General approach to high energy collisions in strong gravitational field

From first concrete calculations to bird's-eye view

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New wave of interest to high energy collisions
Nature supplies us with accelerators free of charge

M. Banados, J. Silk and S. M. West, Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy, Phys. Rev. Lett. ***103****, 111102*

1) High (unbounded) energy in the centre of mass frame E_c.m.

Black holes, naked singularities, wormholes, flat space-time

BHs: rotating or static electrically charged Collisions outside and inside BH

2) Possibility to get high (unbounded) energies E at infinity (debris after collision) – super-Penrose process

BSW effect versus Penrose process: what can be seen at infinity?

Additional forces

Classification of potential sources of high E_c.m.
For BHs: classification of possible scenarios

Head-on collision near horizon

1975 - 1977 T. Piran, J. Katz and J. Shanam

Two particles move in opposite directions near BH

Almost infinite relative blue shift

Energy in CM frame almost diverges

When particle with finite energy moves away from horizon, this is typical of white, not black hole

BSW 2009 Two particles moving toward horizon of black hole
One of particles is fine-tuned

In black hole physics

Collisions outside horizons. Classification of particles that collide with indefinitely large E_c.m.
General explanation of BSW effect: geometric, kinematic

Principle of kinematic censorship

Rotating BHs versus static charged BHs

Collisions inside horizon

Schwarzschild black hole and high energy collisions

Key quantity: energy in centre of mass frame

1 particle $$m^2 = \left| P_\mu P^\mu \right|$$

2 particles colliding in some point

$$E^2{}_{cm} = \left| P_\mu P^\mu \right|$$

Total momentum $$P_\mu = p^{(1)}{}_\mu + p^{(2)}{}_\mu$$

$$P_a = (E_{c.m.}, 0, 0, 0) \qquad u^\mu u_\mu = -1$$

Individual E finite, energy in CM frame unbounded

Two different kinds of energy

Killing energy $E = -p_\mu \xi^\mu$ ξ^μ Killing vector

E conserved, integral of motion since metric is static or stationary

Energy in the CM frame $E_{c.m.}$

not conserved. Moreover, it is defined in one point only.
point of collision

Kinematic censorship

In any collision (and in any event in general) the energy cannot be infinite. It can be as large as one likes but, anyway, it remains finite.

The simplest case. Let collision that formally give infinite energy:

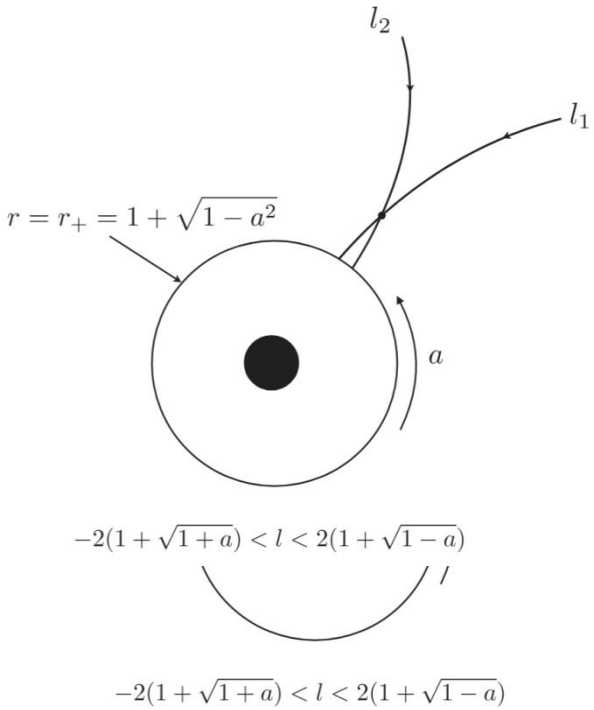
$$\lim_{r \to r_+} E_{c.m.} = \infty$$

Possible ways of resolution:

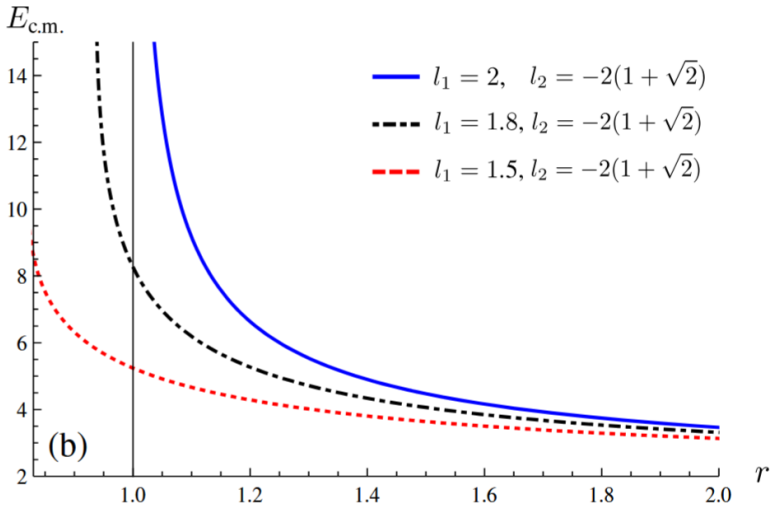
1) The proper time diverges (such collision never occurs)
2) Event of collision itself is impossible

Case 1) is realized in background of extremal BH
Case 2) is realized under horizon

Two particles falling towards horizon. Kerr metric



For some special value of angular momentum, energy in the center-of-mass frame diverges near ***extremal*** horizon



M. Banados, J. Silk and S. M. West, Kerr Black Holes as Particle Accelerators to Arbitrarily High Energy, Phys. Rev. Lett. ***103****, 111102*

