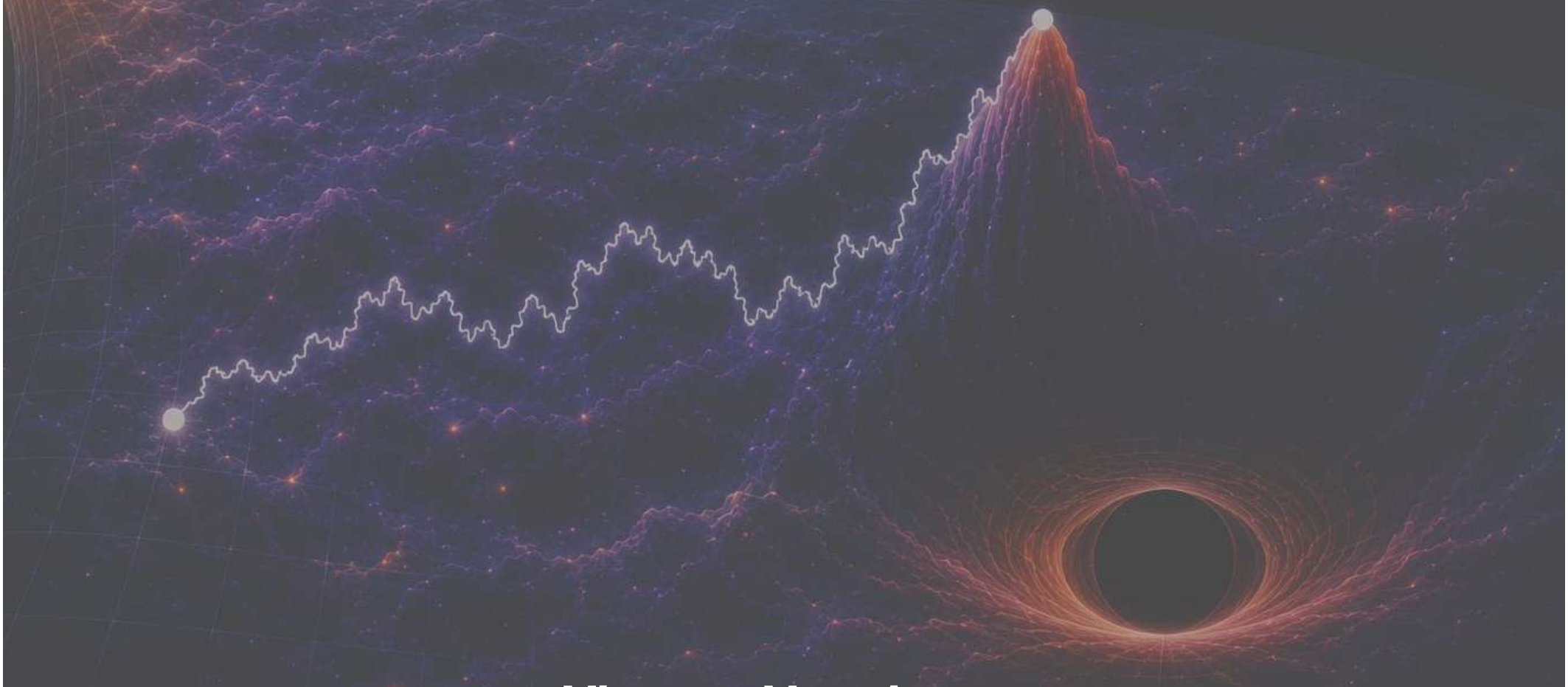
LPENS
LABORATOIRE DE PHYSIQUE DE L'ÉCOLE NORMALE SUPÉRIEURE

cnrs

Harvesting primordial black holes from stochastic trees



Vincent Vennin

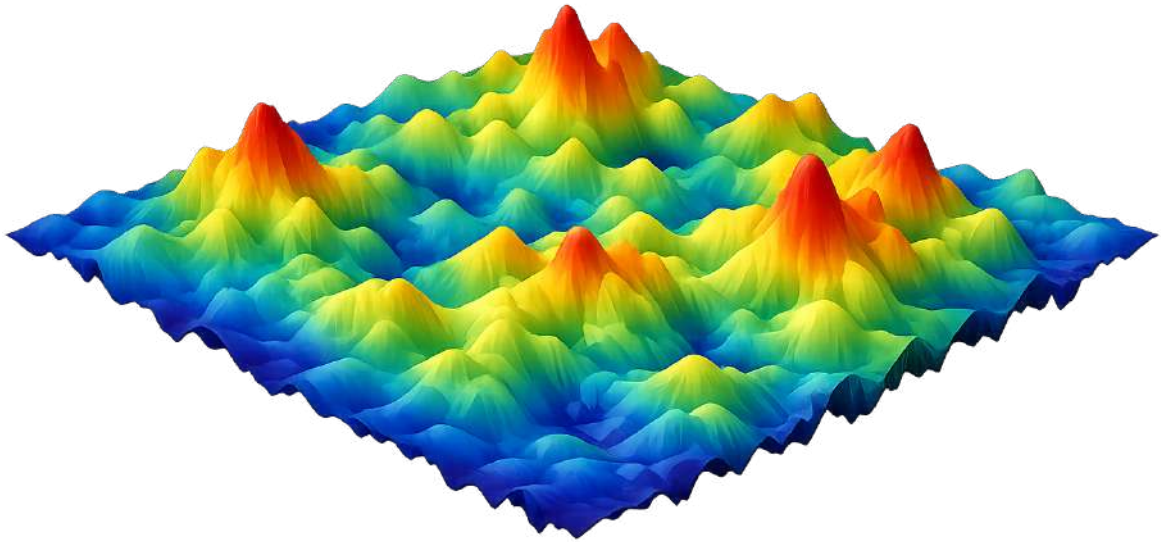
Spontaneous Workshop XVIII, 25 September 2026, Cargèse

Harvesting primordial black holes from stochastic trees

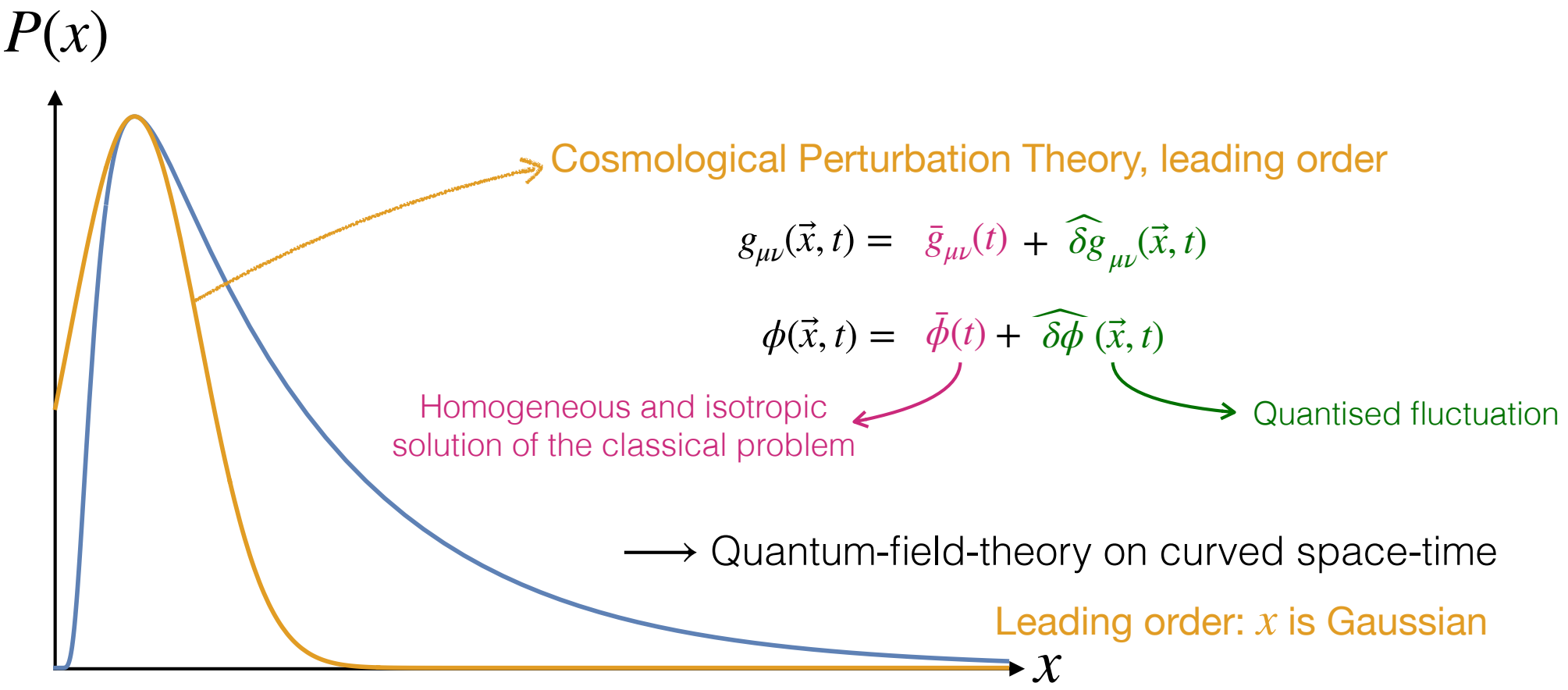
- Primordial black holes collapse from large density fluctuations in the primordial universe.
- They are rare and extreme events, hence they originate from the far tails of distribution functions.
- Perturbative methods are efficient to describe the bulk (near-maximum region) of distribution functions, which shape the n-point correlation functions.
- Primordial black holes require non-perturbative methods.

