Acceleration Mechanisms
Part I

From Fermi to DSA

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Suggested hashtag for social media

#PACR2019

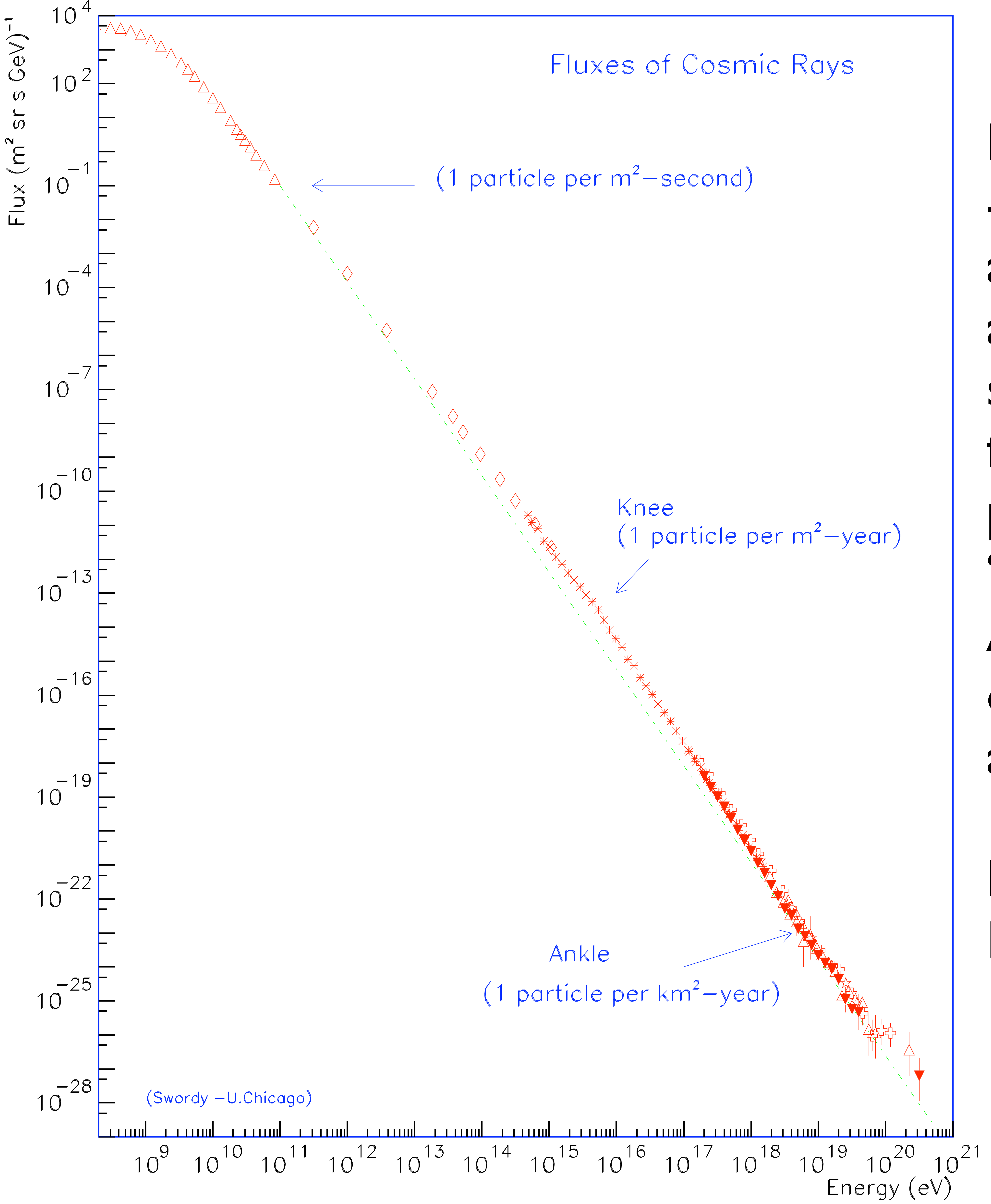
- Will discuss astrophysical acceleration mechanisms - how do cosmic accelerators work? - concentrating mainly on the class of Fermi processes but also some alternatives. Emphasis will be very much on the underlying physics and less on the mathematical and computational details.

- Motivation comes historically from cosmic ray observations going back to 1912 (and even a bit earlier) indicating the existence of an extremely energetic radiation of extraterrestrial origin as well as evidence from radio astronomy and gamma-ray astronomy pointing to a largely non-thermal universe.

“When, in 1912, I was able to demonstrate by means of a series of balloon ascents, that the ionization in a hermetically sealed vessel was reduced with increasing height from the earth (reduction in the effect of radioactive substances in the earth), but that it noticeably increased from 1,000 m onwards, and at 5 km height reached several times the observed value at earth level, I concluded that this ionization might be attributed to the penetration of the earth's atmosphere from outer space by hitherto unknown radiation of exceptionally high penetrating capacity, which was still able to ionize the air at the earth's surface noticeably. Already at that time I sought to clarify the origin of this radiation, for which purpose I undertook a balloon ascent at the time of a nearly complete solar eclipse on the 12th April 1912, and took measurements at heights of two to three kilometres. As I was able to observe no reduction in ionization during the eclipse I decided that, essentially, the sun could not be the source of cosmic rays, at least as far as undeflected rays were concerned.”

From Victor Hess’s nobel prize acceptance speech, December 12, 1936

Viktor Hess's desk and some of his electroscopes, preserved in ECHOPhysics, the European Centre for the History of Physics in Schloss Pöllau, Austria.



Extraordinary energy range - from below a GeV to almost ZeV energies - and a remarkably smooth spectrum with only minor features, the most prominent being the "knee" and "ankle" regions. Almost perfect power-law over ten decades in energy and 30 decades in flux!

How and where does Nature do it?

Quick primer on CR physics

- Solar wind effects (“modulation”) and local sources are dominant below 1 GeV or so.
- Except at the very highest energies the arrival directions are isotropic to $\delta \approx 10^{-3}$
- Composition is well established at low energies and consists of atomic nuclei with some electrons, positrons and antiprotons.
- Clear evidence of secondary particle production (spallatogenic nuclei such as Li, Be, B; antiprotons) from interaction with ISM - grammage $x \approx 5\,\mathrm{g\,cm^{-2}}$

- Secondary to primary ratios (e.g. Boron to Carbon, sub Iron to Iron) decrease as functions of energy around a few GeV
- All primary nuclei appear to have very similar rigidity spectra (momentum/charge) - but recent data show softer protons!
- Some radioactive secondary nuclei (eg $^{10}\mathrm{Be}$) have partially decayed indicating an "age" of around $10^7\,\mathrm{yr}$, again at a few GeV.

- Energy density is similar to other ISM energy densities and mainly in low energy (GeV) particles $\approx 1\,\mathrm{eV\,cm^{-3}}$

- CRs observed at the Solar system appear to be fairly typical of whole Galaxy (gamma-ray observations) with a slight radial gradient.