Acceleration of particles as universal property of rotating black holes

Role of horizon

Universality of black hole physics

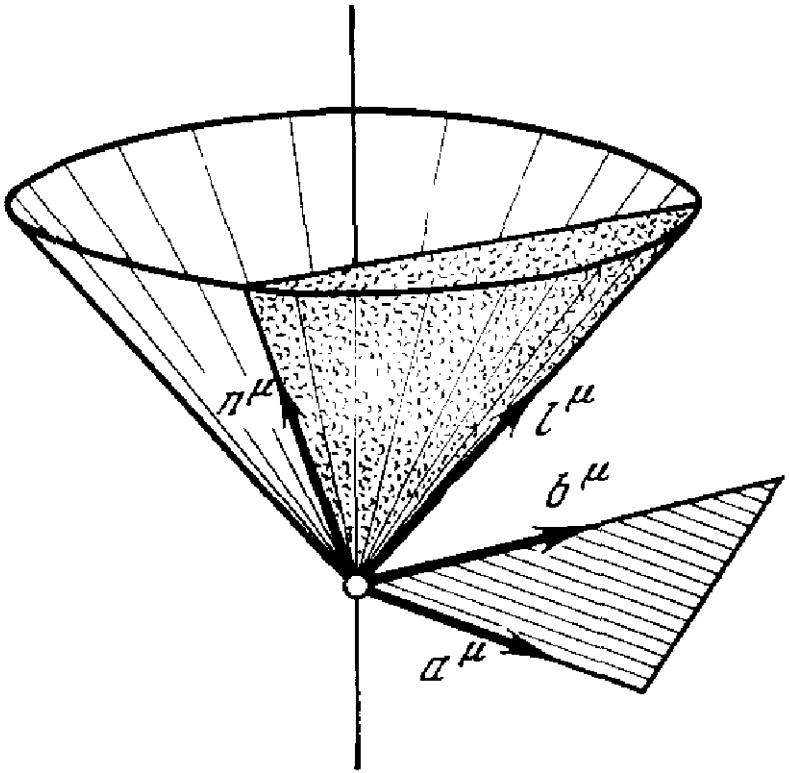
Unified approach to different types of horizon

Geometric explanation

$g_{\alpha\beta} = -l_\alpha N_\beta - l_\beta N_\alpha + \sigma_{\alpha\beta}$ $\sigma_{\alpha\beta} = a_\alpha a_\beta + b_\alpha b_\beta$ lightlike vectors l^μ and N^μ

spacelike vectors a^μ, b^μ orthogonal to them

,



Four-velocity

$$u_i^\mu = \frac{l^\mu}{2\alpha_i} + \beta_i N^\mu + s_i^\mu,\ s_i^\mu = A_i a^\mu + B_i b^\mu$$

$$-(u_1 u_2) = \frac{1}{2}\left(\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1}\right) - (s_1 s_2).$$

$\alpha = 0$

Case 1
always

For head-on collision means that particle cannot cross horizon

$$E_{c.m.}^2 = m_1^2 + m_2^2 - 2m_1 m_2 (u_1 u_2)$$

Case 2
Special condition

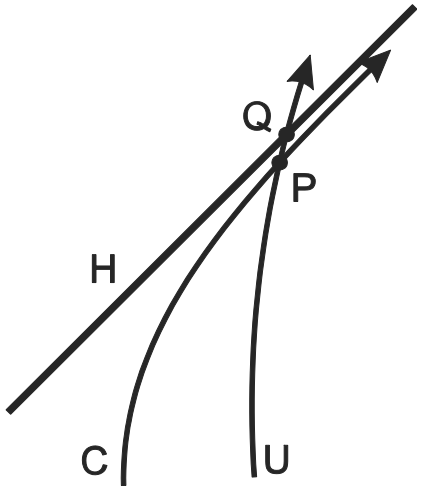
$$E_{c.m.}^2 = m_1^2 + m_2^2 + m_1 m_2 \left[\frac{\beta_1}{\alpha_2} + \frac{\beta_2}{\alpha_1} - 2(s_1 s_2)\right].$$

Case 2

$\alpha_1 \to 0.$

Now, special condition

$E^2_{c.m.} \to \infty$



Kruskal-like coordinates

$$ds^2 = -CdUdV + \gamma_{ab}dx^a dx^b$$

$$Cu^X u^Y = 1$$

$$u^X \sim \alpha \to 0. \qquad \tau \sim -\ln X \to \infty$$

Proper time grows unbound (T. Jacobson, Grib and Pavlov, O. Z.)

FOUR-VELOCITIES AND CLASSIFICATION OF TRAJECTORIES

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_{\theta\theta} d\theta^2$$

$$u^\mu = \left(\frac{\mathcal{X}}{N^2}, \frac{\mathcal{L}}{g_\varphi} + \frac{\omega \mathcal{X}}{N^2}, \sigma \frac{\sqrt{A}}{N} P, 0 \right) \qquad \sigma = \pm 1$$

$$P = \sqrt{\mathcal{X}^2 - N^2 \left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}} \right)}, \qquad \mathcal{X} = \varepsilon - \omega \mathcal{L},$$

Energy of collision

$$E^2_{c.m.} = m^2_1 + m^2_2 + 2m_1 m_2 \gamma$$

$$\gamma = -u_{1\mu} u_2{}^{\mu}$$

$$\gamma = \frac{\mathcal{X}_1 \mathcal{X}_2 - P_1 P_2}{N^2} - \frac{\mathcal{L}_1 \mathcal{L}_2}{g_{\varphi\varphi}}.$$

Kinematic explanation

$$E^2_{c.m.} = -(p_1^\mu + p_2^\mu)(p_{1\mu} + p_{2\mu}) = m_1^2 + m_2^2 - 2m_1 m_2 u_1^\mu u_{2\mu}.$$

$$\gamma = -u_1^\mu u_{2\mu} = \frac{1}{\sqrt{1-w^2}}$$

BSW effect occurs if $w \to 1$ w is relative velocity

$$w^2 = 1 - \frac{(1-v_1^2)(1-v_2^2)}{[1 - v_1 v_2(\vec{n}_1\vec{n}_2)]^2} \qquad \vec{v}_1 = v_1\vec{n}_1$$

The most interesting case: $v_1 < 1, \quad v_2 \to 1$

Collision of rapid particle with target

Relative velocity close to c

Attached to observer

$$h_{(0)\mu} = -N(1,0,0,0),$$

$$h_{(1)\mu} = (0,1,0,0),$$

$$h_{(2)\mu} = \sqrt{g_{zz}}\,(0,0,0,1),$$

$$h_{(3)\mu} = \sqrt{g_{\phi\phi}}\,(-\omega,0,0,1)$$

$$-u_\mu h^\mu_{(0)} = \frac{E-\omega L}{N},$$

$$u_\mu h^\mu_{(3)} = \frac{L}{\sqrt{g_{\phi\phi}}}.$$

If

$$V_\mu = h_{\mu(0)}$$

then

$$V_\mu \xi^{(3)\mu} = 0$$

ZAMO

$$X \equiv E - \omega L = \frac{mN}{\sqrt{1-v^2}},$$

Horizon limit $N \to 0$

1) Usual particle, $E \neq \omega_+ L$ $\quad v \to 1$

2) Critical particle $E = \omega_+ L$ $\quad v \to v_0 < 1$

More on kinematics

$$u^r = \pm \frac{\sqrt{A}}{mN} P \qquad P = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g_\phi})}$$

$$v^{(1)} = \sqrt{1 - \frac{N^2}{X^2}(m^2 + \frac{L^2}{g_\phi})} \qquad v^{(3)} = \frac{LN}{X\sqrt{g_\phi}}$$

$$v^2 = 1 - \frac{N^2 m^2}{X^2} \qquad u^r = \pm \frac{\sqrt{A}}{\sqrt{1-v^2}} v^{(1)}$$