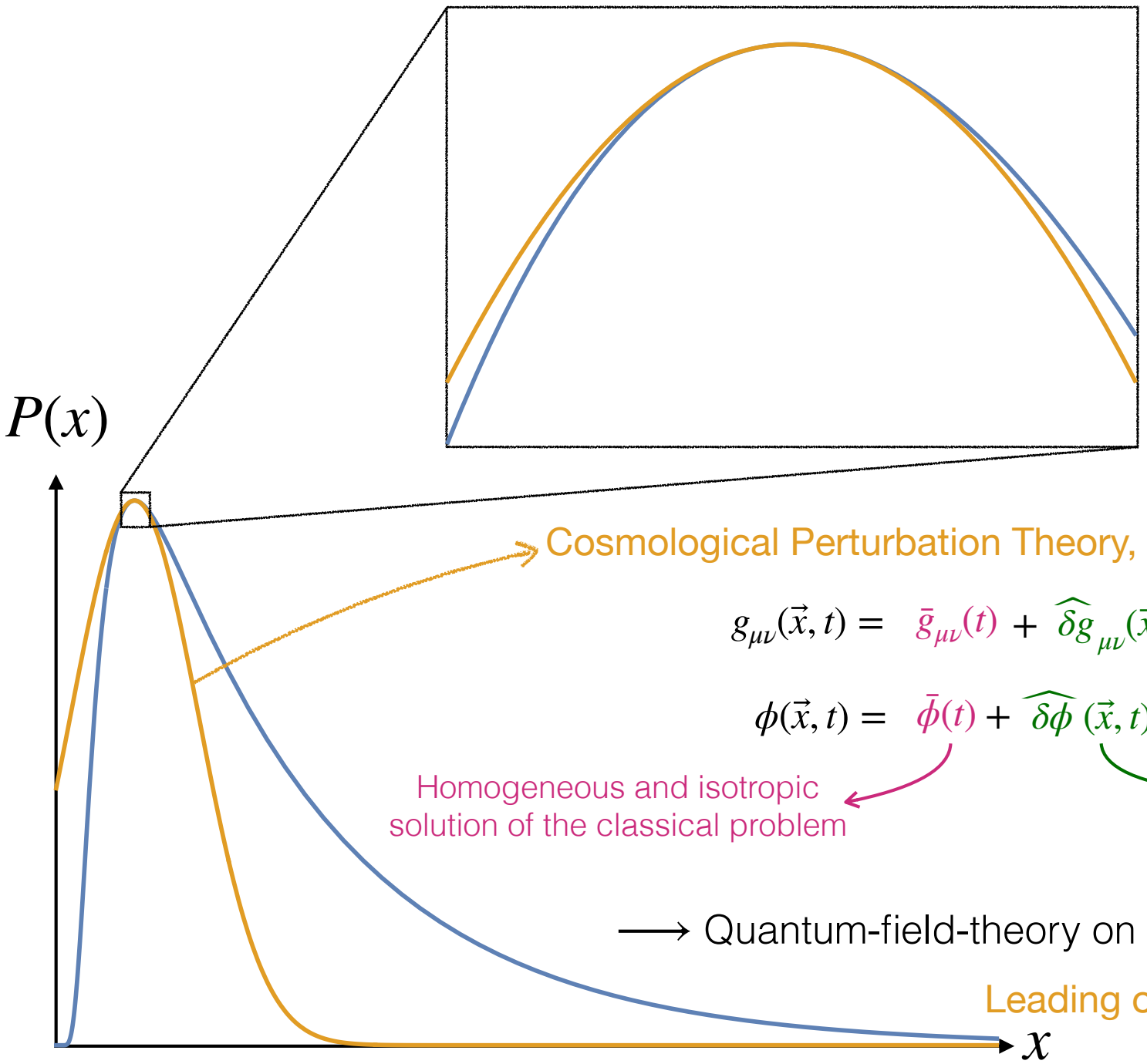
Black holes form in tails

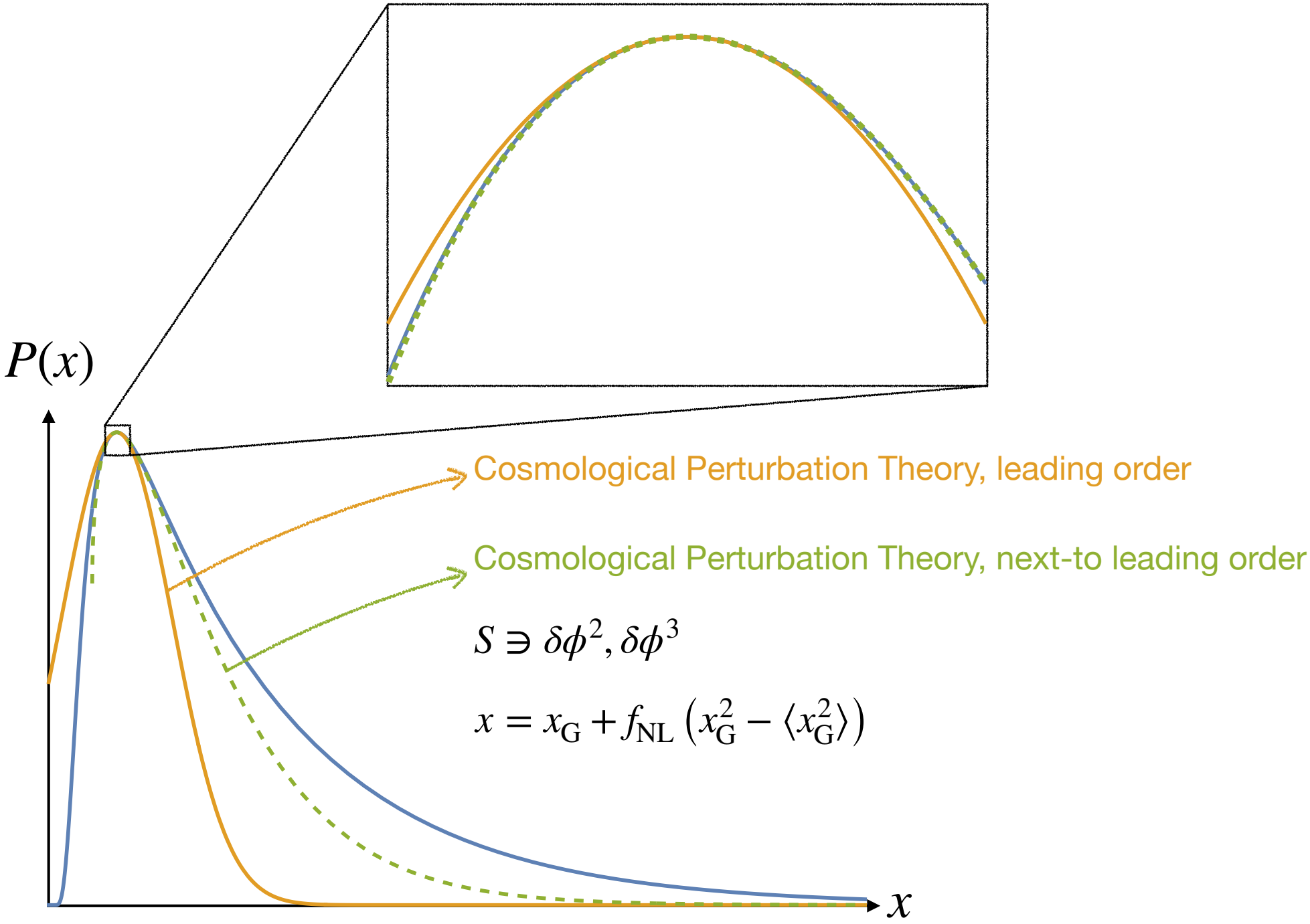
How likely is it to form a given cosmological structure?