- Total CR luminosity of the Galaxy is then of order $10^{41}\,\mathrm{erg\,s^{-1}} = 10^{34}\,\mathrm{W}$

- This is the power needed to run the cosmic accelerator in our Galaxy - a few % of the mechanical energy input from SNe.

$$p + A \to \pi^0 + \ldots \to \gamma + \gamma$$

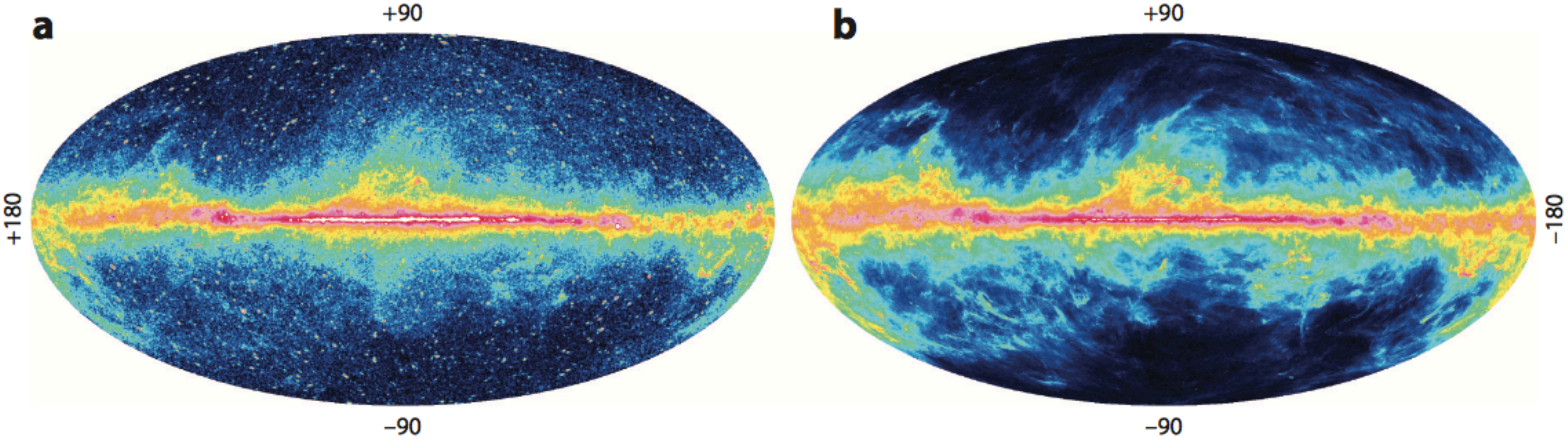


Figure 5

Sky maps of (*a*) the γ-ray intensity recorded by *Fermi*-LAT above 1 GeV in six years of observations (**http://fermi.gsfc.nasa.gov/ssc/**) and of (*b*) the dust optical depth measured at 353 GHz from the *Planck* and IRAS surveys (Planck Collab. et al. 2014b). Both maps broadly trace the same total gas column densities, weighted by the ambient cosmic-ray density in γ rays and by the ambient dust-to-gas mass ratio and starlight heating rate in the dust map. They exhibit striking similarities in details of the gas features. The γ-ray map also contains numerous point sources and faint non-gas-related diffuse components.

I. Grenier, J. Black and A. Strong: Annual Reviews Astronomy and Astrophysics 2015. 53

Basic Power Estimate

- Local energy density and "grammage" for mildly relativistic CRs are both very well constrained by observations at a few GeV/nucleon.
- Gives a more or less model independent estimate of the cosmic ray power needed to maintain a steady state cosmic ray population in the Galaxy.

Confinement time

Target mass

Energy density

Grammage $$g = \frac{\tau c M}{V}$$

Luminosity $$L_{\mathrm{CR}} = \frac{\mathcal{E}_{\mathrm{CR}} V}{\tau}$$

Confinement volume

$$L_{\mathrm{CR}} \approx \mathcal{E}_{\mathrm{CR}} \frac{cM}{g}$$

$$\mathcal{E}_{\mathrm{CR}} \approx 1.0\,\mathrm{eV\,cm^{-3}}$$

$$M \approx 5 \times 10^9 \mathrm{M}_\odot$$

$$g \approx 5\,\mathrm{g\,cm^{-2}}$$

$$\implies L_{\mathrm{CR}} \approx 10^{41}\,\mathrm{erg\,s^{-1}} = 10^{34}\mathrm{W}$$

NB does not depend on ^{10}Be age etc.

- Production spectrum of secondary nuclei is know from observed flux of primaries, the ISM density and nuclear cross-sections, roughly $Q_2 \propto J_1 \sigma c n \propto E^{-2.6}$

- Observed flux of secondaries has a softer energy spectrum, $J_2/J_1 \propto E^{-0.6}$

- Infer that Galactic propagation softens spectra and that the true production spectrum of primaries must be harder than the observed flux, perhaps as much as E^{-2}

- NB - Exact source spectrum depends on details of propagation model (see talk by David and others) - in particular whether reacceleration is significant at low energies.

- Based largely on low-energy composition data and then extrapolated over at least another four decades in energy!

In summary, need

- A very efficient Galactic accelerator
- Producing a hard power law spectrum over many decades
- Accelerating material of rather normal composition
- Not requiring very exotic conditions

Astrophysical Accelerators

- Major problem - most of the universe is filled with conducting plasma and satisfies the ideal MHD condition $E + U \wedge B = 0$
- Locally no E field, only B
- B fields do no work, thus no acceleration!

Two solutions

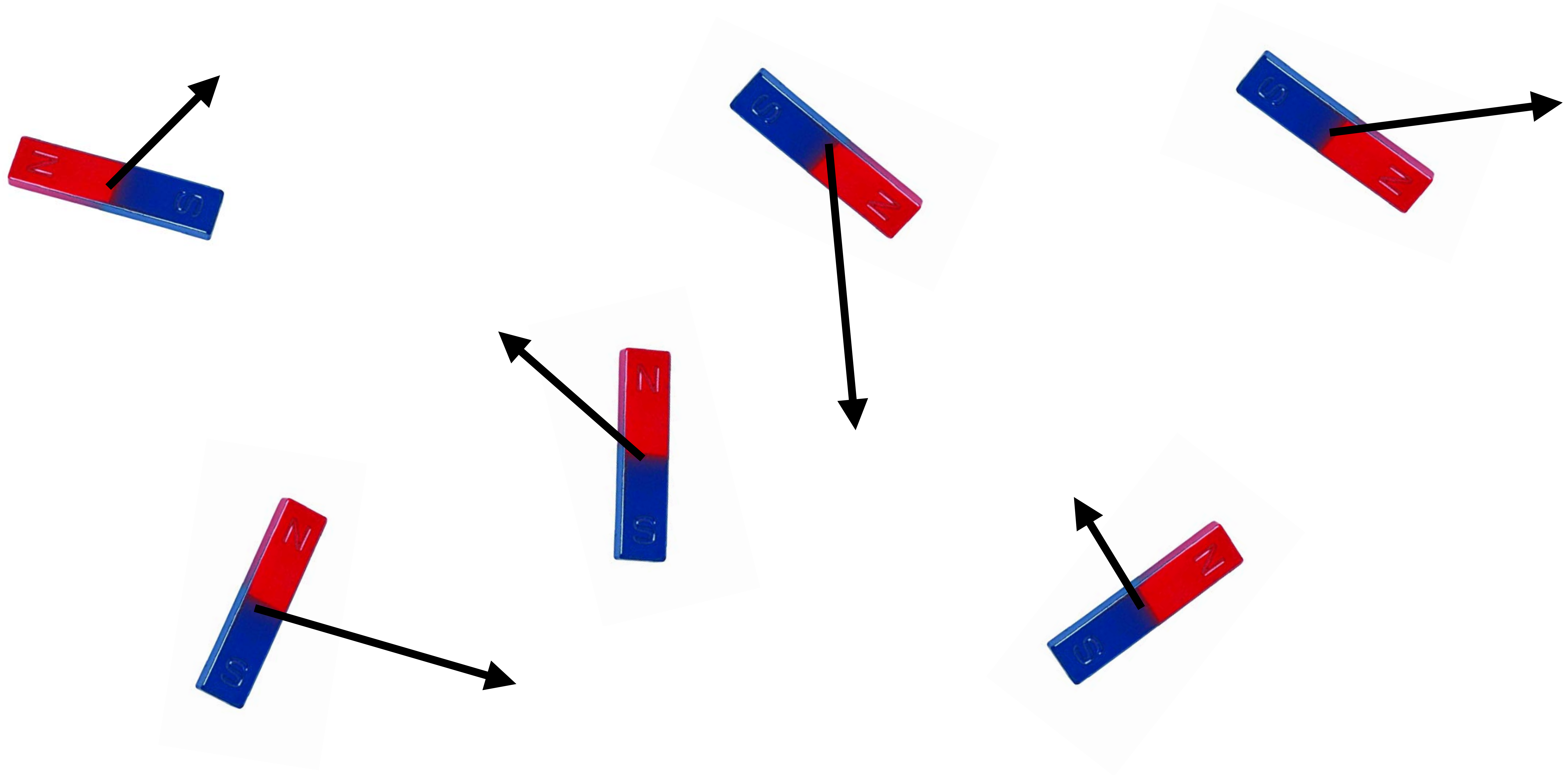
- Look for sites where ideal MHD is broken (magnetic reconnection, pulsar or BH environment, etc).
- Recognise that E only vanishes locally, not globally, if system has differential motion - this is the class of Fermi mechanisms on which I will concentrate.