Horizon limit

Usual: $X \to X_H \neq 0$ $\quad v \to 1$ $\quad u^r \neq 0$

Critical $X = O(N) \to 0$ $\quad v \to v_c < 1$ $\quad u^r \to 0$

Collision between massive and massless particles (electron and photon)

$$E - \omega L = N E_{kin}$$

$$E_{kin} = \frac{m}{\sqrt{1 - V^2}}$$

E_{kin} Measured by local observer

$$\nu_0 - \omega L = \nu N$$

ν Measured by local observer

E conserved

Usual: $V \to 1 \quad E_{kin} \to \infty$

Critical: $V \to V_+ < 1$

E_{kin} finite

ν_0 conserved

Usual: $\nu \to \infty$

Critical:

ν finite

Play of two factors: Doppler effect (DE) and gravitational blue shift (GB)

Case 1: electron is critical, photon is usual	Finite redshift - DE . GB infinite Effect exists
Case 2: electron is usual, photon is critical	DE infinite, GB finite Effect exists
Case 3: both particles are critical	DE and GB finite, no effect
Case 4: both particles are usual	Infinite DE compensated by infinite GB Outcome is finite, no effect

Extremal versus nonextremal

Problems with attaining extremality, a=0,998 (Thorne)

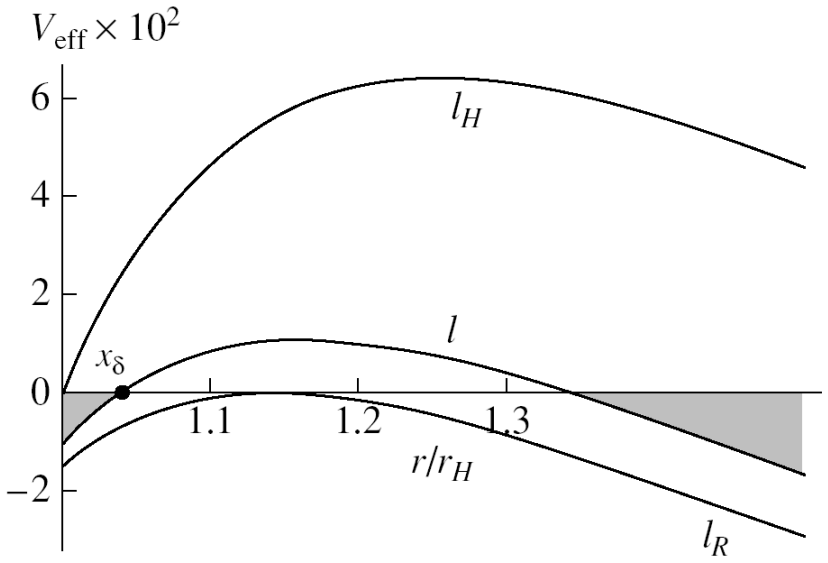
Jacobson et al, Berti at al: difficulties in realization

Grib and Pavlov: nonextremal Kerr

Extremal case: collision near horizon

$$P = \sqrt{\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)}, \qquad X = O(N^2)$$

$$E_{c.m.} \approx \frac{m}{\sqrt{\delta}}\sqrt{\frac{2(L_H - L_2)}{1-\sqrt{1-a^2}}} \qquad L_1 = L_{(H)} - \delta$$



The effective potential for $A = 0.95$ and $l_R \approx 2.45$, $l = 2.5$, $l_H \approx 2.76$. Allowed zones for $l = 2.5$ are shown by the gray color.

Critical particle cannot penetrate forbidden region near horizon.
However, if there is a near-critical particle there from the very beginning, there is BSW effect

generalization?

BSW effect holds for any axially symmetric spacetimes

This may happen for charged particles and non-rotating spacetimes

Analogous phenomenon is observed when a black hole is non-extremal (but particle has to be nearly-critical).

What happens with the presence of a generic force? For other types of horizons?

Types of horizon: nonextremal, extremal (double), ultraextremal (triple, etc.)

Near-horizon expansion of metric

$$N^2 = \kappa_p v^p + o(v^p), \qquad A = A_q v^q + o(v^q),$$

$$\omega = \omega_H + \hat{\omega}_k v^k + \cdots + \hat{\omega}_{l-1} v^{l-1} + \omega_l(\theta) v^l + o(v^l),$$

$$g_a = g_{aH} + g_{a1} v + o(v).$$

$$u^r = (u^r)_c v^c + o(v^c). \qquad \nu = r - r_h$$

$$\mathcal{X} \approx \mathcal{X}_s \nu^s,$$

p, q. k metrics

s

What we aim to find

We introduce such a classification of types of particles $\mathcal{X} \approx \mathcal{X}_s \nu^s$,

	Type	s
1	Usual	0
2	Subcritical	$\frac{p-q}{2} + c,\ 0 < s < \frac{p}{2}$
3	Critical	$\frac{p}{2}$
4	Ultracritical	$\frac{p}{2}$

Usual - X is constant on the horizon

Subcritical - X tends to zero, but with the rate, smaller then N

$$P = \sqrt{\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)},$$

Critical - X tends to zero with the same rate as N

Ultracritical - X tends to zero with the same rate as N, but several first terms in the expansion of P cancel

For subcritical particles

$$\mathcal{X} \approx \frac{N}{\sqrt{A}} |u^r| \to s = \frac{p-q}{2} + c$$

For critical and ultracritical particles $s = \frac{p}{2}$

for ultracritical particles, there exists another restriction

$$(u^r)^2 = \frac{A}{N^2}\left(\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)\right) \sim v^{2c}, \qquad \text{where } c > \tfrac{q}{2}.$$

$$\mathcal{X}^2 = N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right) \text{plus } v^{2c+p-q} \text{ terms}$$

$$ds^2 = -N^2 dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + \frac{dr^2}{A} + g_{\theta\theta} d\theta^2$$

$$u^r = \pm \frac{\sqrt{A}}{N} P \qquad P = \sqrt{X^2 - N^2 (m^2 + \frac{L^2}{g_\phi})}$$

Near-horizon limit: $N \to 0$, $A \to 0$

$$u^r \approx \frac{\sqrt{A}}{N}$$

usual

If $|u^r|$ changes slower than $\sqrt{A}$

subcritical

but faster than $\frac{\sqrt{A}}{N}$

if $|u^r| \sim \sqrt{A}$

critical

if faster than $\sqrt{A}$

ultracritical

$$u_O^{\varphi} = (u_O^{\varphi})_H + (u_O^{\varphi})_b v^b + o(v^b).$$

$$\mathcal{L} \approx \sqrt{g_{\varphi\varphi}}(u_O^{\varphi})_H + \sqrt{g_{\varphi\varphi}}(u_O^{\varphi})_b v^b.$$

$$u^t \sim v^{-\beta},$$

$$\tau = O(v^{-\alpha}) \qquad \alpha = c - 1$$

What we aim to find

Central aim of several our works was to find:
For what types of particles and for what horizons does the high-energy collision effect survive while the forces, acting on such particles, remain finite?