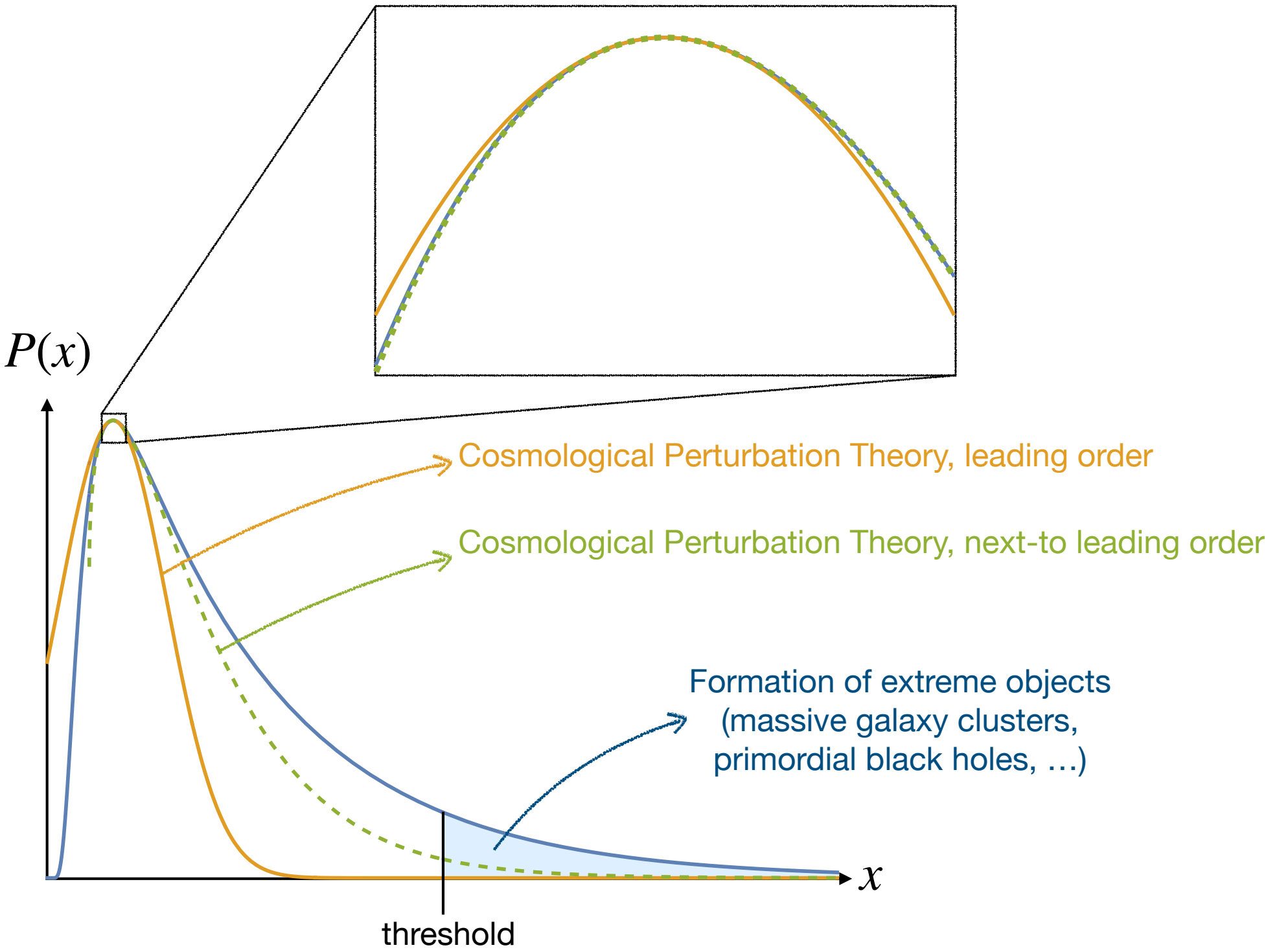


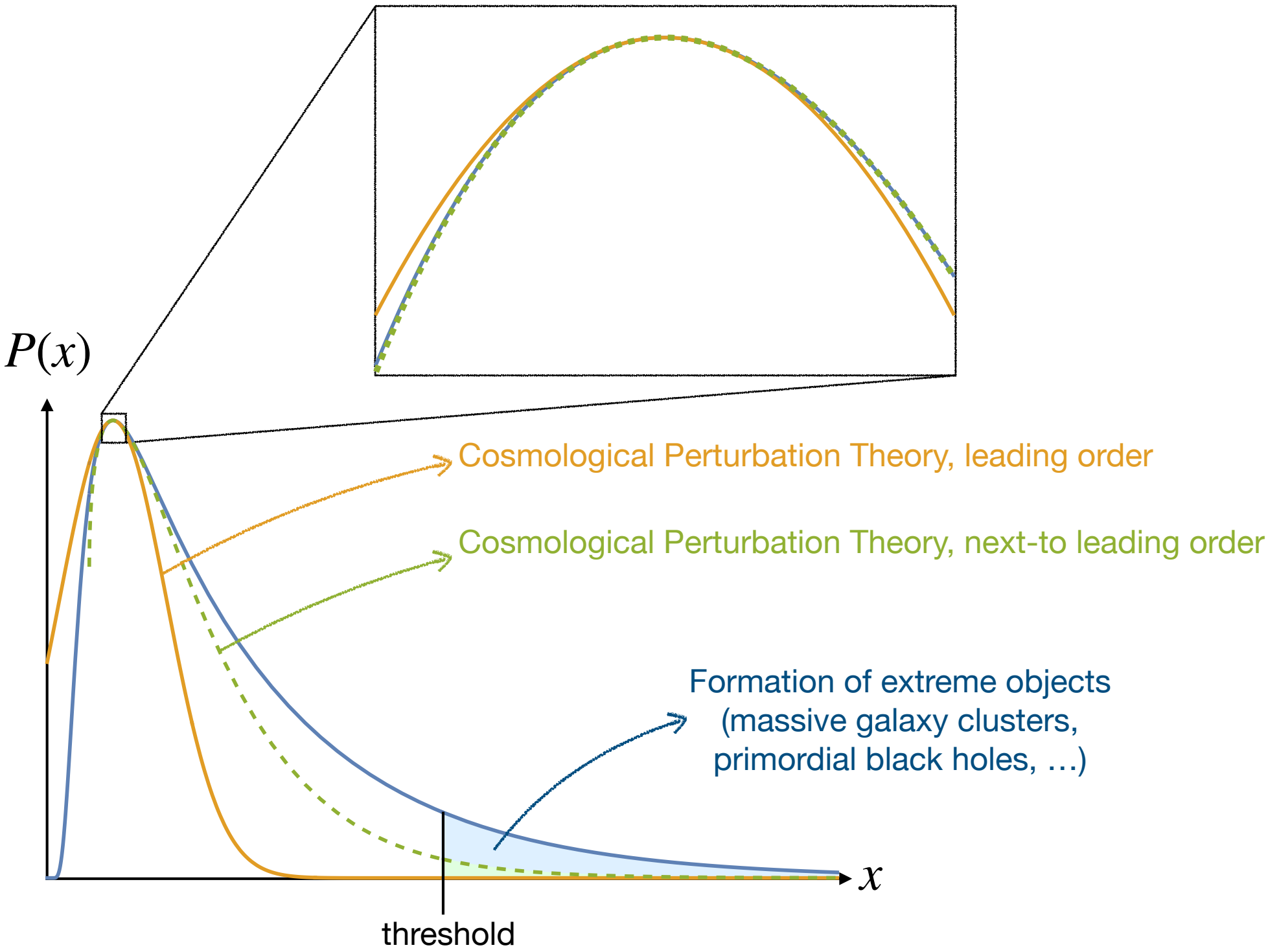
Black holes form in tails

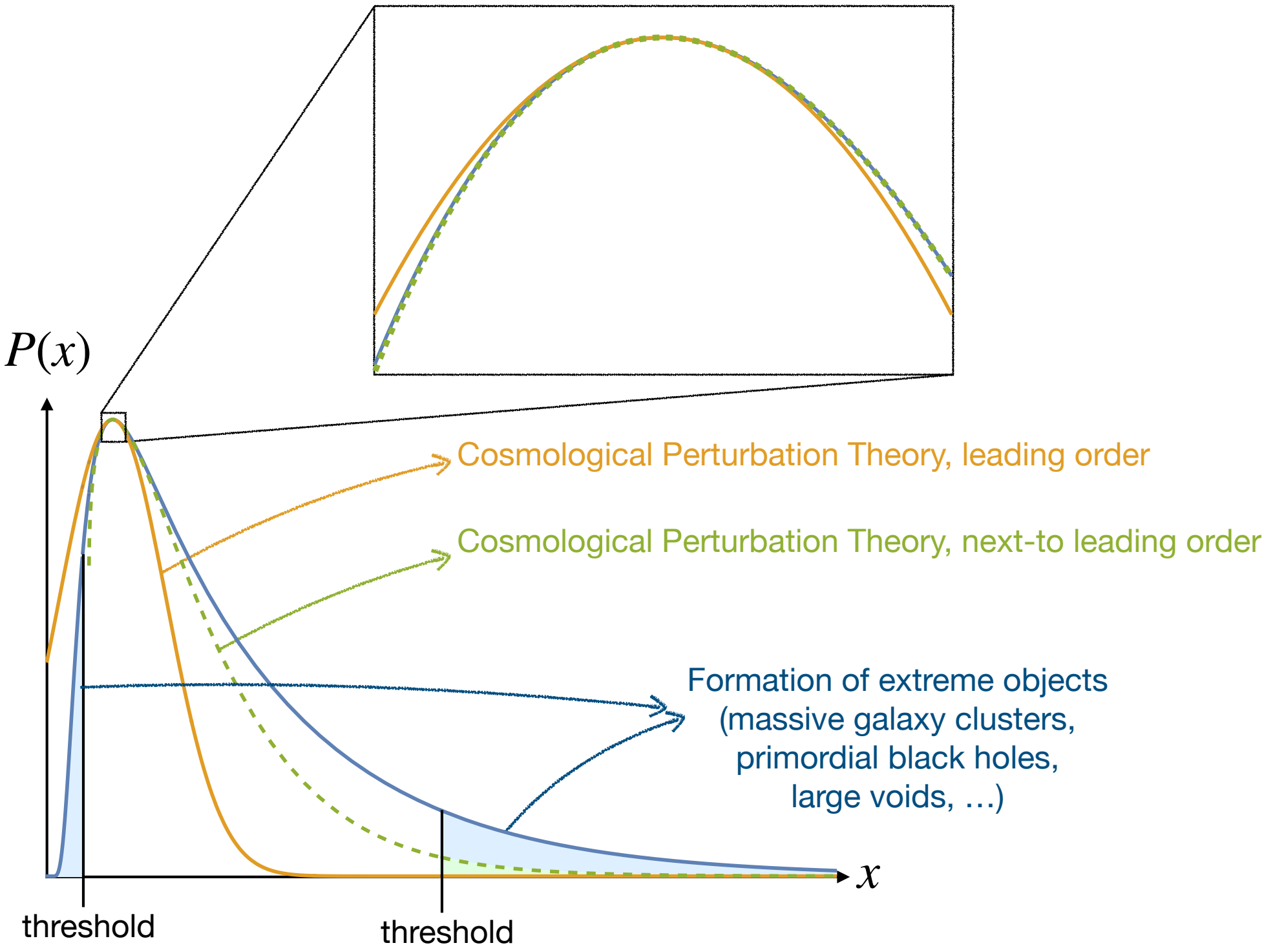



$P(x)$
Cosmological Perturbation Theory, leading order
$g_{\mu\nu}(\vec{x},t) = \bar{g}_{\mu\nu}(t) + \widehat{\delta g}_{\mu\nu}(\vec{x},t)$
$\phi(\vec{x},t) = \bar{\phi}(t) + \widehat{\delta\phi}\,(\vec{x},t)$
Homogeneous and isotropic
solution of the classical problem
Quantised fluctuation
⟶ Quantum-field-theory on curved space-time
Leading order: x is Gaussian
x


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
$S \ni \delta\phi^2, \delta\phi^3$
$x = x_{\rm G} + f_{\rm NL}\left(x_{\rm G}^2 - \langle x_{\rm G}^2 \rangle\right)$
x


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold


$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes, …)
x
threshold

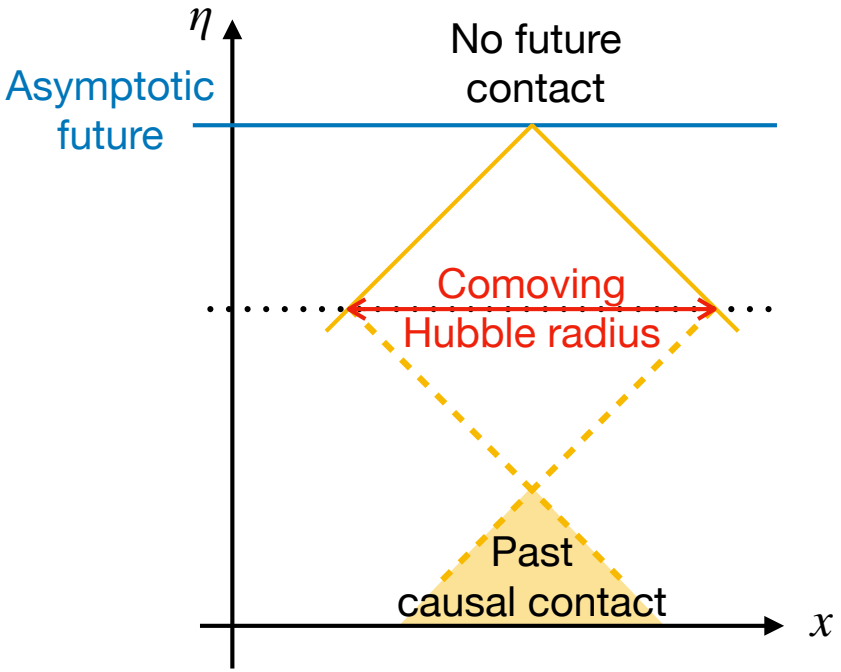

$P(x)$
Cosmological Perturbation Theory, leading order
Cosmological Perturbation Theory, next-to leading order
Formation of extreme objects
(massive galaxy clusters,
primordial black holes,
large voids, …)
x
threshold
threshold

