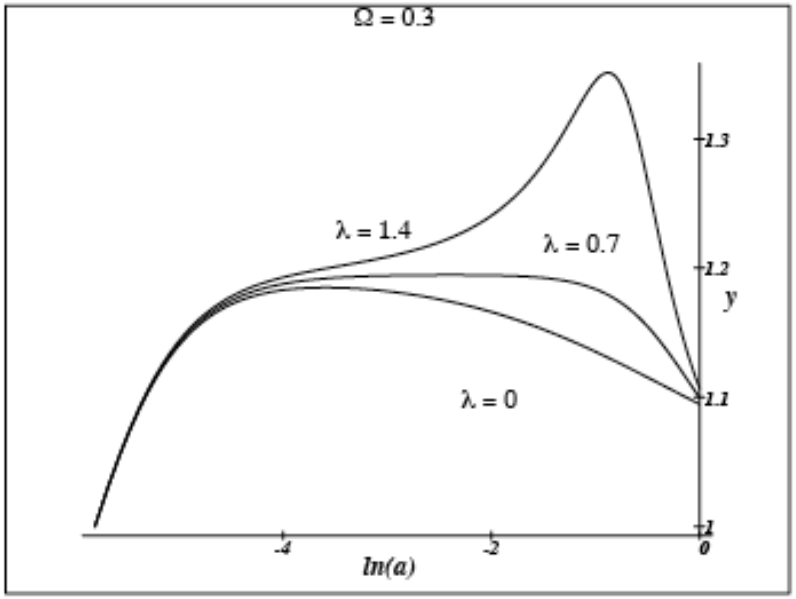
$$\beta = \frac{4\pi G}{a^3}\left(\frac{\rho_c}{3} - \frac{(\rho_S)_c}{2x}\right)$$

Magnification & Expansion Rate

$$\vec{v}_c = \alpha\vec{x}, \qquad \vec{g}_c = \beta\vec{x}$$

$$X = \frac{x}{x_i}, \qquad Y = \frac{\alpha}{H_\circ}$$

y : Corrective factor to Hubble expansion



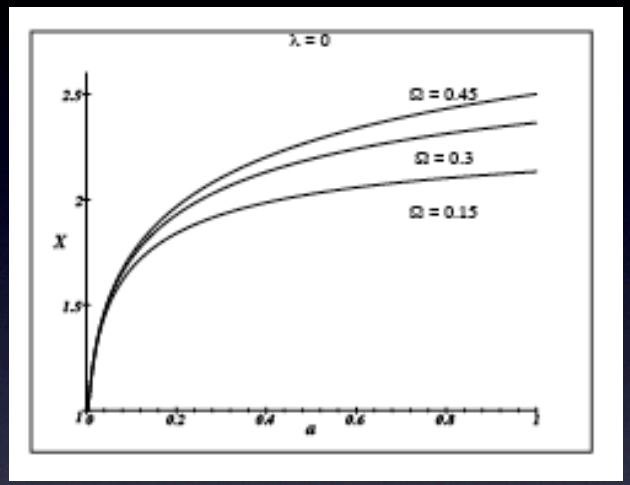
$$\vec{v} = yH\vec{r}, \qquad y = 1 + \frac{Y}{h}, \qquad h = \frac{H}{H_\circ}$$

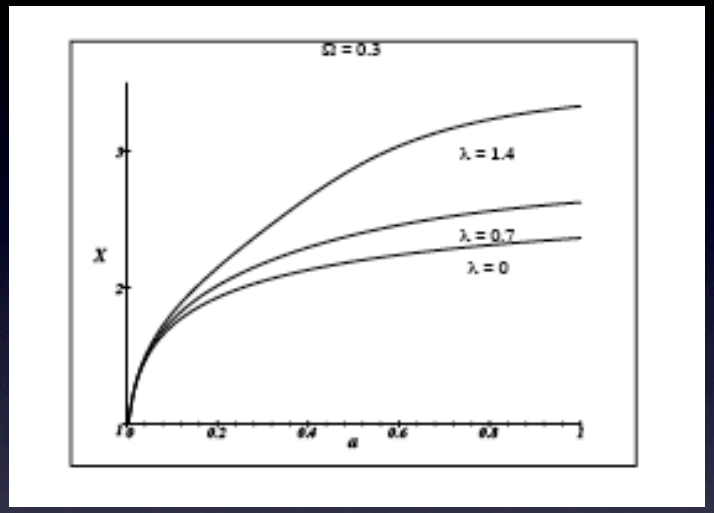
$$\lambda_\circ = \frac{\Lambda}{3H_\circ^2}, \qquad \Omega_\circ = \frac{8\pi G\rho_\circ}{3H_\circ^2}, \qquad k_\circ = \frac{K_\circ}{H_\circ^2}$$