- 1-st case: equatorial motion

In this case, the 4 velocity is given by

$$u^{\mu}=\left(\frac{\mathcal{X}}{N^2},\frac{\mathcal{L}}{g_{\varphi}}+\frac{\omega\mathcal{X}}{N^2},\sigma\frac{\sqrt{A}}{N}P,0\right).$$

Now, *X* is not a constant and is defined by the external force

$$\mathcal{X}=\varepsilon-\omega\mathcal{L},\qquad P=\sqrt{\mathcal{X}^2-N^2\left(1+\frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)},$$

As it is not a constant, we assume that near the horizon X behaves as

$$\mathcal{X}\approx\mathcal{X}_s\nu^s,$$

TABLE I. Characteristics of the near-horizon behavior of u^r, u^t, $\mathcal{X}$, and the proper time τ. Here, the proper time changes as $\tau \sim v^{-\alpha}$. The value $\alpha = 0$ means that the proper time logarithmically diverges $\tau \sim |\ln v|$.

	Type	c	β	s	$\alpha = c - 1$
1	Usual	$\frac{q-p}{2}$	p	0	$\frac{q-p-2}{2}$
2	Subcritical	$\frac{q-p}{2} < c < \frac{q}{2}$	$\frac{p+q}{2} - c$	$\frac{p-q}{2} + c$, $0 < s < \frac{p}{2}$	$\frac{q-p-2}{2} < \alpha < \frac{q-2}{2}$
3	Critical	$\frac{q}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\frac{q-2}{2}$
4	Ultracritical	$c > \frac{q}{2}$	$\frac{p}{2}$	$\frac{p}{2}$	$\alpha > \frac{q-2}{2}$

Acceleration

Tetrad components in ZAMO frame

$$a_o^{(t)} = N a^t, \quad a_o^{(r)} = \frac{a^r}{\sqrt{A}},$$

$$a_o^{(\varphi)} = \sqrt{g_{\varphi\varphi}}(a^\varphi - \omega a^t), \quad a_o^{(\theta)} = \sqrt{g_{\theta\theta}} a^\theta.$$

$$a_o^{(t)} = (a_o^{(t)})_{n_0} v^{n_0} + o(v^{n_0}),$$

$$a_o^{(r)} = (a_o^{(r)})_{n_1} v^{n_1} + o(v^{n_1}),$$

$$a_o^{(\varphi)} = (a_o^{(\varphi)})_{n_2} v^{n_2} + o(v^{n_2}),$$

From metrics and acceleration (force) to possibility of BSW effect

Given n_0, n_1, n_2 p, q, k

Properties of near-horizon collisions? γ

$$N^2 = \kappa_p v^p + o(v^p), \qquad A = A_q v^q + o(v^q),$$

$$\omega = \omega_H + \hat{\omega}_k v^k + \cdots + \hat{\omega}_{l-1} v^{l-1} + \omega_l(\theta) v^l + o(v^l),$$

$$g_a = g_{aH} + g_{a1} v + o(v).$$

BSW effect and types of particles

By analysing the gamma-factor for various types of particles, we obtained such table of possible scenarios

$$\gamma = \frac{\mathcal{X}_1\mathcal{X}_2 - P_1P_2}{N^2} - \frac{\mathcal{L}_1\mathcal{L}_2}{g_{\varphi\varphi}}.$$

TABLE II. The possibility of BSW phenomenon for different types of particles. D means that the gamma factor diverges, and R means that it is regular.

	First particle	Second particle	d	γ
1	Usual	Usual	0	R
2	Usual	Subcritical	s_2	D
3	Subcritical	Subcritical	$\lvert s_1 - s_2\rvert$	D if $s_1 \neq s_2$ R if $s_1 = s_2$
4	Usual or subcritical	Critical or ultracritical	$p/2 - s_1$	D
5	Critical or ultracritical	Critical or ultracritical	0	R

Thus, generally, BSW is possible if one particle is non-usual (while second is usual or subcritical)

What happens with the presence of a generic force? For other types of horizons?

TABLE III. Behavior of the angular component of acceleration.

	Type of trajectory	n_2
1	Usual	$\frac{q-p}{2} + b - 1$
2	Subcritical	$\frac{q-p}{2} + b - 1 < n_2 < b - 1 + \frac{q}{2}$
3	Critical	$\frac{q}{2} + b - 1$
4	Ultracritical	$n_2 > \frac{q}{2} + b - 1$

Results for the finite force

Non-regular horizon

$$0 < k < \frac{p-q}{2} + 1, \quad \text{region I},$$

$$\frac{p-q}{2} + 1 \le k < \frac{p+1-q/2}{2}, \quad \text{region II},$$

$$\frac{p+1-q/2}{2} \le k < \frac{p}{2}, \quad \text{region III},$$

$$k \ge p/2, \quad \text{region IV}.$$

Diverging force

TABLE IV. Classification of near-horizon trajectories for different k regions for $q > 2$ (ultraextremal horizon). The fourth solution in (64) is not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
1	I	Stationary metric For any type of trajectory, n_1 is negative (forces diverge)		
2	II	$2k + \frac{q-2-2p}{2} < n_1 < \frac{q-p}{2} - 1 + k$	$n_1 + 1 + p/2 - k$	Subcritical
		$n_1 = \frac{q-p}{2} - 1 + k$	$q/2$	Critical
		$n_1 = \frac{q-p}{2} - 1 + k$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
3	III	$\max(0, \frac{q-2-2p}{2}) < n_1 \le 2k + \frac{q-2-2p}{2}$	$\frac{2n_1+2+q}{4}$	Subcritical
		$2k + \frac{q-2-2p}{2} < n_1 < \frac{q-p}{2} - 1 + k$	$n_1 + 1 + p/2 - k$	Subcritical
		$n_1 = \frac{q-p}{2} - 1 + k$	$q/2$	Critical
		$n_1 = \frac{q-p}{2} - 1 + k$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
4	IV	$\max(0, \frac{q-2-2p}{2}) < n_1 < \frac{q-2}{2}$	$\frac{2n_1+2+q}{4}$	Subcritical
		$n_1 = \frac{q-2}{2}$	$q/2$	Critical
		$n_1 = \frac{q-2}{2}$ and (28)	Any $c > \frac{q}{2}$	Ultracritical
5	$k = 0$	Static metric Same results as in IV for stationary metric		