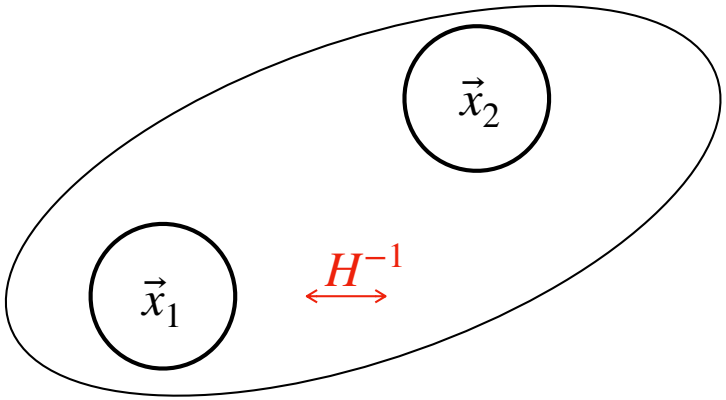
Separate universes

Salopek & Bond; Sasaki & Stewart; Wands, Malik, Lyth & Liddle (90's, 20's)

de-Sitter spacetime: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\mathrm{d}\vec{x}^2 \quad \text{with} \quad a(t) = e^{Ht}, \; -\infty < t < \infty$

$= a^2(\eta)\left(-\mathrm{d}\eta^2 + \mathrm{d}\vec{x}^2\right) \quad \text{with} \quad a(t) = -1/(H\eta), \; -\infty < \eta < 0$





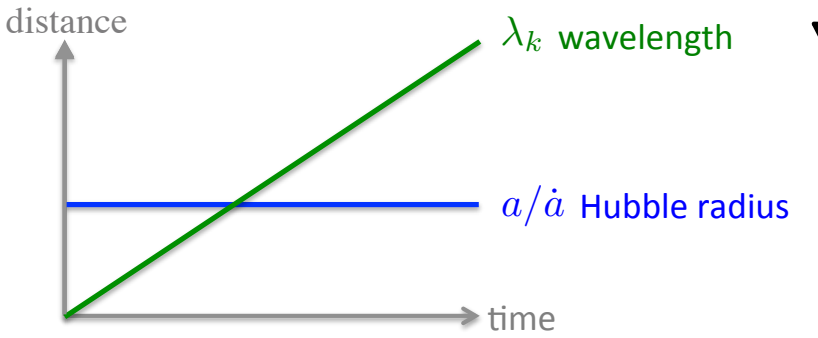
If a classical fluctuation develops at $\vec{x}_1$, this cannot affect the local geometry at $\vec{x}_2$

"separate universes": on large scales, the universe can be described by an ensemble of independent, locally homogeneous and isotropic patches.

$\mathrm{d}s^2 = -\mathrm{d}t^2 + \underbrace{a^2(t)e^{2\zeta(t,\vec{x})}}_{\tilde{a}^2(t,\vec{x})}\mathrm{d}\vec{x}^2 \longrightarrow \zeta = \delta \ln \tilde{a} \equiv \delta N$ where the curvature perturbation $\zeta \propto \delta\rho/\rho, \delta T/T, \text{etc.}$

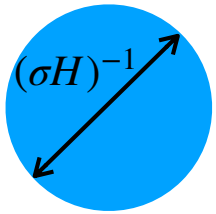
Stochastic inflation

Starobinsky, (1982) 1986



super-Hubble scales —> Stochastic Inflation

sub-Hubble scales —> Perturbation Theory



Coarse-grained field $N = \ln(a)$

$$\hat{\mathbf{\Phi}}_{\rm cg}(N,\vec{x}) = \int_{k<\sigma H a(N)} \mathrm{d}\vec{k}\left[\mathbf{\Phi}_{\vec{k}}(N)e^{-i\vec{k}\cdot\vec{x}}\hat{a}_{\vec{k}} + \mathbf{\Phi}^{\star}_{\vec{k}}(N)e^{i\vec{k}\cdot\vec{x}}\hat{a}^{\dagger}_{\vec{k}}\right]$$

Quantum fluctuations source the **background**

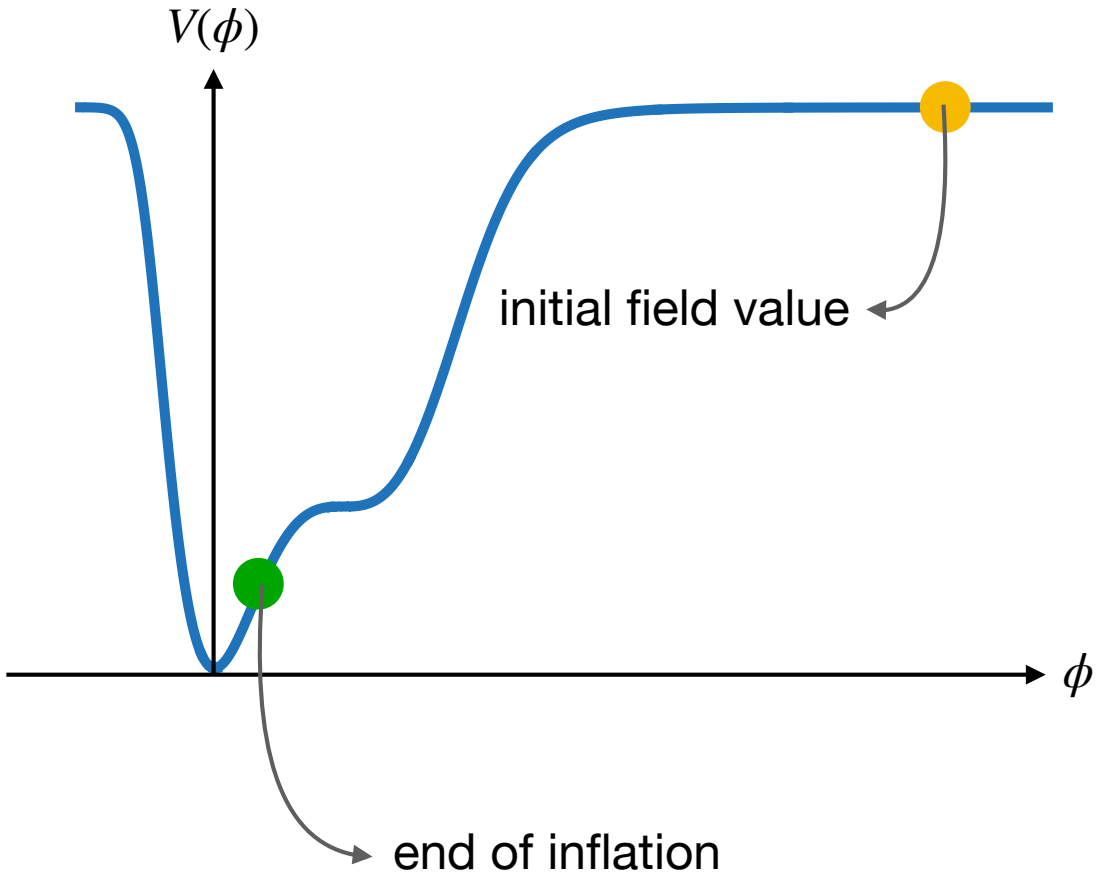
Equation of motion

$$\frac{\mathrm{d}}{\mathrm{d}N}\mathbf{\Phi}_{\rm cg} = \mathscr{F}_{\text{H\&I background}}(\mathbf{\Phi}_{\rm cg}) + \mathscr{G}(\mathbf{\Phi}_{\rm cg})\cdot\boldsymbol{\xi}(N)$$

For a fully quantum derivation, see Christie, Joo, Kaplanek, Vennin, Wands 2606.12636

Stochastic δN formalism

Vennin & Starobinsky (2015); Fujita, Kawasaki, Tada, Takesako (2015)

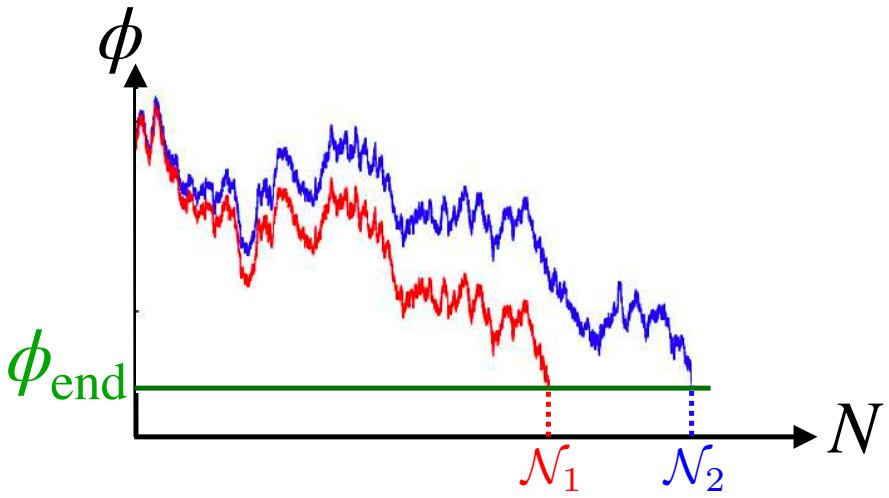


Langevin equation:

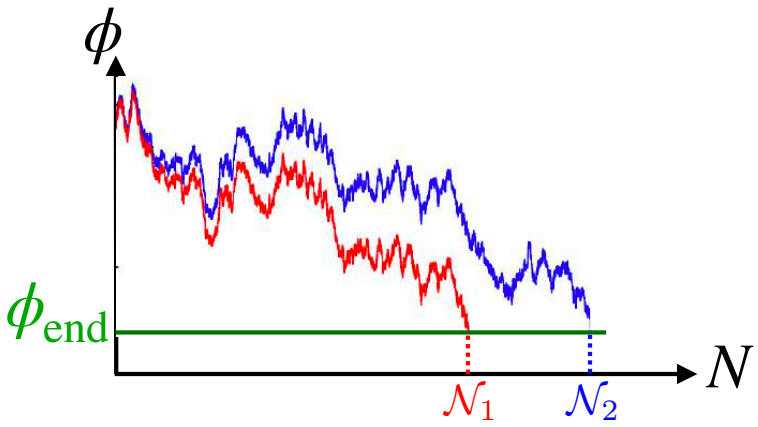
$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'(\phi)}{3H^2} + \frac{H}{2\pi}\xi(N)$$

where $\langle \xi(N)\xi(N') \rangle = \delta(N - N')$

$P(\zeta)$ is nothing but a first-passage-time distribution!