- Close analogy to terrestrial distinction between
  - One shot electrostatic accelerators, e.g. tandem Van der Graf accelerators or classic Cockroft-Walton design.
  - Storage rings with many small boosts, eg LHC at CERN (each RF cavity has only about 2MV, but LHC reaches several TeV energies).

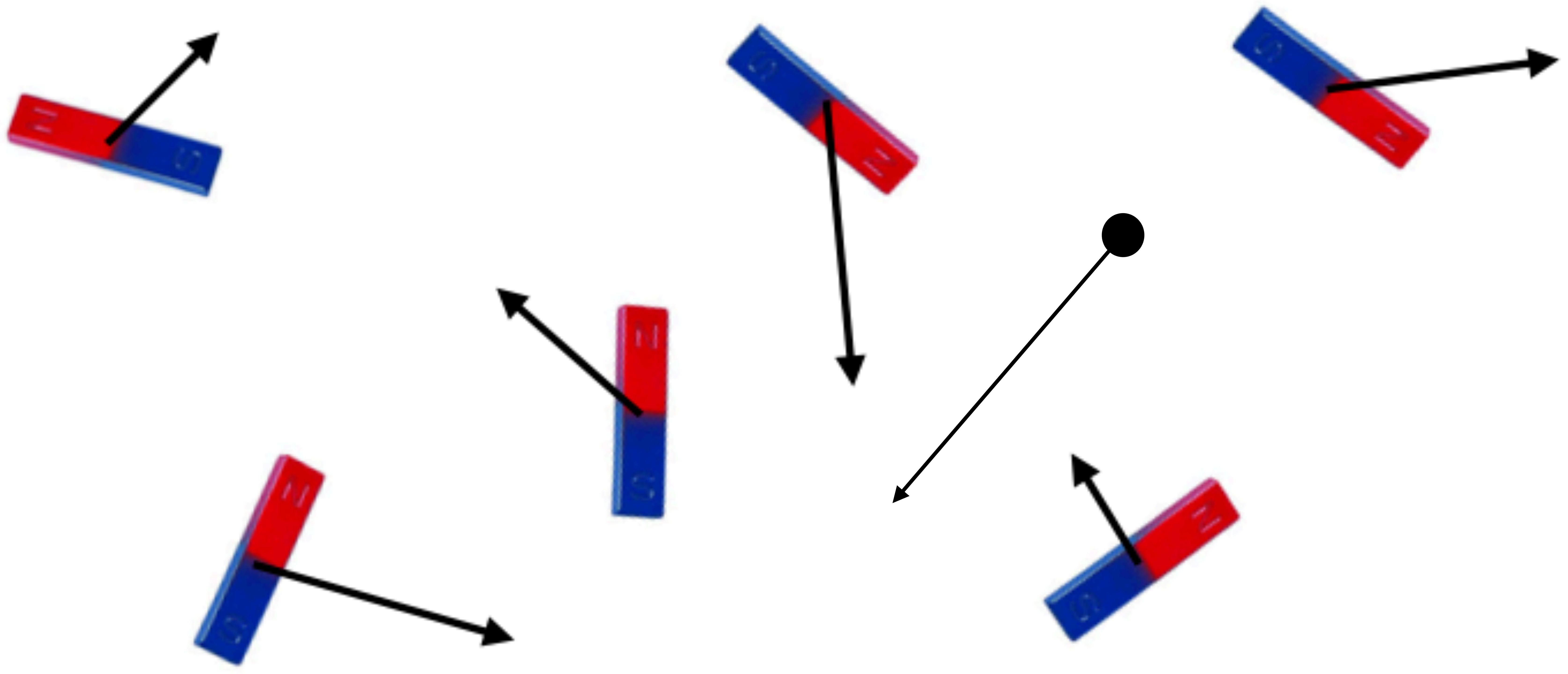
Fermi 1949

- Galaxy is filled with randomly moving clouds of gas.
- The clouds have embedded magnetic fields.
- High-energy charged particles can “scatter” off these magnetised clouds.
- The system will attempt to achieve “energy equipartition” between macroscopic clouds and individual atomic nuclei leading to acceleration of the particles.

Gedanken experiment - imagine a "gas" of bar magnets (massive magnetic dipoles) interacting through their dipole fields only - Maxwellian velocity distribution.



Now drop in one proton. What will happen as the system tries to come into "thermal" equilibrium?



Equipartition of Energy

- Implies mean KE of proton must ultimately approach mean KE of the magnets.
- Attempt to equilibrate macroscopic degrees of freedom of magnet to microscopic ones of proton implies massive acceleration of the proton eventually.
- But how long does it take?

Trivial but very important point; the energy of a particle is not a scalar quantity, but the time-like component of its energy-momentum four vector. If we shift to a different reference frame, the energy changes and so does the magnitude of the momentum.

Shift from lab frame to frame of cloud (or magnet) moving with velocity $\vec{U}$

$$E' = \frac{E + \vec{p} \cdot \vec{U}}{\sqrt{1 - U^2/c^2}}$$

$$
\begin{aligned}
\Delta E &\approx \vec{p}\cdot\vec{U} \\
\Delta p &\approx \frac{E}{c^2 p}\vec{p}\cdot\vec{U} = \frac{1}{v}\vec{p}\cdot\vec{U} \\
&\approx \frac{1}{c}\vec{p}\cdot\vec{U} \qquad p \gg mc \\
&\approx \frac{m}{p}\vec{p}\cdot\vec{U} \qquad p \ll mc
\end{aligned}
$$

Lab frame $\longrightarrow$ Cloud frame $\longrightarrow$ Lab frame

$$\Delta p \approx \beta p \left(\cos\vartheta_1 - \cos\vartheta_2\right)$$

for relativistic particles scattering off clouds
with dimensionless peculiar velocity $\beta = \dfrac{U}{c}$

Mean square change in momentum is

$$\langle \Delta p^2 \rangle = \frac{2}{3}\beta^2 p^2$$

Particle makes a random walk in momentum space with steps of order βp at each scattering.

Corresponds to diffusion process,

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right) + Q - \frac{f}{T}$$

with diffusion coefficient of order

$$D_{pp} \approx \frac{\beta^2 p^2}{\tau}$$

where τ is the mean time between scatterings.

Fermi pointed out that if the scattering and loss time scales are both energy independent this produces power law spectra with exponent

$$q = \sqrt{\frac{\tau}{\beta^2 T} + \frac{9}{4}} - \frac{3}{2}$$

Beautiful but wrong

- Too slow - acceleration time scale is diffusion time scale

$$\begin{aligned}
\frac{p^2}{D_{pp}} &\approx \frac{\tau}{\beta^2} \\
\beta \le 10^{-4}, \qquad & \qquad \tau \ge 1\,\mathrm{yr} \\
&\ge 10^8\,\mathrm{yr}
\end{aligned}$$

- Requires unnatural fine-tuning of collision time and loss time to produce a power-law.