TABLE VI. Classification of near-horizon trajectories for different k regions for $q < 2$ (nonextremal horizon). Regions II and III in this case are absent, so they are not presented in this table. The fourth solution in (64) is also not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
1	I and IV	Stationary metric For any type of trajectory, n_1 is negative (forces diverge)		
2	$k = 0$	Static metric For any type of trajectory, n_1 is negative (forces diverge)		

TABLE V. Classification of near-horizon trajectories for different k regions for $q = 2$ (extemal horizon). Regions II and III in this case are absent, so they are not presented in this table. The fourth solution in (64) is also not presented in this table. Definitions of different k regions are given in Fig. 1.

	k Region	n_1 Range	c	Type of trajectory
1	I	Stationary metric For any type of trajectory, n_1 is negative (forces diverge)		
2	IV	$n_1 = 0$	1	Critical
		$n_1 = 0$ and (28)	Any $c > 1$	Ultracritical
3	$k = 0$	Static metric Same results as in IV for stationary metric		

Results for the finite force

TABLE VIII. Classification of cases when forces are finite for different types of horizons and trajectories.

	Type of horizon	Type of trajectory	Region of k
1	Nonextremal	All types	Absent
2	Extremal	Subcritical Critical Ultracritical	Absent $k \geq p/2$ or $k = 0$
3	Ultraextremal	Subcritical Critical Ultracritical	$k \geq \frac{p-q}{2} + 1$ or $k = 0$

Force remains finite only for extremal and ultraextremal horizons

For critical particles the proper time diverges

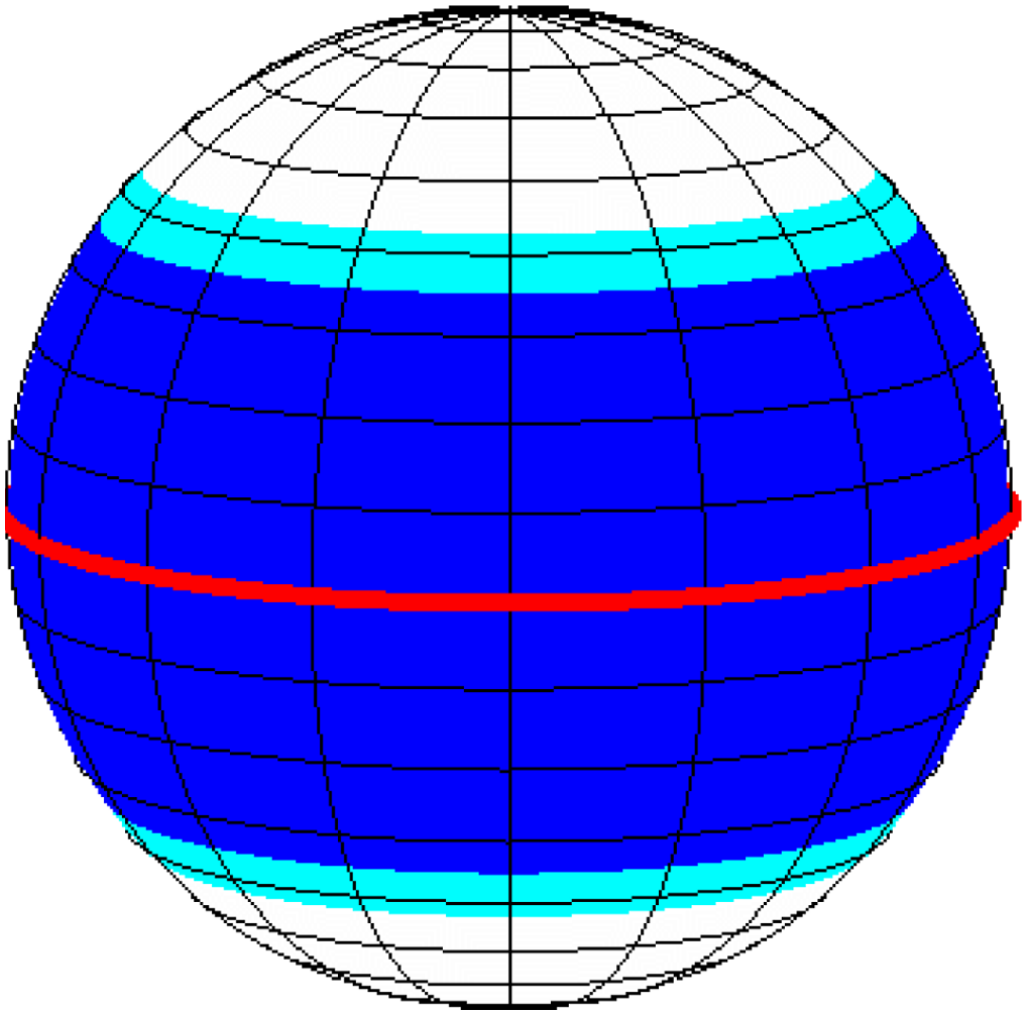
We were able to prove that *for any non-usual particle with the finite proper time either the force is infinite, or the horizon is non regular*.

Thus, for all the cases when the BSW effect is possible, proper time of reaching the horizon diverges (***kinematic censorship***). Is there any way that allows bypassing this issue?

Kinematic censorship

In any event only finite energy can be released. It can be unbounded but it cannot be literally infinite.

Nonequatorial motion



From Harada and Kimura,
PRD 2011
Kerr metric

Nonequatorial motion

The scenarios of high energy particle collisions, found previously in the context of the BSW effect for equatorial motion, do not change qualitatively in the nonequatorial case
Classification of different cases survives.