First-passage time analysis



- Stock prices and barrier options in finance
- Nucleation theory
- Neuronal firing
- Conformal changes of proteins
- Dynamics of epidemics
- etc

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\longrightarrow$$

$$\frac{\mathrm{d}}{\mathrm{d}N}P(\phi;N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right)$$

Fokker-Planck equation

$$\downarrow$$

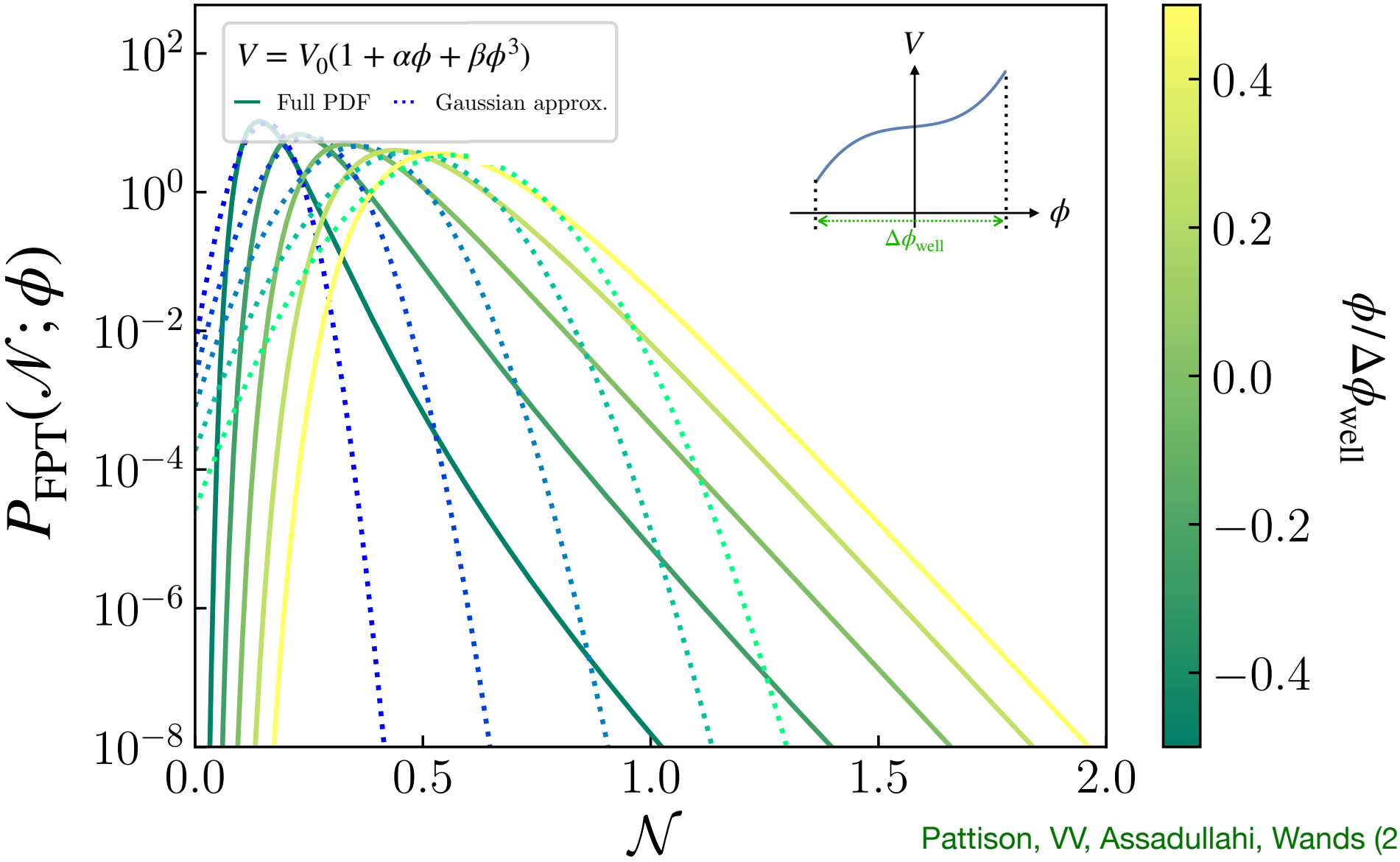
$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}P_{\rm FPT}(\mathcal{N};\phi) = \mathscr{L}^{\dagger}_{\phi}\cdot P_{\rm FPT}$$

Adjoint FP for the first passage time

$$\longleftarrow$$

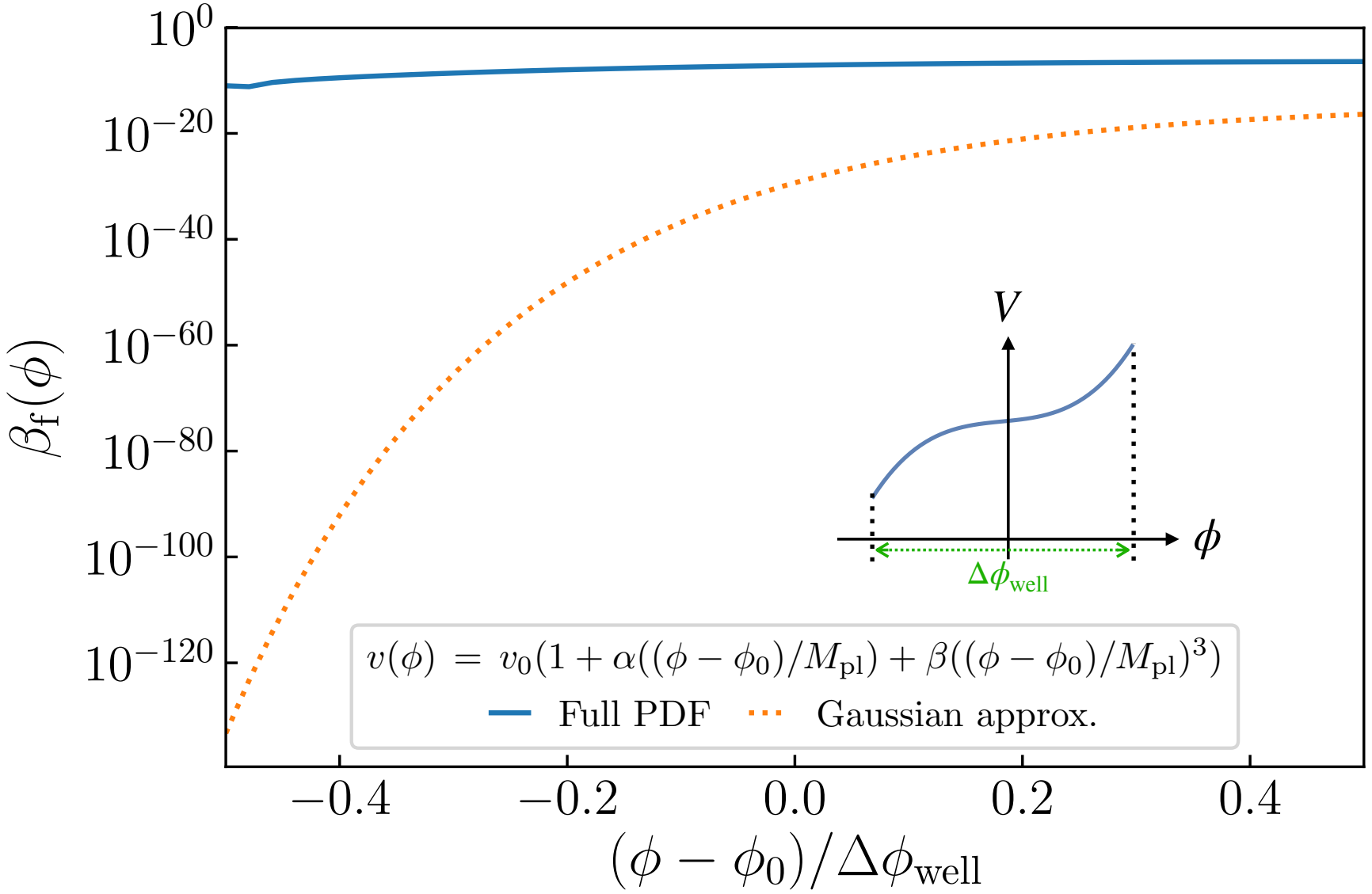
$$P_{\rm FPT}(\mathcal{N};\phi) = \sum_n a_n(\phi)e^{-\Lambda_n\mathcal{N}}$$

First-passage time analysis



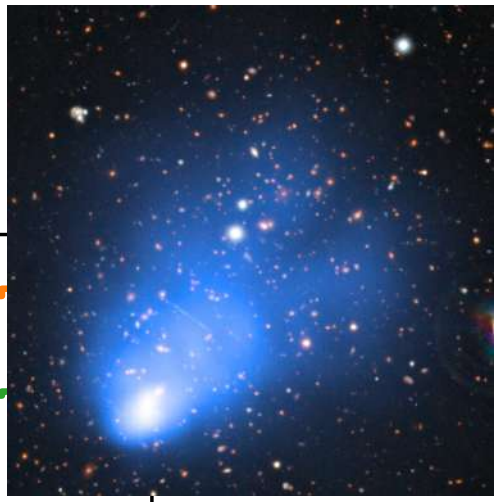
Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