- Requires an additional injection process to get particles to relativistic energies (very high energy loss rate for non-relativistic charged particles).

- Would imply that higher energy particles are older, contrary to the observed secondary to primary ratios.

- Has difficulty with the chemical composition.

But...

- Must occur at some level (eg reacceleration models of CR propagation).
- Is historically very important.
- Contains valuable physical insight - macroscopic differential motion can couple to individual charged particles in such a way that acceleration occurs.
- Also Fermi drew attention to very long ionisation loss time scales for relativistic ions in typical ISM - important reason for existence of CRs.

General cosmic ray transport equation

$$
\begin{aligned}
\frac{\partial f}{\partial t} \quad + \quad & U \cdot \nabla f = && \text{Convective derivative} \\
& \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 D_{pp}\frac{\partial f}{\partial p}\right) && \text{Momentum diffusion} \\
+ \quad & \nabla \cdot (D_{xx}\nabla f) && \text{Spatial diffusion} \\
- \quad & \frac{1}{3}\nabla \cdot U\, p\frac{\partial f}{\partial p} && \text{Adiabatic compression} \\
+ \quad & Q - \frac{f}{T} && \text{Sources and sinks}
\end{aligned}
$$

Key Assumptions

- Distribution function is close to isotropic (strong scattering by magnetic fields)

$$f(\vec{p}) \approx f(p), \qquad p = |\vec{p}|$$

- Mixed coordinate system, particle momentum p measured in local fluid frame, fluid velocity U in global reference system.

- Motion is non-relativistic $\qquad U \ll c$

If the same scattering gives rise to both the momentum and spatial diffusion, the two coefficients are related roughly by

$$D_{pp} \approx \frac{\beta^2 p^2}{\tau}$$

$$D_{xx} \approx \frac{\lambda^2}{\tau} = c^2 \tau$$

$$D_{pp} D_{xx} \approx V^2 p^2$$

where V is the random velocity of the scattering centres, often taken to be Alfvén waves. Thus if one is large, the other is small and *vice versa*.

Shock acceleration

- Major breakthrough in 1977/1978
- Four independent publications of same essential idea by
  - G. F. Krymsky
  - R. Blandford and J. Ostriker
  - I. Axford, E. Leer and G. Skadron
  - A.Bell

Доклады Академии наук СССР
1977. Том 234, № 6

УДК 523.165+523.72 *ФИЗИКА*

Г. Ф. КРЫМСКИЙ

РЕГУЛЯРНЫЙ МЕХАНИЗМ УСКОРЕНИЯ ЗАРЯЖЕННЫХ ЧАСТИЦ НА ФРОНТЕ УДАРНОЙ ВОЛНЫ

(Представлено академиком С. Н. Верновым 18 X 1976)

G. F. Krymsky, Regular mechanism of charged particle acceleration on the shock wave front, *Dokl. Akad. Nauk SSSR*, 1977, Volume 234, Number 6, 1306–1308

Collisionless Shocks

- Shocks, sudden jumps in velocity and density, appear whenever flow hits an obstacle, flows collide, flows converge.
- Physically appear as 1-D dissipative structures in which KE of bulk motion is transferred to micro-scale random motion.
- Dissipation in collisionless shock comes from collective plasma processes (not, as in gas dynamics, from 2-body collisions).

Keep advection, adiabatic compression and spatial diffusion terms in transport equation,

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f = \nabla(\kappa \nabla f) + \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p}$$

and apply it to the flow through a shock

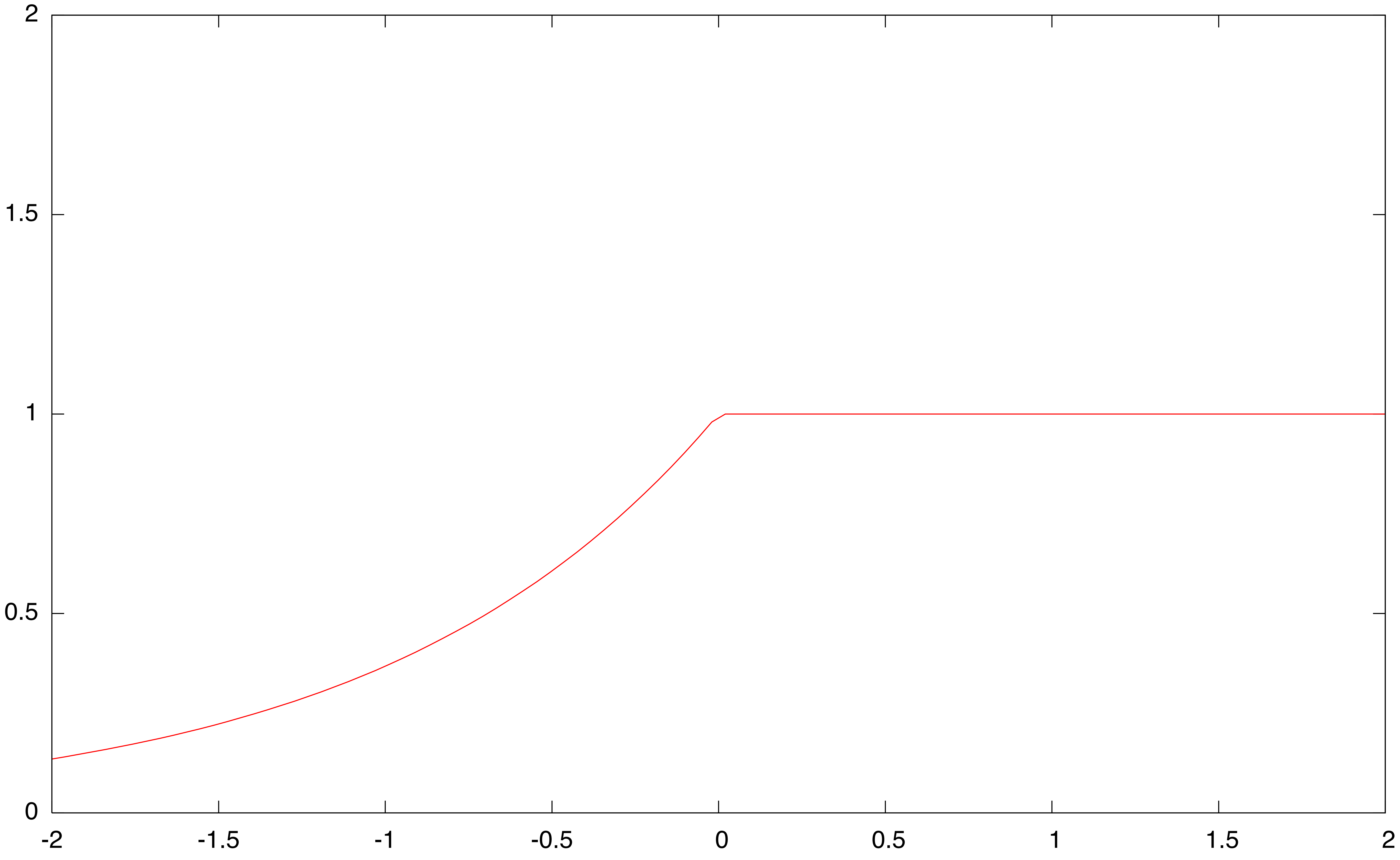
$$U(x) = \begin{cases} U_1, & x < 0 \\ U_2, & x > 0 \end{cases}$$

Look for steady solutions in upstream and downstream regions...