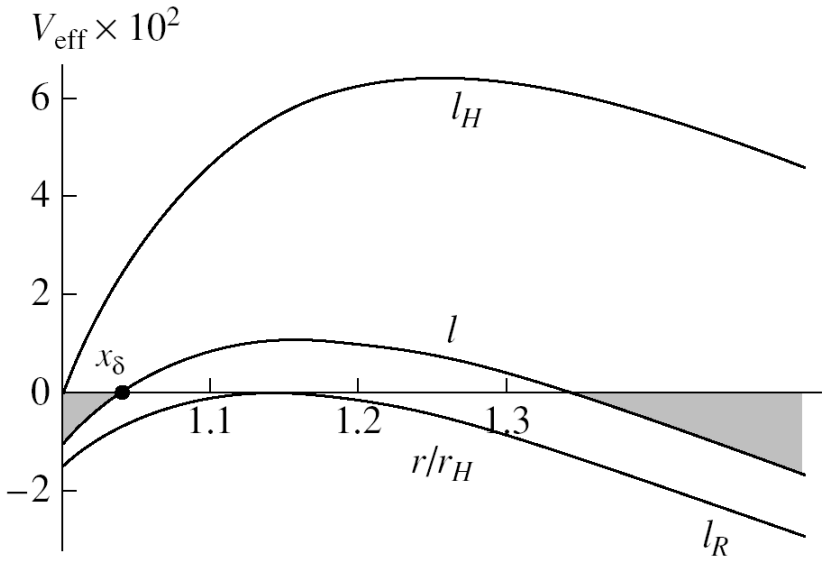
Extremal versus nonextremal . Near-fine-tuned particles

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Critical particle cannot penetrate forbidden region near horizon.
However, if there is a near-critical particle there from the very beginning, there is BSW effect

Near-fine-tuned particles

Original BSW: extremal BH horizon
Nonextremal BH: critical particle cannot reach horizon Is BSW possible?

Yes! To this end, has to consider near-fine-tuned particle

$$\mathcal{X} = \delta + X_s v^s + o(v^s), \quad s > 0,$$

where δ is some small parameter

Thus we have two small parameters: and the point of collision v_c

Near-fine-tuned particles

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$$\mathcal{X} = \delta + X_s v^s + o(v^s), \quad s > 0,$$

where δ is some small parameter

Thus we have two small parameters: and the point of collision v_c

Near-fine-tuned particles

Thus we have two small parameters: δ and the point of collision v_c

The case $\delta << v_c$ effectively corresponds to subcritical, critical and ultracritical particles

$$\mathcal{X} = \delta + X_s v^s + o(v^s),$$

0

The case $\delta >> v_c$ effectively corresponds to usual particles

$$\mathcal{X} = \delta + X_s v^s + o(v^s),$$

0

The case δ of order of v_c gives new results

$$\mathcal{X} = \delta + X_s v^s + o(v^s),$$

Of the same order

Classification of particles

TABLE I. Table showing classification of different near-fine-tuned particles.

Condition	Type of particle with nonzero $\delta \ll 1$	Abbreviation
$s < p/2$	Near subcritical	NSC
$s = p/2$	Near critical	NC
$s = p/2$ and (24)	Near ultracritical	NUC
$s > p/2$	Near overcritical	NOC

Thus, we have two small parameters: δ and the point of collision v_c

Near subcritical (NSC) - X tends to δ, but with the rate, smaller than N

Near critical (NC) - X tends to δ with the same rate as N

Near ultracritical (NUC) - X tends to δ with the same rate as N, but several first terms in the expansion of P cancel

Near overcritical (NOC) - X tends to δ with the rate, higher than N.

$$P = \sqrt{\mathcal{X}^2 - N^2\left(1 + \frac{\mathcal{L}^2}{g_{\varphi\varphi}}\right)},$$

Gamma-factor

TABLE II. Table showing behavior of γ factor for $v_c \sim v_{e,t}$ in a case when first particle is fine-tuned and second is near-fine-tuned. Here d is defined by relation $\gamma \sim v_c^{-d}$.

	First particle	Second particle	BSW is possible
1	U or SC	NSC	$\lvert s_1 - s_2\rvert$ or $\lvert s_1 - r_2\rvert$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)} v_c^r$
2	C or UC	NSC	$\frac{p}{2} - s_2$ or $\frac{p}{2} - r_2$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)} v_c^r$
3	U or SC	NC, NUC, or NOC	$\frac{p}{2} - s_1$
4	C or UC	NC, NUC, or NOC	0
			BSW is impossible

TABLE III. Table showing behavior of γ factor for $v_c \sim v_{e,t}$ in a case when both particles are near-fine-tuned. Here d is defined by relation $\gamma \sim v_c^{-d}$.

	First particle	Second particle	BSW is possible
1	NSC	NSC	$\lvert s_1 - s_2\rvert$ $\lvert r_1 - s_2\rvert$ if $\delta_1 = -X_s^{(1)} v_c^s + B_r^{(1)} v_c^r$ $\lvert s_1 - r_2\rvert$ if $\delta_2 = -X_s^{(2)} v_c^s + B_r^{(2)} v_c^r$ $\lvert r_1 - r_2\rvert$ if $\delta_{1,2} = -X_s^{(1,2)} v_c^s + B_r^{(1,2)} v_c^r$
2	NSC	NC, NUC, or NOC	$\frac{p}{2} - s_1$
3	NC, NUC, or NOC	NC, NUC, or NOC	0
			BSW is impossible

Finiteness of force

TABLE IV. Classification of near-horizon trajectories for different k regions for $q > 2$ (ultraextremal horizon). The fourth solution in (108) is not presented in this table.

	k region	n_1 range	s	Type of trajectory
		Stationary metric		
1	I	For any type of trajectory n_1 is negative (forces diverge)		
2	II and III	$\max(0, \frac{q-p}{2} - 1) < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NSC
		$n_1 = k + \frac{q-p}{2} - 1$	Second in (108)	NC and NOC
		$n_1 = k + \frac{q-p}{2} - 1$ and (24)	Second in (108)	NUC
3	IV	$\max(0, \frac{q-p}{2} - 1) < n_1 < \frac{q-2}{2}$	First in (108)	NSC
		$n_1 = \frac{q-2}{2}$	First in (108)	NC
		$n_1 = \frac{q-2}{2}$ and (24)	First in (108)	NUC
		$\frac{q-2}{2} < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NOC
		Static metric		
4	$k = 0$	Same results as in IV for stationary metric		NSC, NC, and NUC
		$n_1 > \frac{q-2}{2}$	Third in (108)	NOC

TABLE V. Classification of near-horizon trajectories for different k regions for $q = 2$ (ultraextremal horizon). The fourth solution in (108) is not presented in this table.

	k region	n_1 range	s	Type of trajectory
		Stationary metric		
1	I	For any type of trajectory n_1 is negative (forces diverge)		
2	IV	$n_1 = 0$	First in (108)	NC
		$n_1 = 0$ and (24)	First in (108)	NUC
		$0 \leq n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NOC