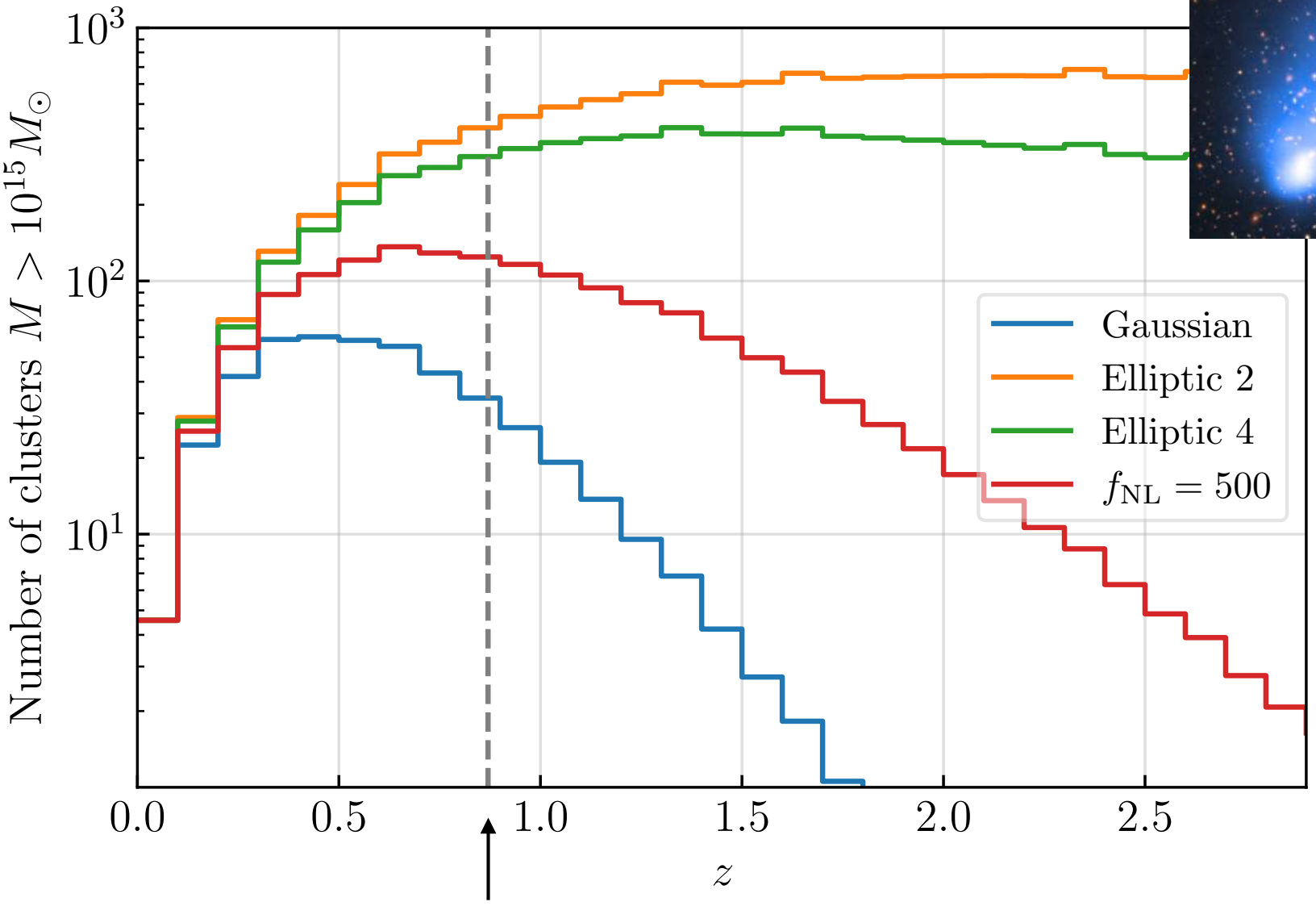
Impact on PBHs



Pattison, VV, Assadullahi, Wands (2017)
Ezquiaga, Garcia-Bellido, VV (2020)

Impact on LSS

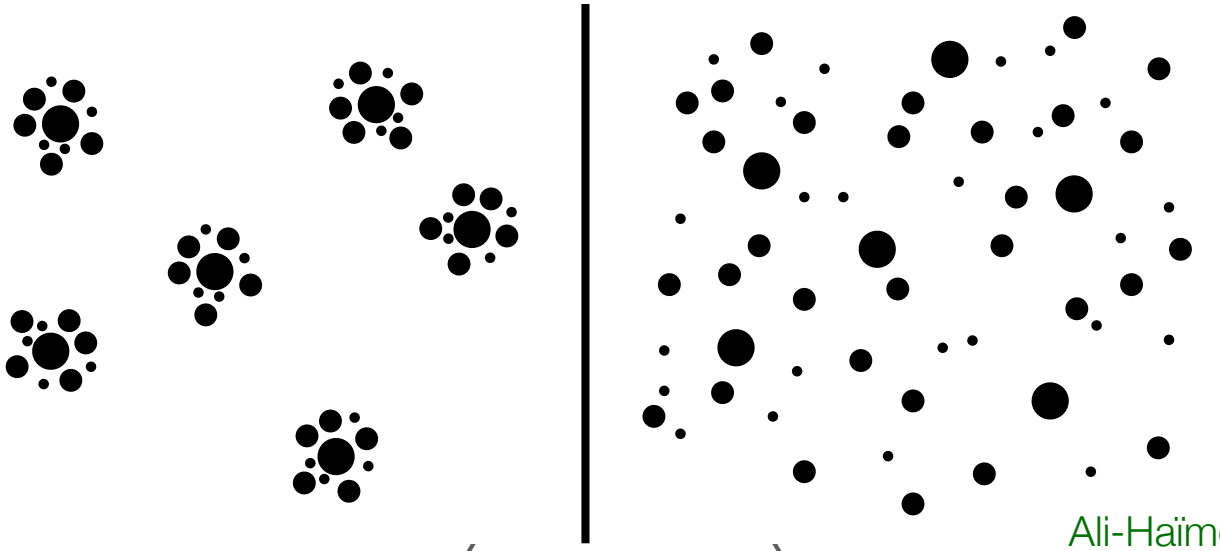




Ezquiaga, Garcia-Bellido, VV (2022)

Reconstructing spatial information

- One realisation of the Langevin equation follows the worldline ending up in one Hubble patch at the end of inflation.
- By simulating many such realisations, we reconstruct the curvature perturbation $\zeta = \delta N$ in many such patches.
- However, information regarding the spatial arrangement of these patches is lost.
- In particular, can we determine whether PBHs are uniformly distributed in space or form in clusters?

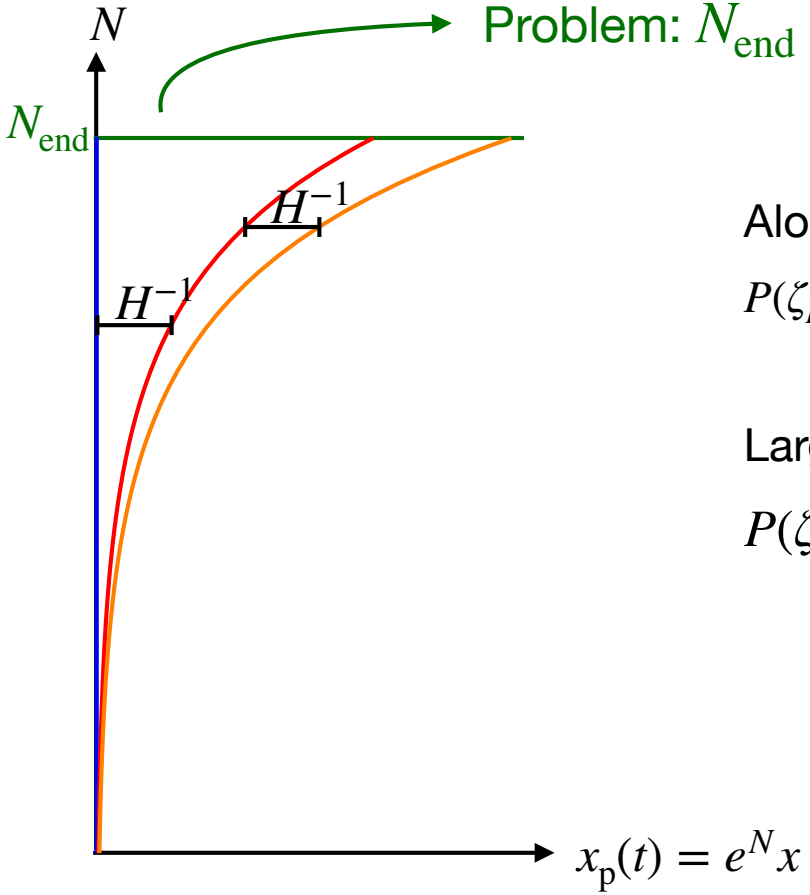


Would lead to larger merger rates

Ali-Haïmoud (2018)

Expected from large peaks of Gaussian random fields

Spatial reconstruction



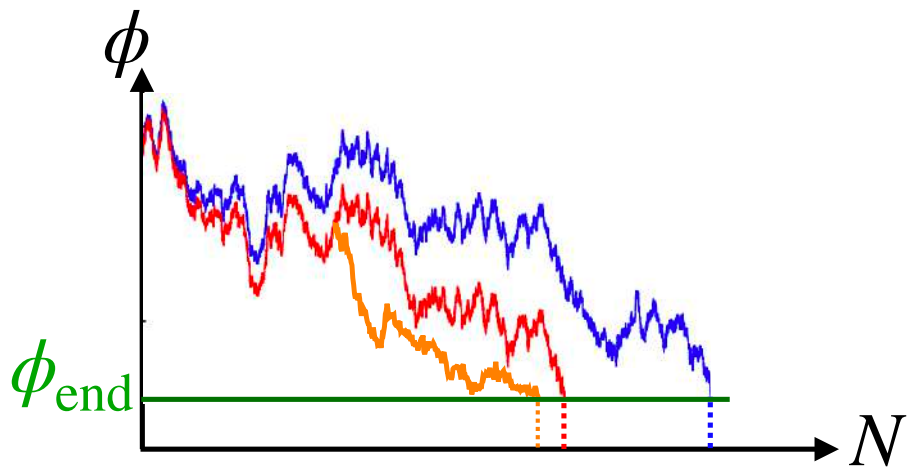
Along a reference trajectory (Tada & VV 2021), at small distances

$$P(\zeta_R) = \int_\Omega \mathrm{d}\mathbf{\Phi}_* P_{\rm bw}\left[\mathbf{\Phi}_* \,|\, N_{\rm bw}(R)\right] P_{\rm FPT,\mathbf{\Phi}_0\to\mathbf{\Phi}_*}\left[\zeta_R - \langle\mathcal{N}(\mathbf{\Phi}_*)\rangle + \langle\mathcal{N}(\mathbf{\Phi}_0)\rangle\right]$$