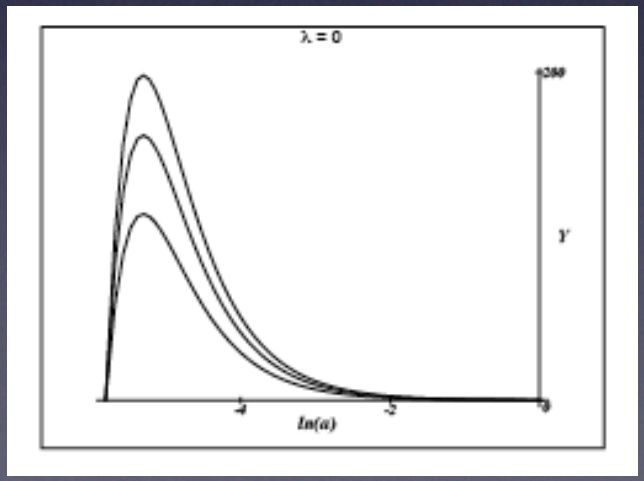
Observational Signature of $K_\circ$

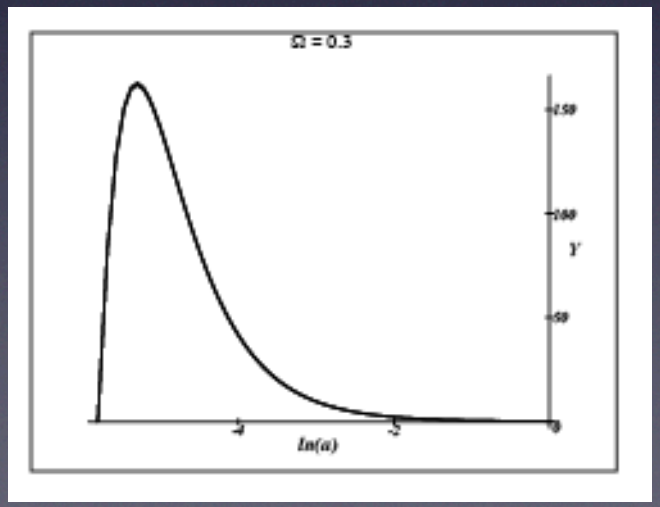
$$H_m = H_\infty\sqrt{1 - \frac{K_\circ^3}{\Lambda\left(4\pi G\rho_\circ\right)^2}}, \qquad H_\infty = \lim_{a\to\infty} H = \sqrt{\frac{\Lambda}{3}}$$

Dependence on Cosmological Parameters









GR Model

$$\mathrm{d}s_{\pm}^2 = -\mathrm{d}t_{\pm}^2 + a_{\pm}^2\left[\mathrm{d}\chi_{\pm}^2 + f_{\pm}^2(\chi_{\pm})\mathrm{d}\Omega^2\right], \qquad f_{\pm}(\chi_{\pm}) = \begin{cases} \sin\chi_{\pm} & (k_{\pm}=1) \\ \chi_{\pm} & (k_{\pm}=0) \\ \sinh\chi_{\pm} & (k_{\pm}=-1) \end{cases}$$