TABLE VI. Classification of near-horizon trajectories for different k regions for $q < 2$ (ultraextremal horizon). The fourth solution in (108) is not presented in this table.

	k region	n_1 range	s	Type of trajectory
		Stationary metric		
1	I	For any type of trajectory n_1 is negative (forces diverge)		
2	IV	$0 < n_1 \leq k + \frac{q-p}{2} - 1$	First and second in (108)	NOC
		Static metric		
3	$k = 0$	$0 < n_1$	Third in (108)	NOC

Non-regular horizon

$$0 < k < \frac{p-q}{2} + 1, \quad \text{region I},$$

$$\frac{p-q}{2} + 1 \leq k < \frac{p+1-q/2}{2}, \quad \text{region II},$$

$$\frac{p+1-q/2}{2} \leq k < \frac{p}{2}, \quad \text{region III},$$

$$k \geq p/2, \quad \text{region IV}.$$

Diverging force

Finite force

This part was absent for fine-tuned particles!

Finiteness of force

TABLE VII. Classification of cases when forces are finite for different types of horizons and trajectories.

	Type of horizon	Type of trajectory	Region of k
1	Nonextremal	NOC	IV or static
2	Extremal	NC, NUC, and NOC	IV or static
3	Ultraextremal	NSC, NC, NUC, and NOC	II, III, IV, or static

May achieve infinity

Cannot achieve infinity

Also, for such particles the proper time is large, but finite!

Finiteness of force

Type of horizon	1-st particle's type and range			2-nd particles's type and range			
Extremal	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{p/2}$	4
	U						
	U			C, UC			
Ultraextremal	NSC, NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NSC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{s}$	2
	U						
	U			SC			
	NSC, NC, NUC, NOC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{p/2}$	NSC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{s}$	3
	SC						
	SC			SC			
	NSC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{s}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{p/2}$	4
	U						
	U			C, UC			
	NSC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{s}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{p/2}$	4
	SC						
	SC			C, UC			

Original BSW

Finiteness of force

Type of horizon	1-st particle's type and range			2-nd particle's type and range			
Non-extremal	NOC	$v_c \ll v_t^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NOC	$v_c \sim v_t^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(89)
	U						
Extremal	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(89)
	U						
Ultraextremal	NSC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{s}$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{s}$	(83)
	U						
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^{s}$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{s}$	(91)
	NSC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{s}$	NSC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{s}$	(83)
	SC						
	NSC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{s}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(89)
	U						
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^{s}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(92)
	NSC	$v_c \sim v_{e,t}^{(1)}$	$\delta_1 \sim v_c^{s}$	NC, NUC	$v_c \gg v_{e,t}^{(2)}$	$\delta_2 \ll v_c^{p/2}$	(85)
				C, UC			
	NSC	$v_c \gg v_{e,t}^{(1)}$	$\delta_1 \ll v_c^{s}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(89)
	SC						
	NC, NUC, NOC	$v_c \ll v_{e,t}^{(1)}$	$\delta_1 \gg v_c^{p/2}$	NC, NUC, NOC	$v_c \sim v_{e,t}^{(2)}$	$\delta_2 \sim v_c^{p/2}$	(89)
	U						

Wide variety of various cases!

Acceleration of particles by nonrotating charged black holes

O. Z. JETP Letters 2010

Role of rotation $L_1 = \frac{E_1}{\omega_H}$ If $\omega_H \to 0$, $L_1 \to \infty$,

Angular momentum versus charge

Reissner-Nordström

Pure radial motion

not only $\omega_H = 0$ but also $L_1 = L_2 = 0$ for both colliding particles

particles charged, nongeodesic motion

$$ds^2 = -dt^2 f + \frac{dr^2}{f} + r^2 d\omega^2 \qquad f = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

$$mu^0 = m\dot{t} = \frac{1}{f}(E - \frac{qQ}{r}),$$

$$m^2\dot{r}^2 = (E - \frac{qQ}{r})^2 - m^2 f.$$

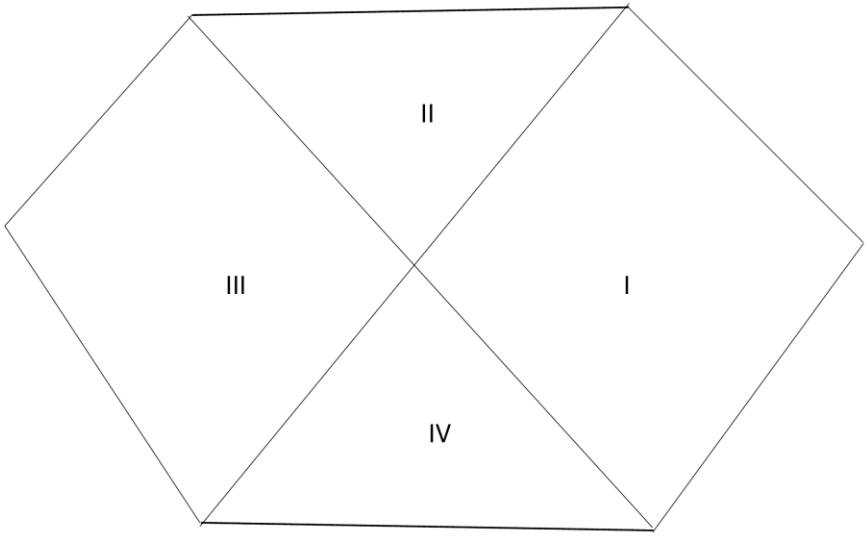
$$\frac{E_{cm}^2}{2m^2} = 1 + \frac{X_1 X_2 - Z_1 Z_2}{f m^2} \qquad X_i = E_i - \frac{q_i Q}{r},\ Z_i = \sqrt{X_i^2 - m^2 f}.$$

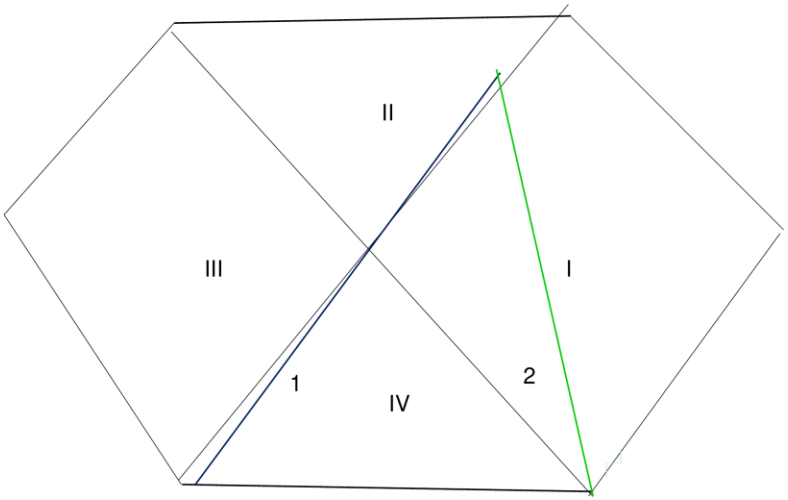
Schwarzschild BH and high energy collisions