$$\cancel{\frac{\partial f}{\partial t}} + \vec{U} \cdot \nabla f = \nabla(\kappa \nabla f) + \frac{1}{3} \cancel{(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p}}$$

$$f(x,p) = f_0(p) \exp \int \frac{U}{\kappa} dx \qquad x \leq 0$$

$$f(x,p) = f_0(p) \qquad x \geq 0$$


2
1.5
1
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-2
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-0.5
0
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1
1.5
2

$$\frac{\partial f}{\partial t} + \vec{U} \cdot \nabla f - \frac{1}{3}(\nabla \cdot \vec{U}) p \frac{\partial f}{\partial p} = \nabla(\kappa \nabla f)$$

Advection in x-space
with spatial velocity

$$\vec{U}$$

Advection in p-space
with velocity given by

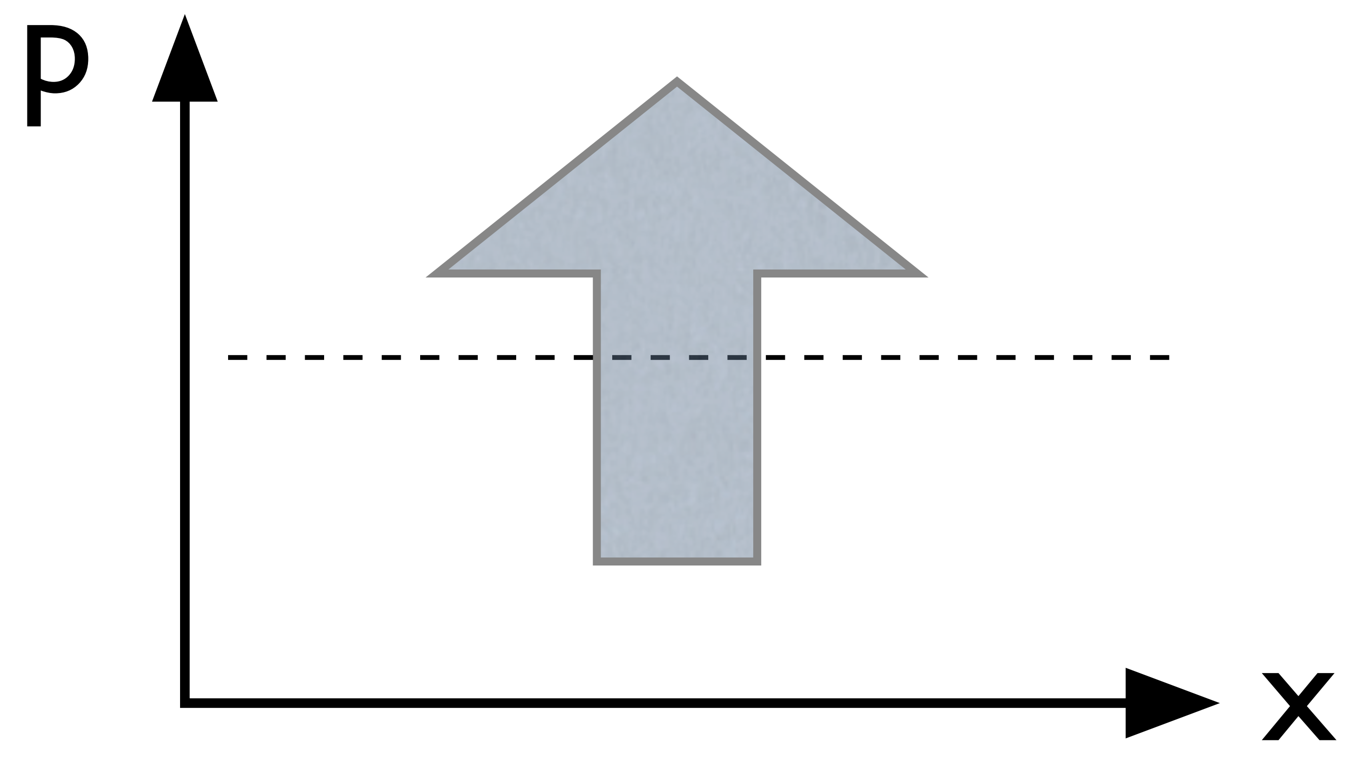
$$-\frac{1}{3}\left(\nabla \cdot \vec{U}\right) p$$

Acceleration from compression in shock front!

Useful to think in terms of the acceleration flux,

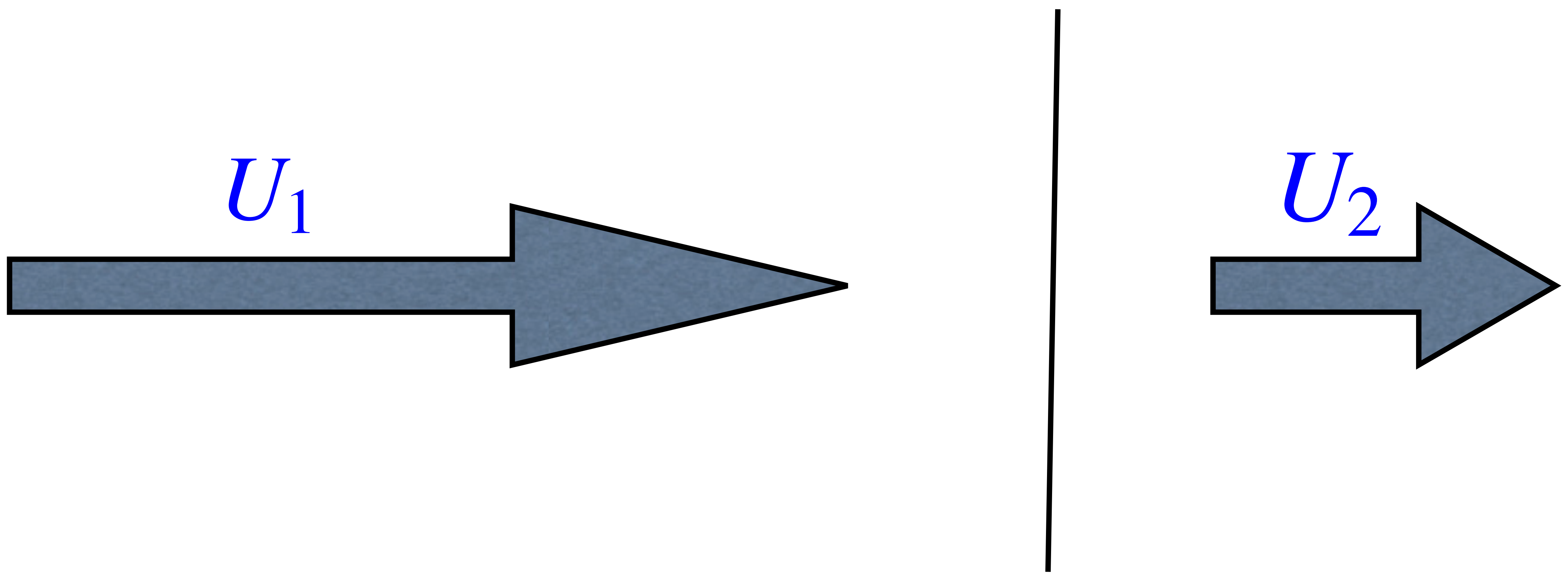
$$\Phi(p) = \int \frac{4\pi p^3}{3} f(p)(-\nabla \cdot \vec{U})\, d^3x$$

Rate at which particles are being accelerated through a given momentum (or energy) level.



If compression occurs only at the shock, then

$$\Phi(p) = \frac{4\pi p^3}{3} f_0(p)\,(U_1 - U_2)$$



and is localised at the shock.