$$\mathrm{d}s_{\Sigma}^2 = -\mathrm{d}\tau^2 + R^2(\tau)\mathrm{d}\Omega^2$$

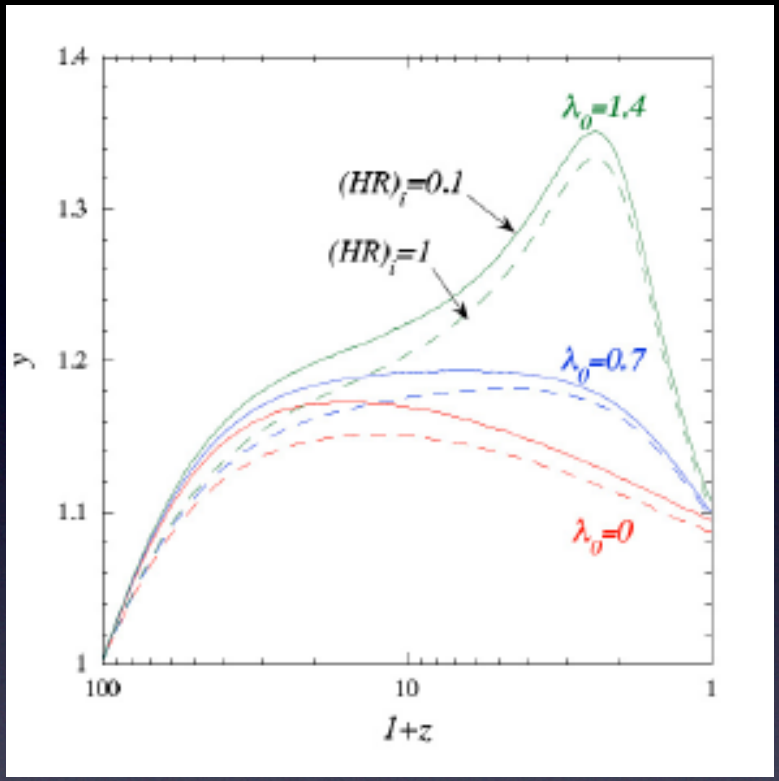
$$H_{\pm}^2 + \frac{k_{\pm}}{a_{\pm}^2} = \frac{8\pi G}{3}\rho_{\pm}, \qquad \text{with} \qquad H_{\pm} = \frac{\dot{a}_{\pm}}{a_{\pm}}, \quad \text{and} \quad \rho_{\pm} = \rho_{\mathrm{vac}} + \rho_{\pm}^{(\mathrm{m})}$$

$$\rho_{\mathrm{vac}} = \Lambda/(8\pi G)$$

$$R = a_+ f_+\left(\chi_+^{(\Sigma)}\right) = a_- f_-\left(\chi_-^{(\Sigma)}\right)$$

$$d\tau^2 = dt_+^2 - a_+^2(t_+)\left(d\chi_+^{(\Sigma)}\right)^2 = dt_-^2 - a_-^2(t_-)\left(d\chi_-^{(\Sigma)}\right)^2$$

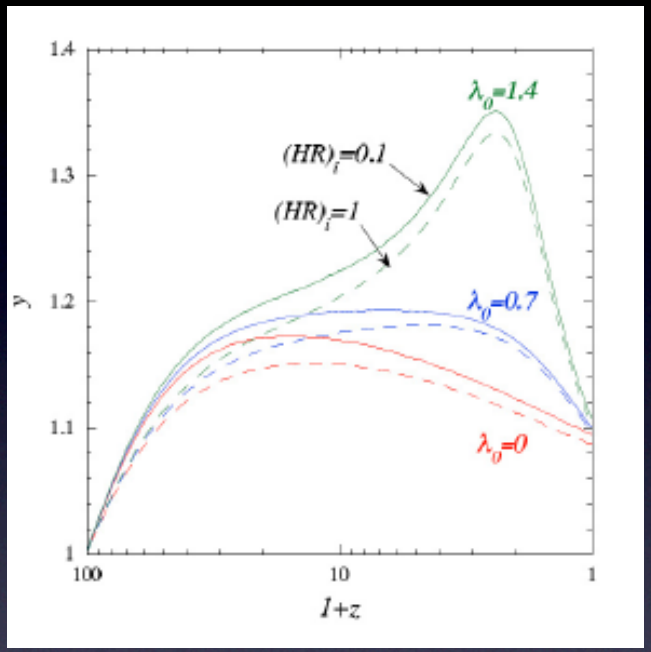
GR Model



1. The voids always grow faster than Hubble expansion (*i.e.*, $y > 1$) along their evolution. The peculiar velocity starts with a huge burst and decreases asymptotically toward Hubble velocity; the present value is higher than the Hubble flow by about 10%.

2. The smaller the radius gets, the higher the peculiar velocity is. Namely, the void with the largest initial size $R_i = H_i^{-1}$ grows slightly slower, which provides us with the minimal peculiar velocity.