Large-volume limit (Animali & VV 2024), at large distances

$$P(\zeta_R) = \int_{\mathcal{S}_*=\{\mathbf{\Phi}_*,\langle V\rangle(\mathbf{\Phi}_*)=R^3\}} \mathrm{d}\mathbf{\Phi}_* P^{\rm V}_{\rm FPTL,\mathbf{\Phi}_0\to\mathcal{S}_*}(\mathcal{N}_{\mathbf{\Phi}_0\to\mathcal{S}_*} = \zeta_R - \langle\mathcal{N}_{\mathbf{\Phi}_*}\rangle_{\rm V} + \langle\mathcal{N}_{\mathbf{\Phi}_0}\rangle_{\rm V}, \mathbf{\Phi}_*)$$

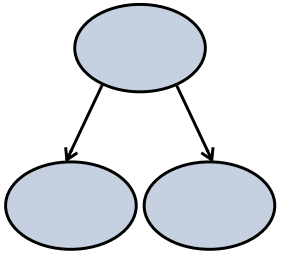
The final distance between two worldlines is encoded in the time at which they become stochastically independent.



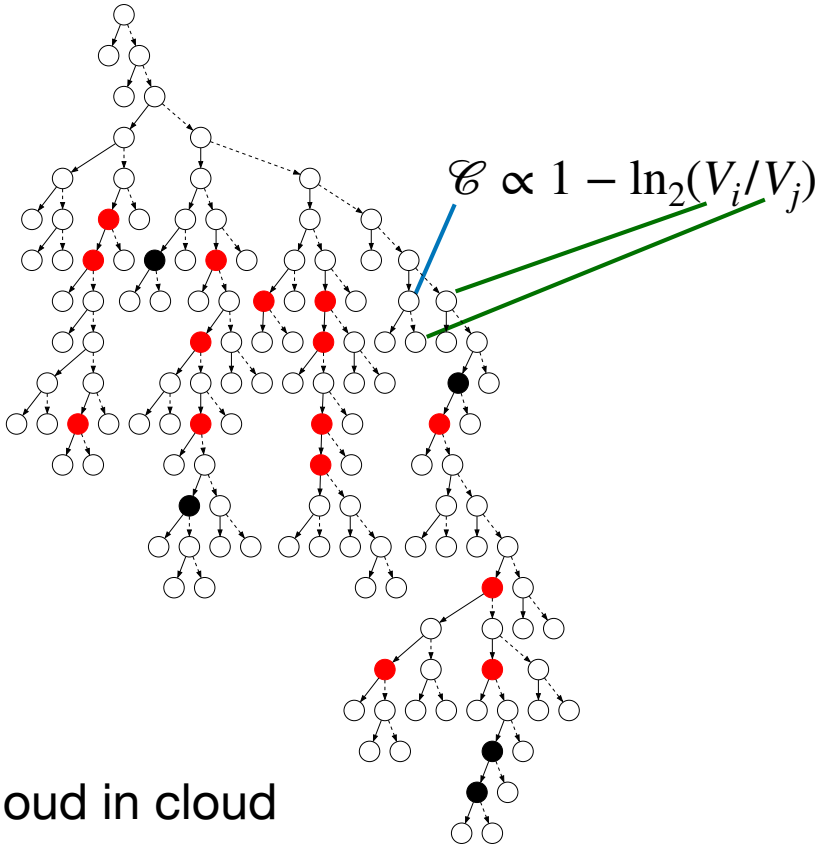
Stochastic trees

Animali, Auclair, Blachier, Vennin (2025)
Animali, Auclair, Blachier, Tomberg, Vennin (2026)

- As one Hubble patch expands during inflation, it gives rise to two Hubble patches.



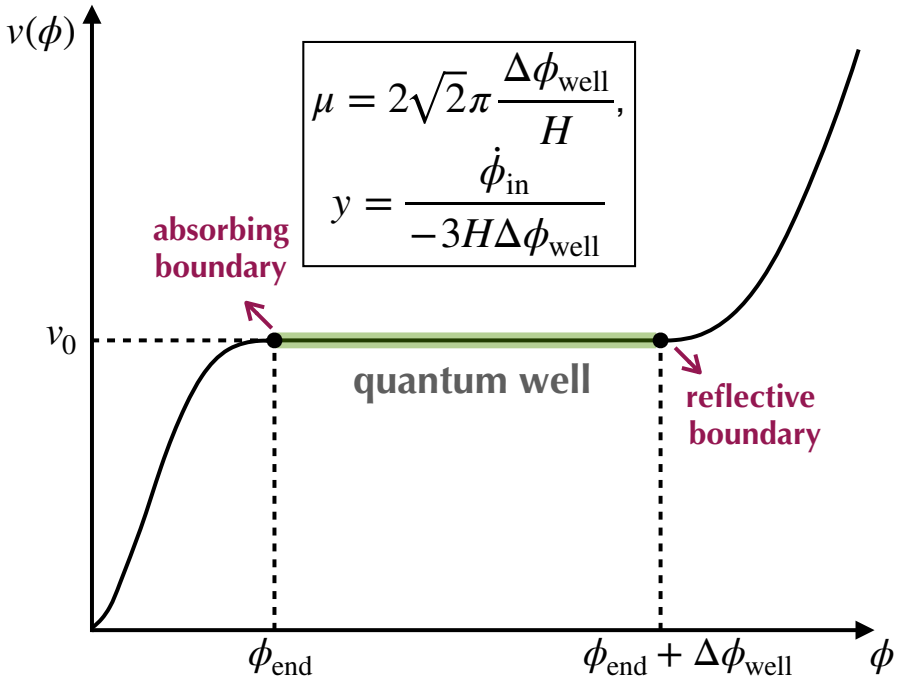
- When this structure is repeated iteratively, this gives rise to a tree.

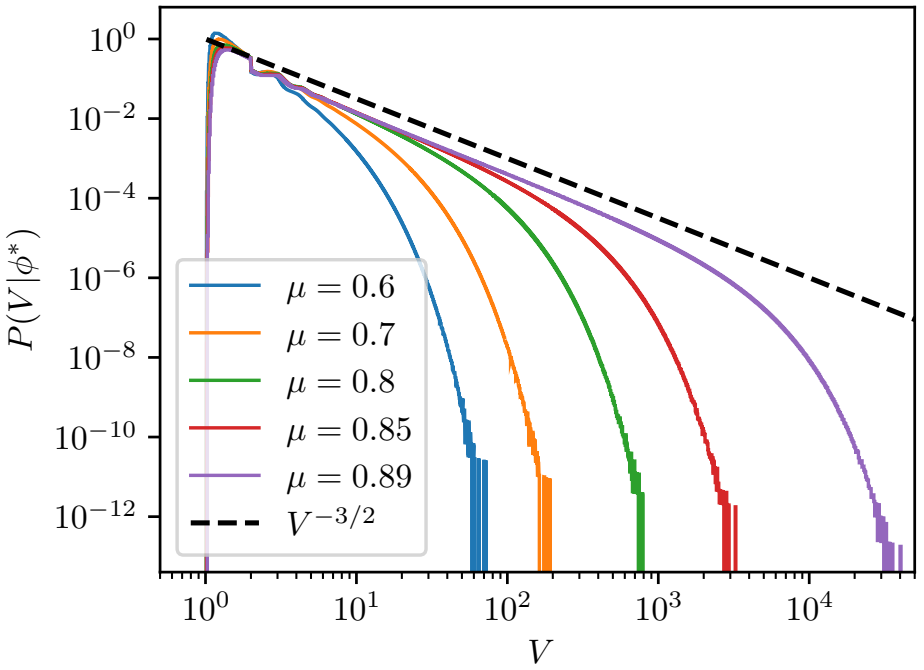


- Automatically accounts for cloud in cloud

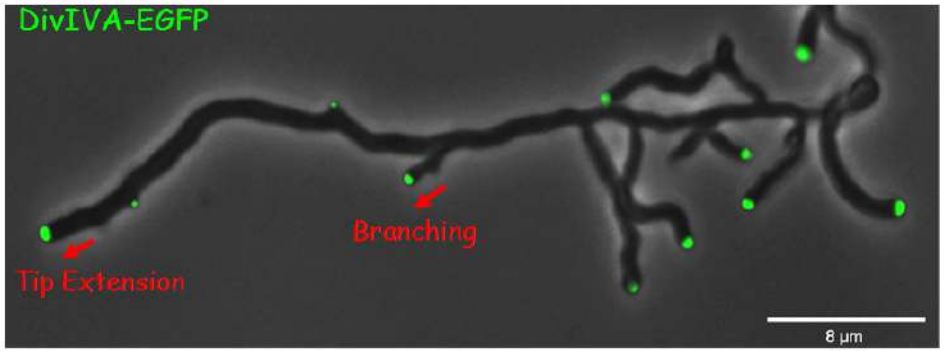
PBH mass distribution

Animali, Auclair, Blachier, Vennin (2025)





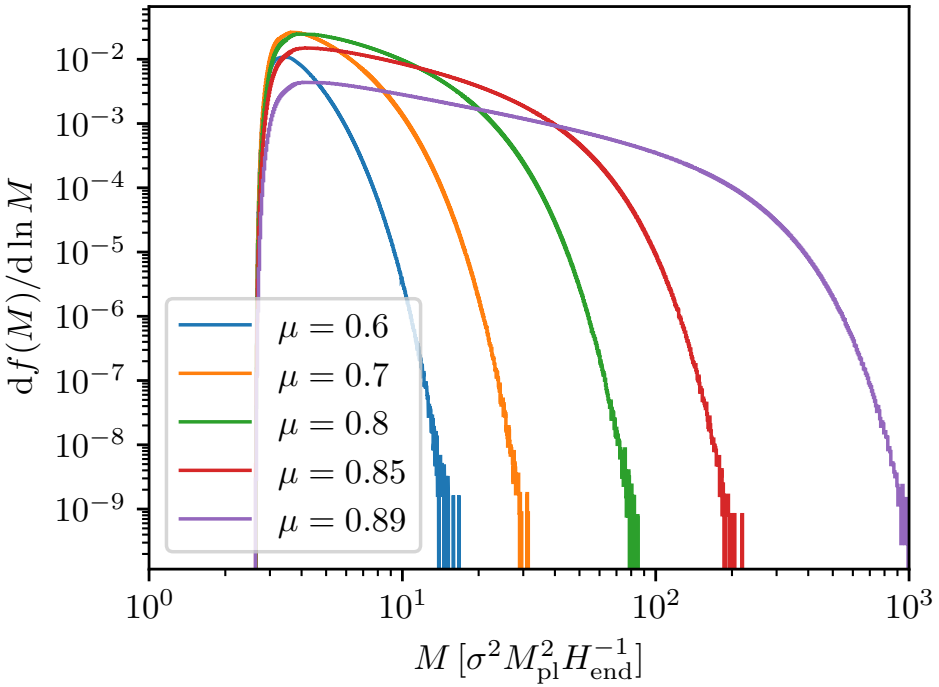
- μ controls the width of the field region dominated by quantum diffusion
- $P(V) \propto V^{-3/2}$ is expected from bacteria models (Galton-Watson processes)
- Bacteria non-extinction ↔ eternal inflation!