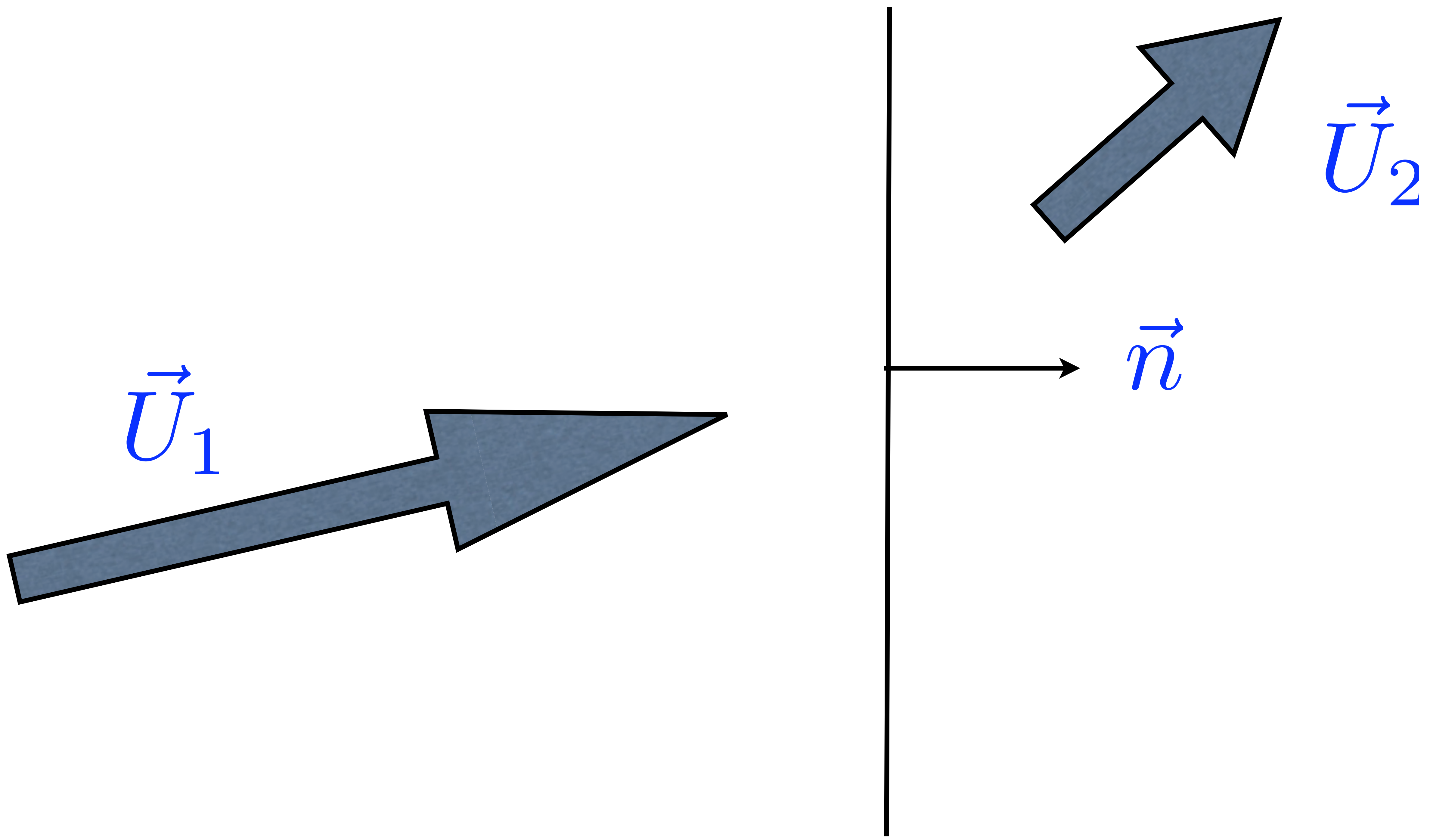
Formally follows from putting

$$-\nabla \cdot U = (U_1 - U_2)\delta(x)$$

in the transport equation, but can be seen more directly by looking at the kinetic level.

$$\begin{aligned}
\Phi(p) &= \int \frac{1}{v}\vec{p}\cdot\left(\vec{U}_1 - \vec{U}_2\right)(\vec{v}\cdot\vec{n})\, f(p)p^2\, d\Omega \\
&= p^3 f(p)\,\vec{n}\cdot\left(\vec{U}_1 - \vec{U}_2\right)\int_{-1}^{+1}\mu^2\, 2\pi d\mu \\
&= \frac{4\pi}{3}p^3 f(p)\,\vec{n}\cdot\left(\vec{U}_1 - \vec{U}_2\right)
\end{aligned}$$

This result applies quite generally to oblique MHD shocks and only depends on the near isotropy of the particle distribution at the shock and the condition (related to the isotropy) that the particles are fast relative to the flow.



- Positive, though small, change in momentum each time shock is crossed in either direction of order

$$\Delta U/v$$

- In diffusion regime particles cross shock many times - probability of escape downstream is low,

$$v/4U_2$$

Is it a con trick?

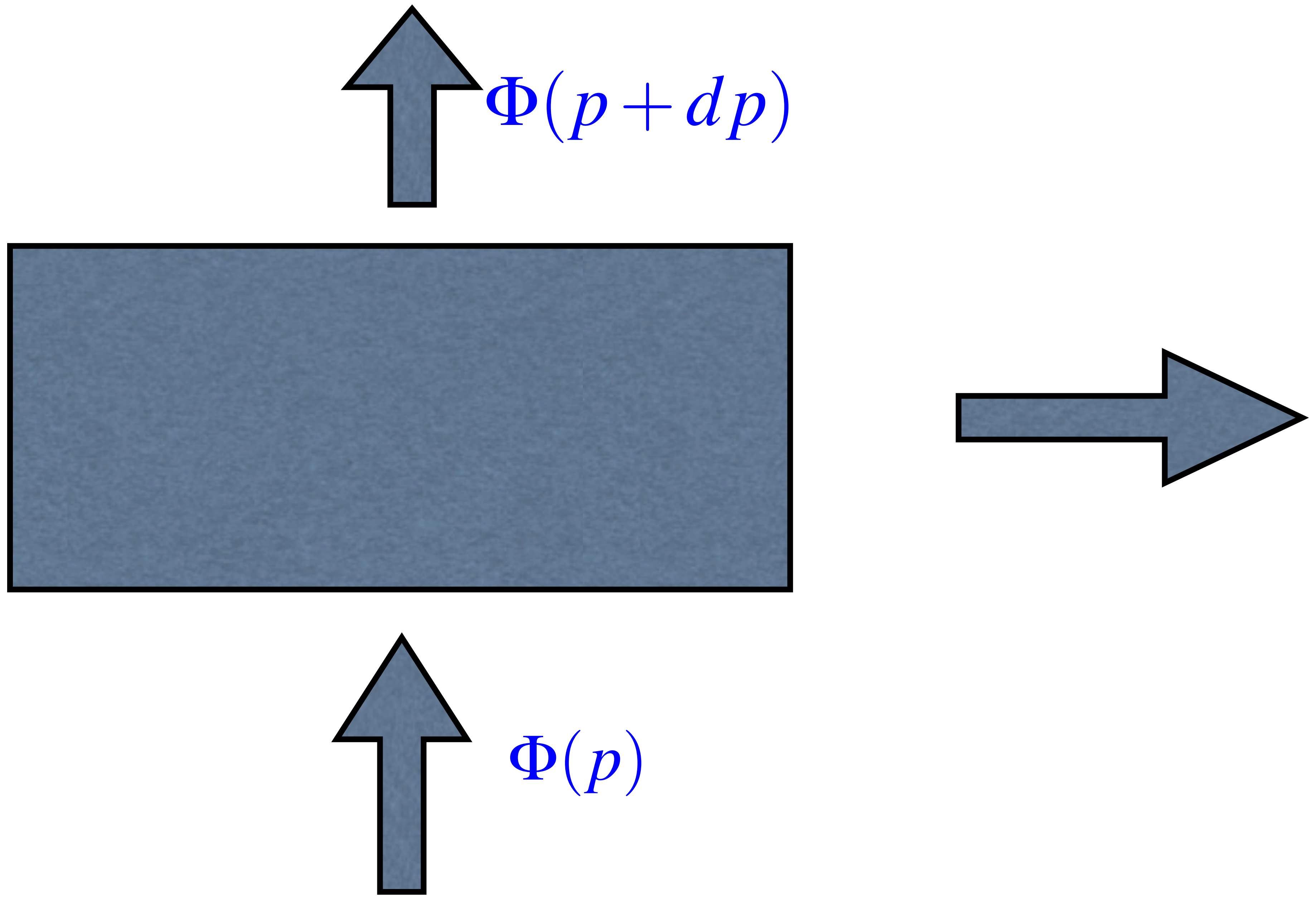
- Nothing happens to particle as it crosses the front - we just change the reference frame.
- But if we were to work in the shock frame, then the scattering processes would all be energy changing.
- Using separate reference frames up and down stream is consistent and greatly simplifies the analysis by concentrating all the effects at the shock.