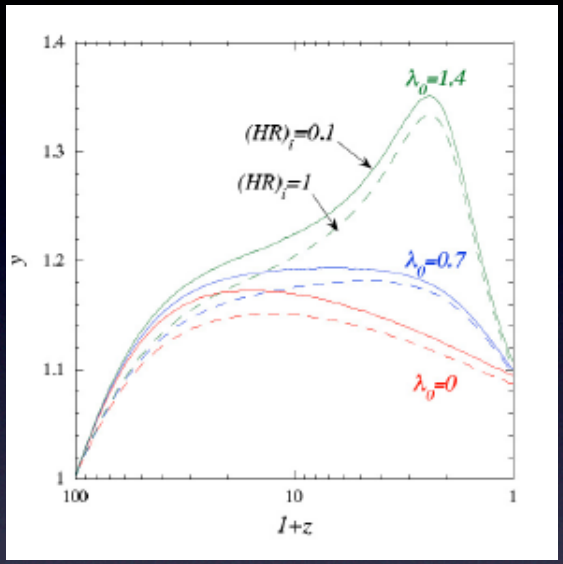
GR Model



3. While the dependence on λ_0 is not significant in the beginning, its effect eventually appears in the evolution, and the correction factor y increases with λ_0. With $\lambda_0 = 0.7$, the peculiar velocity reaches the value higher than the Hubble expansion at $z \sim 10$ by about 20%.

4. The redshift $z^\star$ when y reaches its maximum $y_{\max} = y(z^\star)$ decreases with λ_0. It corresponds to $z^\star \sim 40$ for $\lambda_0 = 0$ and to $z^\star \sim 1.7$ for $\lambda_0 = 1.4$. For the intermediate value $\lambda_0 = 0.7$,for which the corresponding curve shows a plateau, Λ gives the maximal contribution to v at $z^\star \sim 1.7$, which corresponds to 30% of that of the matter density (when it is compared to the case of $\lambda_0 = 0$).

GR Model

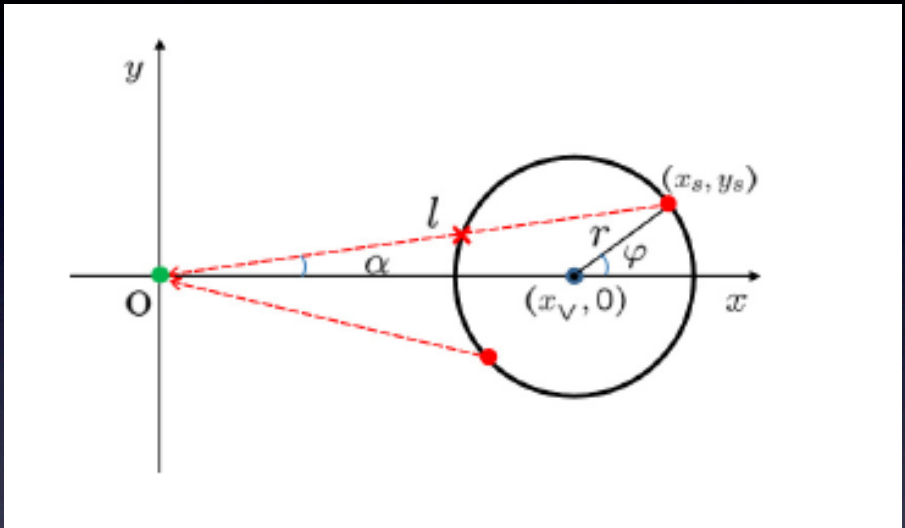


5. Let us pay attention to the presence of a bump on the curve, which becomes visible for $\lambda_0 > 0.7$ (*i.e.* when $k > 0$). It is caused by the fact that the universe experiences a loitering period of cosmological expansion, while the void then continues its own expansion.

6. The present values y_0 related to $\Omega_0 = 0.3$ but to different values for $\lambda_0 \in \{0, 0.7, 1.4\}$ are very close to each other, while the variation of y with time (or with z) depends undeniably on λ_0. In other words, the Λ effect, which accounts for a deviation between these curves, is not substantial nowadays.

7. It is remarkable that the present GR approach for the sub-horizon void confirms the previous result found in the Newtonian dynamics [13]. Even for a relativistic spherical void with a horizon size radius $R_i = H_i^{-1}$, the relativistic effect turns out to be weak.