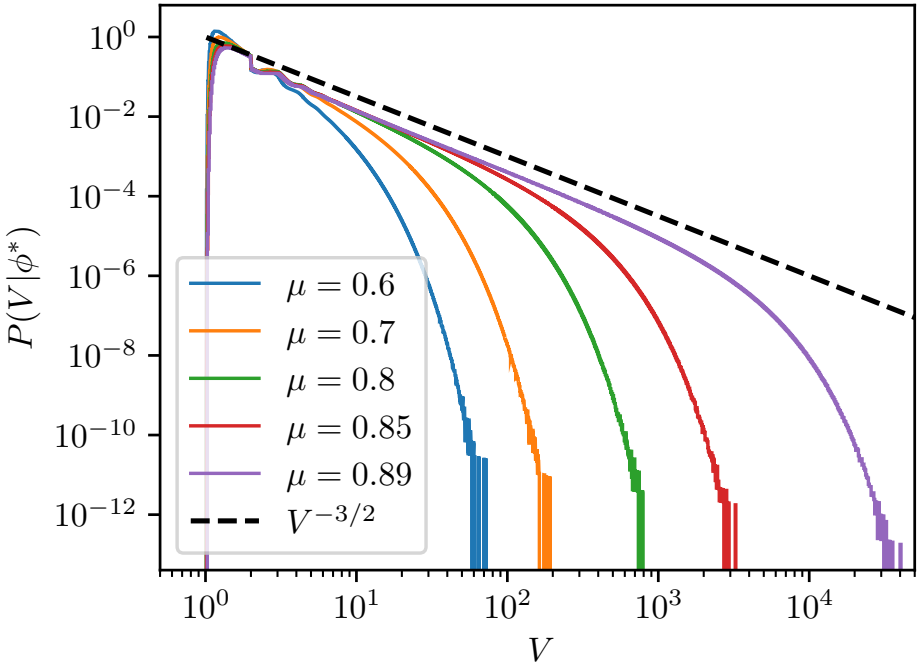


Streptomyces bacteria, Hempel *et al* 2005

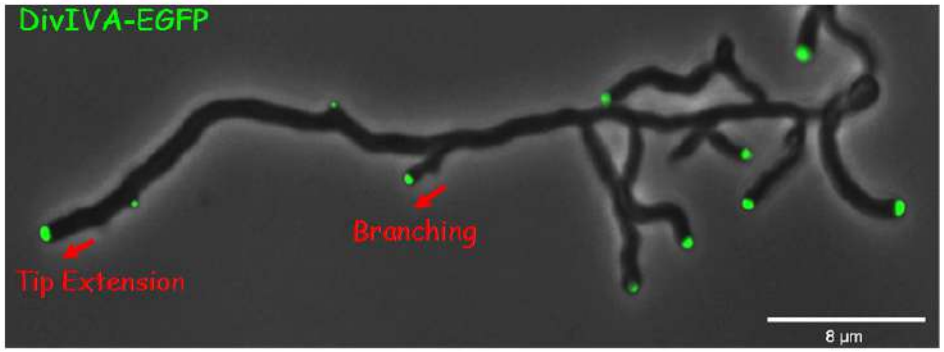
PBH mass distribution

Animali, Auclair, Blachier, Vennin (2025)





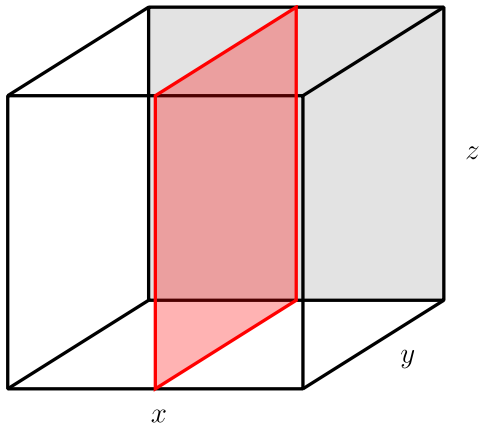
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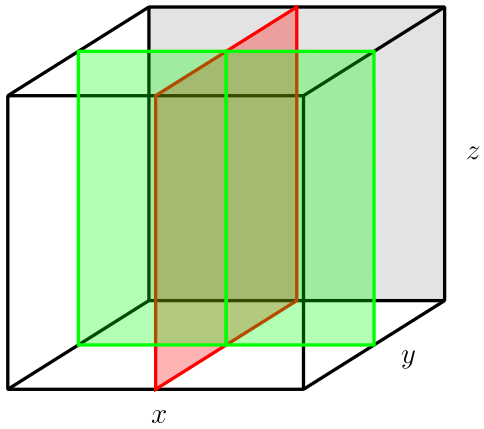


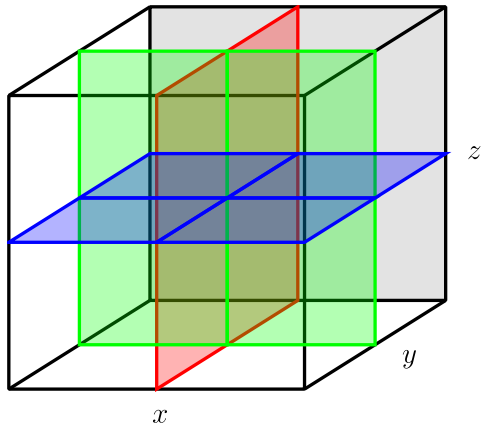
Streptomyces bacteria, Hempel *et al* 2005

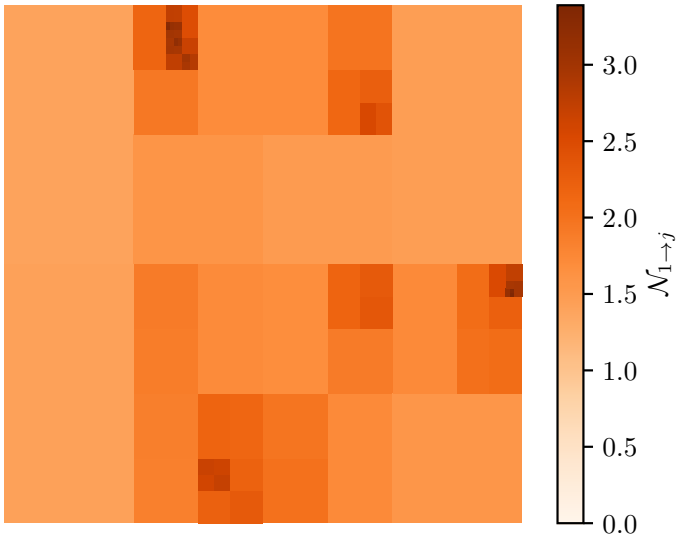
Creating maps

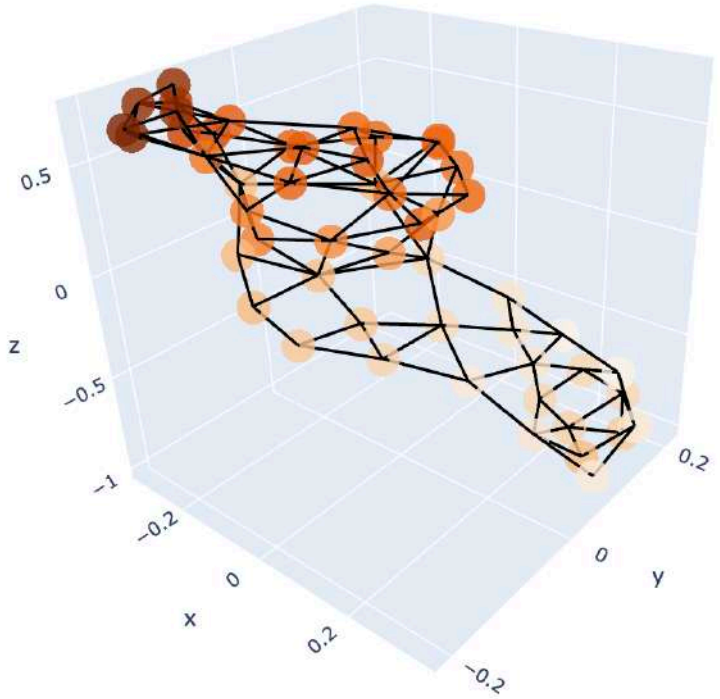
Animali, Auclair, Blachier, Tomberg, Vennin (in progress)











Conclusions

- The backreaction of quantum fluctuations on an inflating background can be incorporated within the formalism of stochastic inflation

- This is necessary to describe regimes leading to large fluctuations, such as those yielding primordial black holes

- Quantum diffusion leads to exponential tails: non-perturbative breakdown of Gaussian statistics

- Most cosmological observables can be reconstructed from first-passage time analysis in multi-type branching processes (power spectrum, mass distributions, n-point functions, etc.)

- Ongoing work:
  - PBH clustering
  - type I versus type II fluctuations
  - eternal regimes