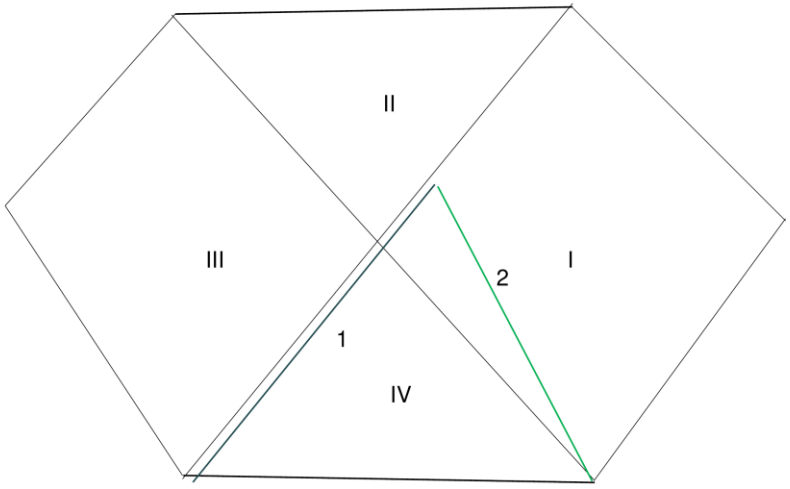
No rotation, no charge. How can such collisions happen?
If two identical particles collide near horizon, the maximum of E_cm is only

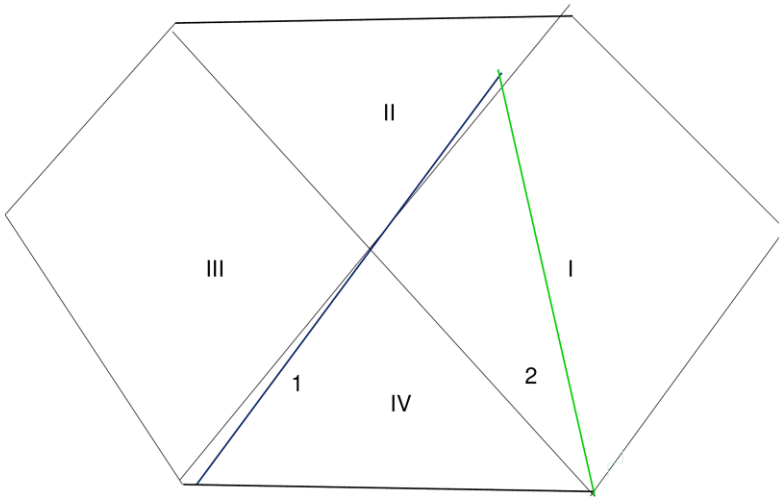
$$m\sqrt{5}$$

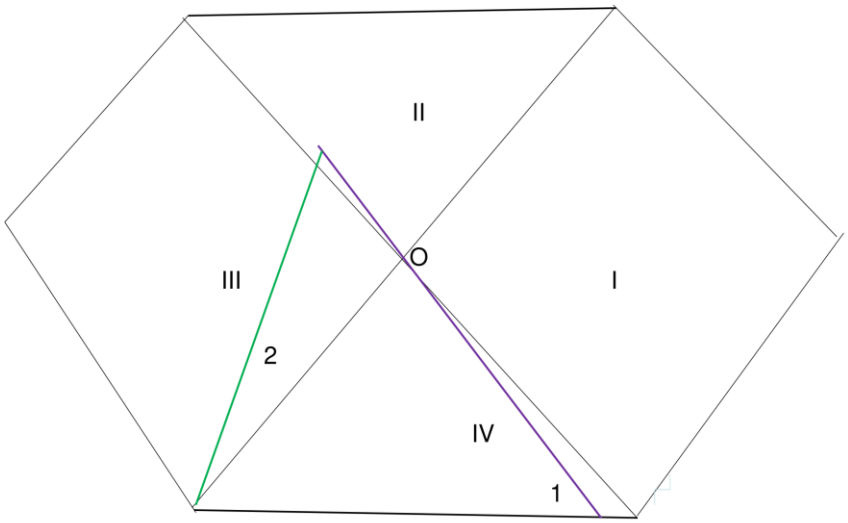
But this is for collisions outside horizon. But full space-time diagrams includes White hole region IV and mirror universe III.



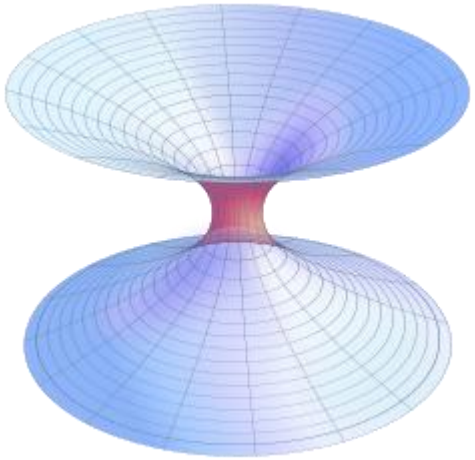

II
III
I
1
IV
2


II
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IV


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1

Collisions in wormholes



Two particles enter diferent mouths and collide near throat

Wormholes and SPP

New scenario of collision: Krasnikov 2018
Wormholes, particles come from opposite mouths and collide near throat.
If N (lapse function) is small there, we have analogue of head-collision near horizon, without difficulties related to black-white issue.
Another problem is that curvature becomes unbounded, so N should be restricted from below (Sushkov and O.Z. 2009)

$$E^2_{c.m.} = \frac{X_1 X_2 + P_1 P_2}{N^2} + m^2 + m^2{}_2 - \frac{L_1 L_2}{g_\phi},$$

Krasnikov scenario gives unbounded energy in CM frame and works even for Schwarzchild-like wormholes.
Answer is YES !



Summary

Possible set of high energy scenarios with finite force depending on type of horizons and trajectories of colliding particles

Validity of kinematic censorship

Thank you for your attention!