Alternative approach

- Fully covariant relativistic formulation of generalised Fermi acceleration due to Martin Lemoine.
- Works entirely in local E=0 frame and explicitly tracks inertial forces due to frame changes.
- arXiv:1903.05917v2 (Phys. Rev. D 99 083006)
- "unified description … applies equally well in sub- and ultra-relativistic settings, Cartesian and non-Cartesian geometries, flat or non-flat space time."

Now write down particle conservation law for balance between rate of advection away from shock region and acceleration

$$\frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$



Particles interacting with the shock fill a “box” extending one diffusion length upstream and downstream of the shock,

$$L = \left(\frac{\kappa_1}{U_1} + \frac{\kappa_2}{U_2} \right)$$

so time dependent particle conservation is

$$\frac{\partial}{\partial t} \left(4\pi p^2 f_0(p) L \right) + \frac{\partial \Phi}{\partial p} = -4\pi p^2 f_0(p) U_2$$

or, substituting for the acceleration flux

$$4\pi p^2 L \frac{\partial f}{\partial t} + 4\pi p^2 f (U_1 - U_2) + \frac{4\pi p^3}{3} (U_1 - U_2) \frac{\partial f}{\partial p} = -4\pi p^2 f U_2$$

and simplifying

$$L \frac{\partial f}{\partial t} + \frac{U_1 - U_2}{3} p \frac{\partial f}{\partial p} = -U_1 f$$

"Box" approximation to shock acceleration -
can be trivially solved by method of characteristics

The single PDE

$$L\frac{\partial f}{\partial t} + \frac{U_1 - U_2}{3} p \frac{\partial f}{\partial p} = -U_1 f$$

is equivalent to the pair of ODEs

$$\begin{aligned} \frac{d\,p}{d\,t} &= \frac{U_1 - U_2}{3L} p \\ \frac{d\,f}{d\,p} &= -3\frac{U_1}{U_1 - U_2}\frac{f}{p} \end{aligned}$$

The first equation says that particles gain energy at rate,

$$t_{\mathrm{acc}} = \frac{p}{\dot{p}} = \frac{3L}{U_1 - U_2}$$

the second that the number of particles decreases
in such a way as to give a power-law spectrum
as a function of momentum,

$$f \propto p^{-3U_1/(U_1-U_2)}$$

Main defect of the box model is that it assumes that all particles gain energy at precisely the same rate, whereas in reality there is considerable dispersion in the acceleration time distribution. However it is a useful simplification that captures much of the physics. Can add synchrotron losses, spherical geometry etc without too much difficulty.

It is actually possible to do a lot analytically with the full transport equation, and it is quite easy to solve numerically, so this linear test-particle theory is very well understood.

Key points

- Process is a pure first-order acceleration (although not if post-shock expansion is included).
- Naturally produces power-law spectra with exponent fixed by kinematics of shock (scale free).
- Spectral exponents are in right ball park.

- Process is relatively fast **if** local turbulence at shock is high (as is expected from plasma instabilities) **and** these scatter particles strongly - usual assumption is Bohm scaling.

$$\kappa \approx \frac{1}{3} r_g v$$

$$t_{\rm acc} \approx 10 \frac{\kappa_1}{U_1^2}$$

$$\dot{E} = v\dot{p} = \frac{vp}{t_{\rm acc}} \approx 0.3 e B U_1^2$$

For typical ISM field of 0.3nT and a young SN shock of velocity 3000 km/s get energy gain of 1000eV/s

$$0.3\mathrm{e} \times (3 \times 10^{-10}\,\mathrm{T}) \times (3 \times 10^{6}\,\mathrm{m\,s^{-1}})^2 = 10^3\,\mathrm{eV/s}$$

Impressive, and easily enough to overcome coulomb losses etc, but do not expect such high shock speeds to last more than a few hundred years. After 300 years maximum energy still only of order 10TeV, well short of the PeV needed for the knee region (Lagage and Cesarsky limit).

One of very general limits on possible accelerators:

$$r_g = \frac{p}{eB} < L \implies E < eBLc$$

If electric field derived from a velocity scale U operating over a length scale L:

$$E \leq eUBL$$

Diffusive shock acceleration in Bohm limit

$$E \leq 0.3eBU^2t = 0.3eBUL$$

Basically about as fast as is physically possible!

Some numbers for the ISM....

$$\left(\frac{B}{3\mu\mathrm{G}}\right)\left(\frac{L}{10\,\mathrm{pc}}\right)\left(\frac{U}{10^4\,\mathrm{km\,s^{-1}}}\right) = \left(\frac{\Phi}{1\,\mathrm{PV}}\right)$$

So-called Lagage-Cesarky limit - hard to accelerate protons to PeV energies in SNRs with conventional parameters.