Void in the redshift-space

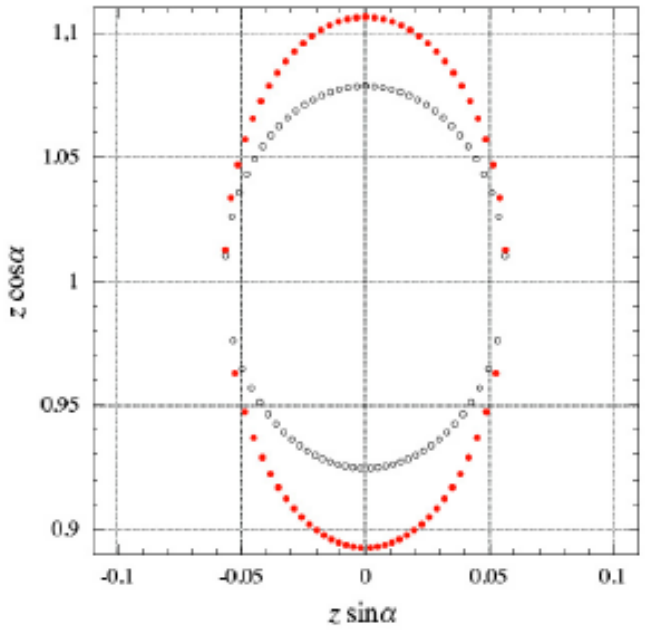


The Sachs-Wolfe effect $\Delta z_s|_{\rm ISW} \sim (HR)^3$

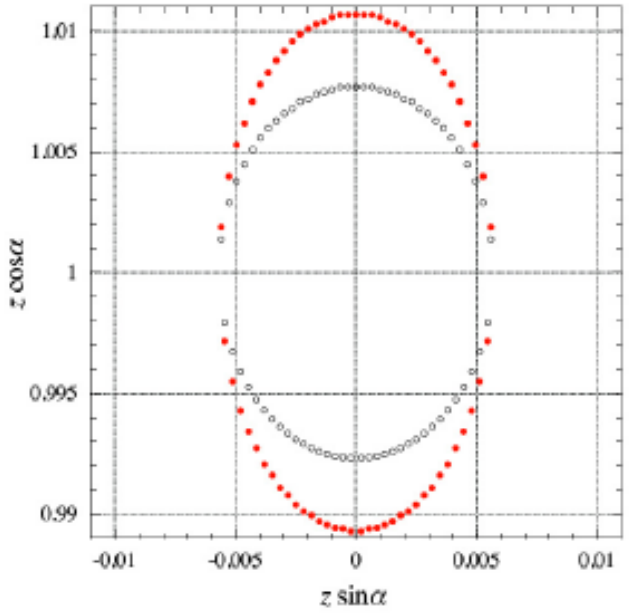
and the gravitational lens effect $\Delta z_s|_{\rm GL} \approx \frac{R(\Delta\alpha)^2}{a x_V} \approx 0.06 \frac{r}{x_V}(HR)^2 \sin(\varphi - \alpha), \quad \text{for} \quad \Omega_0 = 0.3$

are weak compared to the "doppler effect".

Image of a Void in the redshift-space



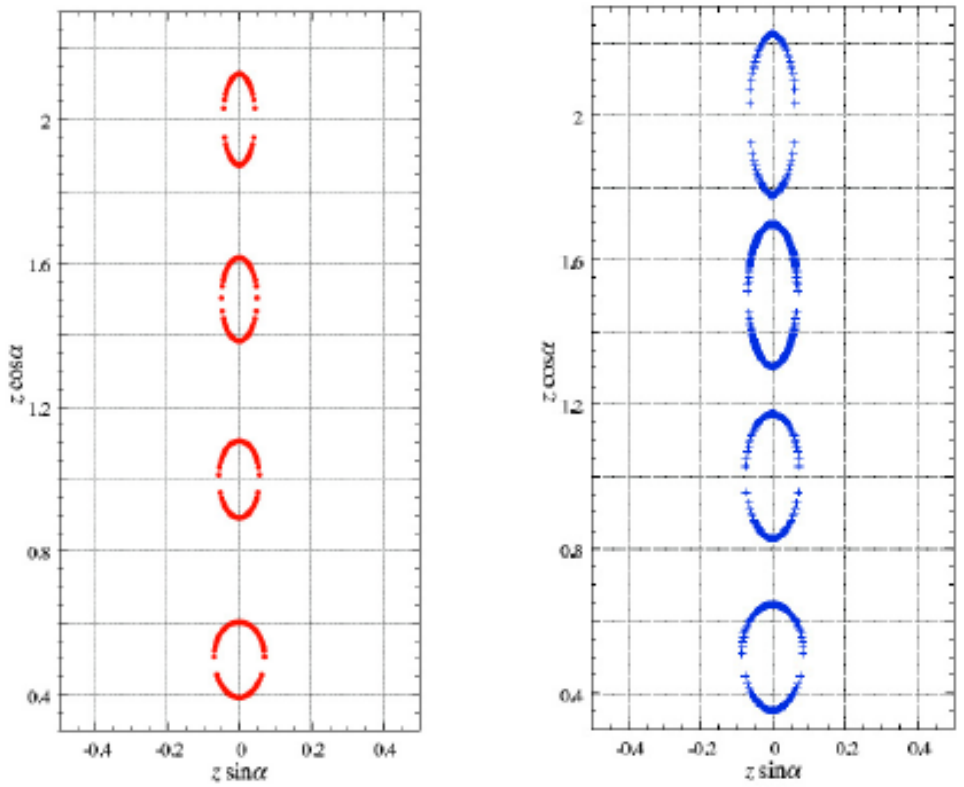
(a) $R_0 = 0.1H_0^{-1}, \quad z_V = 1$



(b) $R_0 = 0.01H_0^{-1}, \quad z_V = 1$

Figure 3. Empty spherical voids in the redshift space. — The images (red dots) of their boundary layers (*i.e.*, the void shell S) are depicted for the case of $\Omega_0 = 0.3$ and $\lambda_0 = 0.7$. The present values of their radii are (a) $R_0 = 0.1H_0^{-1}$ and (b) $R_0 = 0.01H_0^{-1}$. The observed radii at $z_V = 1$ are : (a) $R_V = 0.0437H_0^{-1}$ and (b) $R_V = 0.00437H_0^{-1}$. The images of the standard static spheres (small black dots) are also displayed.

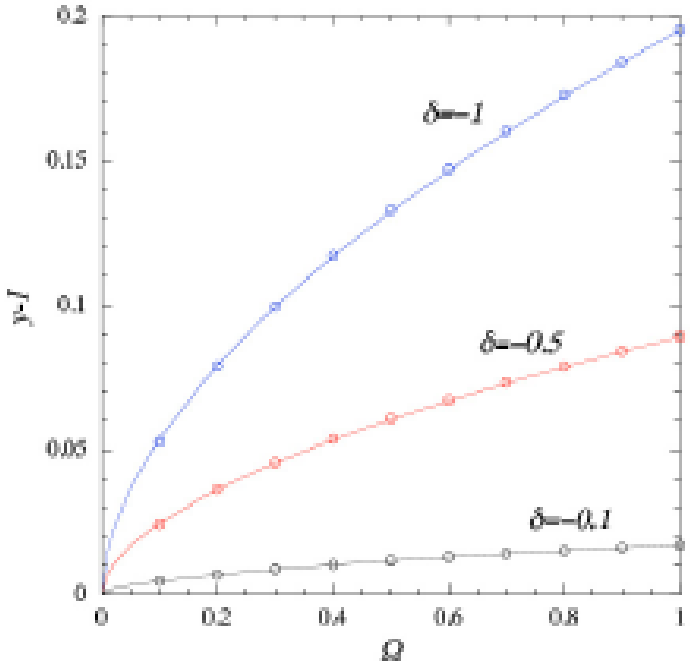
Dependence on Cosmological Parameters



(a) $\Omega_0 = 0.3$ (b) $\Omega_0 = 1$

Figure 5. The images in the redshift space of an empty spherical void with present radius $R_0 = 0.1H_0^{-1}$ at redshift $z_v \in \{0.5, 1, 1.5, 2\}$ for (a) $\Omega_0 = 0.3$ and (b)$\Omega_0 = 1$.

Non Empty Voids



Velocity y versus Ω and δ^-.

Figure 7. The velocity y of a non-empty void. — We assume $\Omega + \lambda = 1$ and $\delta^- \in \{01, -0.5, -0.1\}$. The circles and the continuous lines correspond respectively to our numerical results and to the fitting formula given in Eq. (3.2).

Conclusion

In addition to provide us with a method for estimating the cosmological parameters, this test enables us to characterize clearly the sign of the spatial curvature of the FL universe. In other words, it provides an answer to the question on whether the space is finite or infinite, which is a crucial issue for the primordial cosmology.

Thank you