The "Hillas plot" shown at Moriond

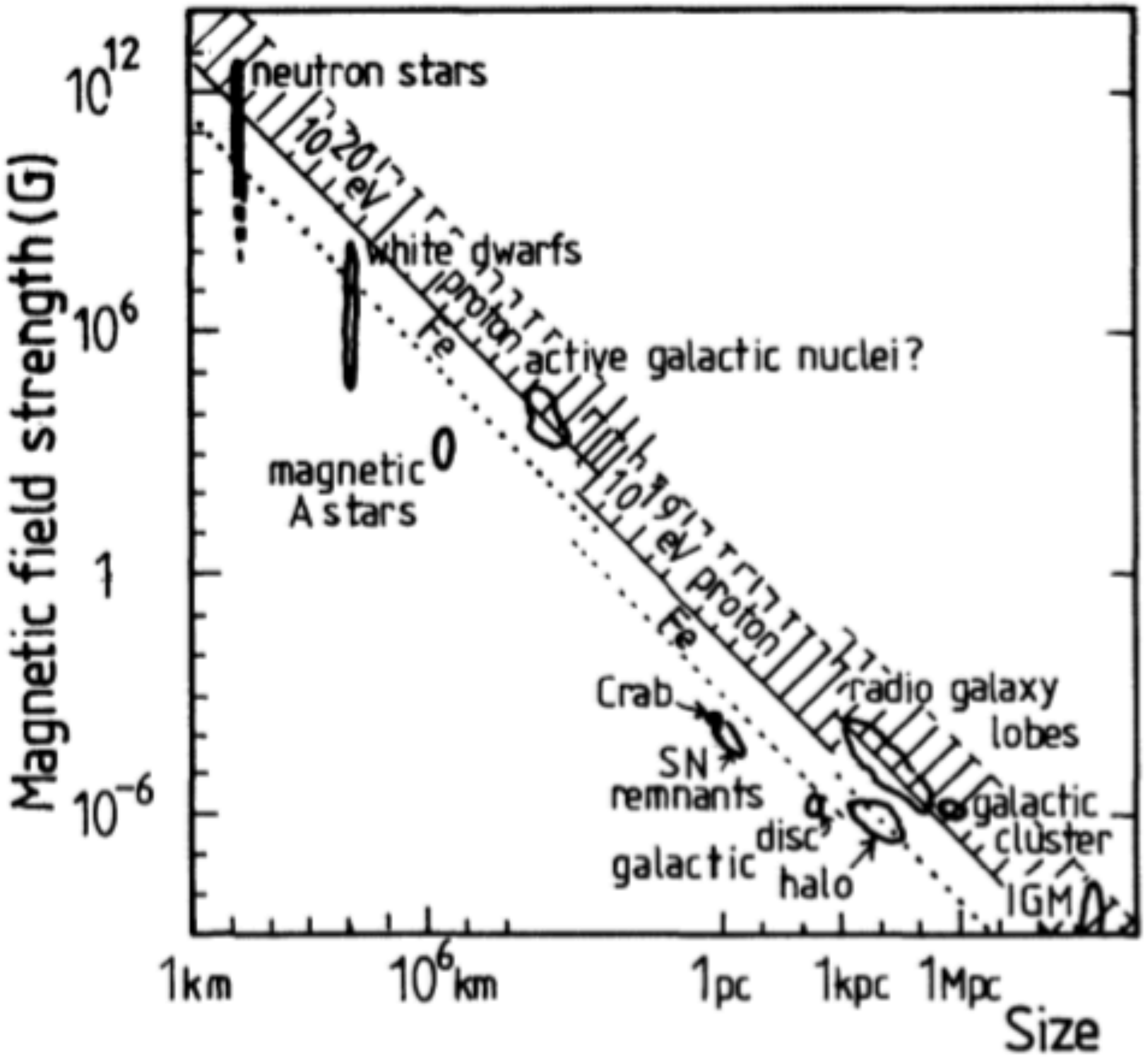
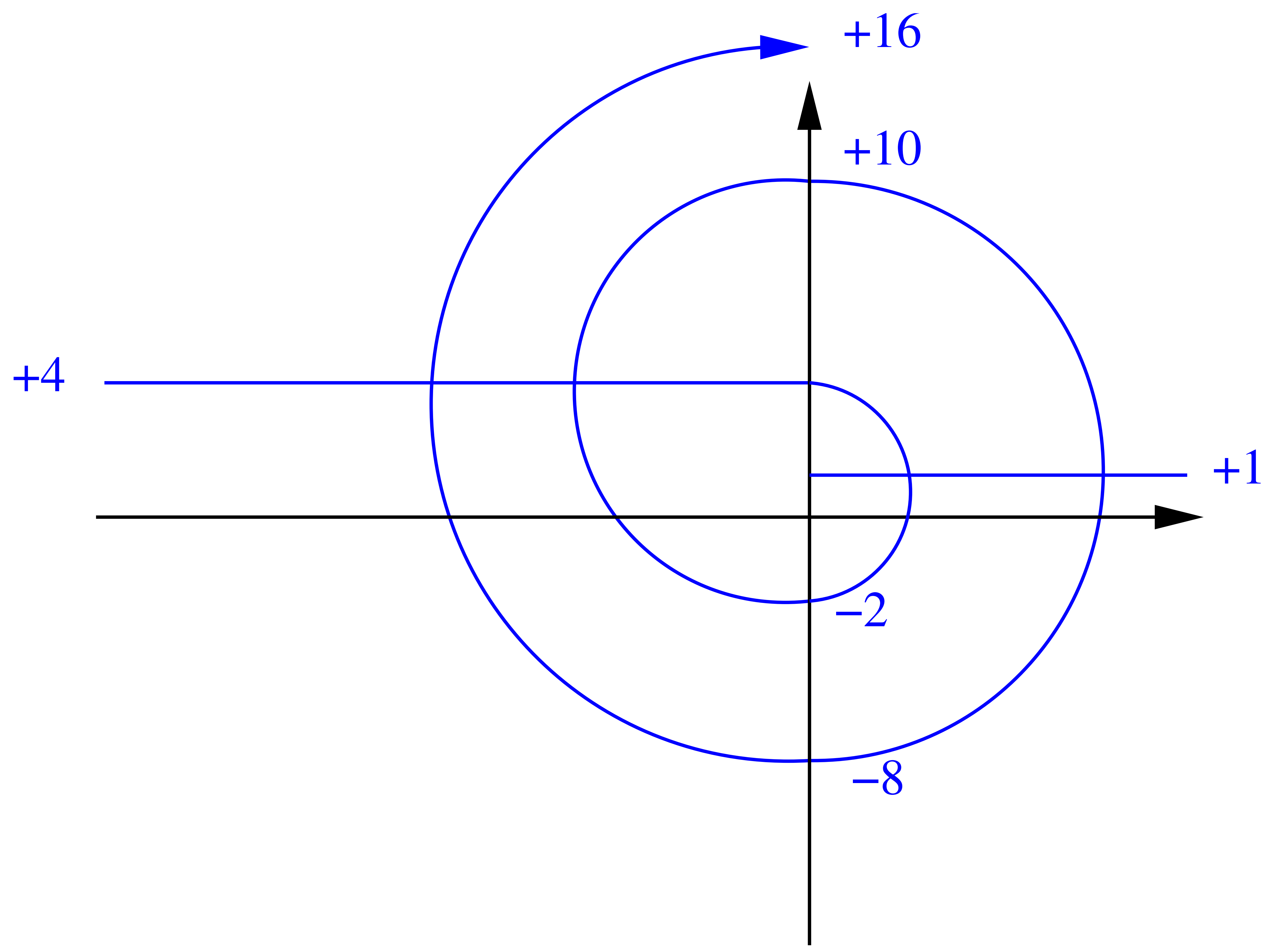


Figure 3. Size and magnetic field strength of sites where cosmic rays might possibly be accelerated to ultra-high energies. To accelerate protons to 10^{20} or 10^{19} eV (as indicated) the object must not lie below the diagonal band (lower edge of band if plasma is relativistic - upper edge if speeds are 1000 km s^{-1}).

Injection

- Second great advantage of DSA is that it does not need a separate injection process - the shock can directly inject particles into the acceleration process.
- Although distributions are anisotropic at these low energies, same basic process of shock crossing and magnetic scattering should occur.



Have back-streaming ions for compression > 2.

- Well known in hybrid simulations of collisionless shocks (and more recently in PIC simulations also).
- Few backstreaming ions then act as seed population for further acceleration.
- NB electron injection is much more complicated, but there are certainly possible processes which can produce sufficiently energetic electrons.

The Astrophysical Journal, 783:91 (17pp), 2014 March 10 doi:10.1088/0004-637X/783/2/91

SIMULATIONS OF ION ACCELERATION AT NON-RELATIVISTIC SHOCKS. I. ACCELERATION EFFICIENCY

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Received 2013 October 9; accepted 2014 January 13; published 2014 February 20

The Astrophysical Journal, 794:46 (12pp), 2014 October 10 doi:10.1088/0004-637X/794/1/46

SIMULATIONS OF ION ACCELERATION AT NON-RELATIVISTIC SHOCKS. II. MAGNETIC FIELD AMPLIFICATION

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Received 2014 January 31; accepted 2014 August 11; published 2014 September 23

The Astrophysical Journal, 794:47 (7pp), 2014 October 10 doi:10.1088/0004-637X/794/1/47

SIMULATIONS OF ION ACCELERATION AT NON-RELATIVISTIC SHOCKS. III. PARTICLE DIFFUSION

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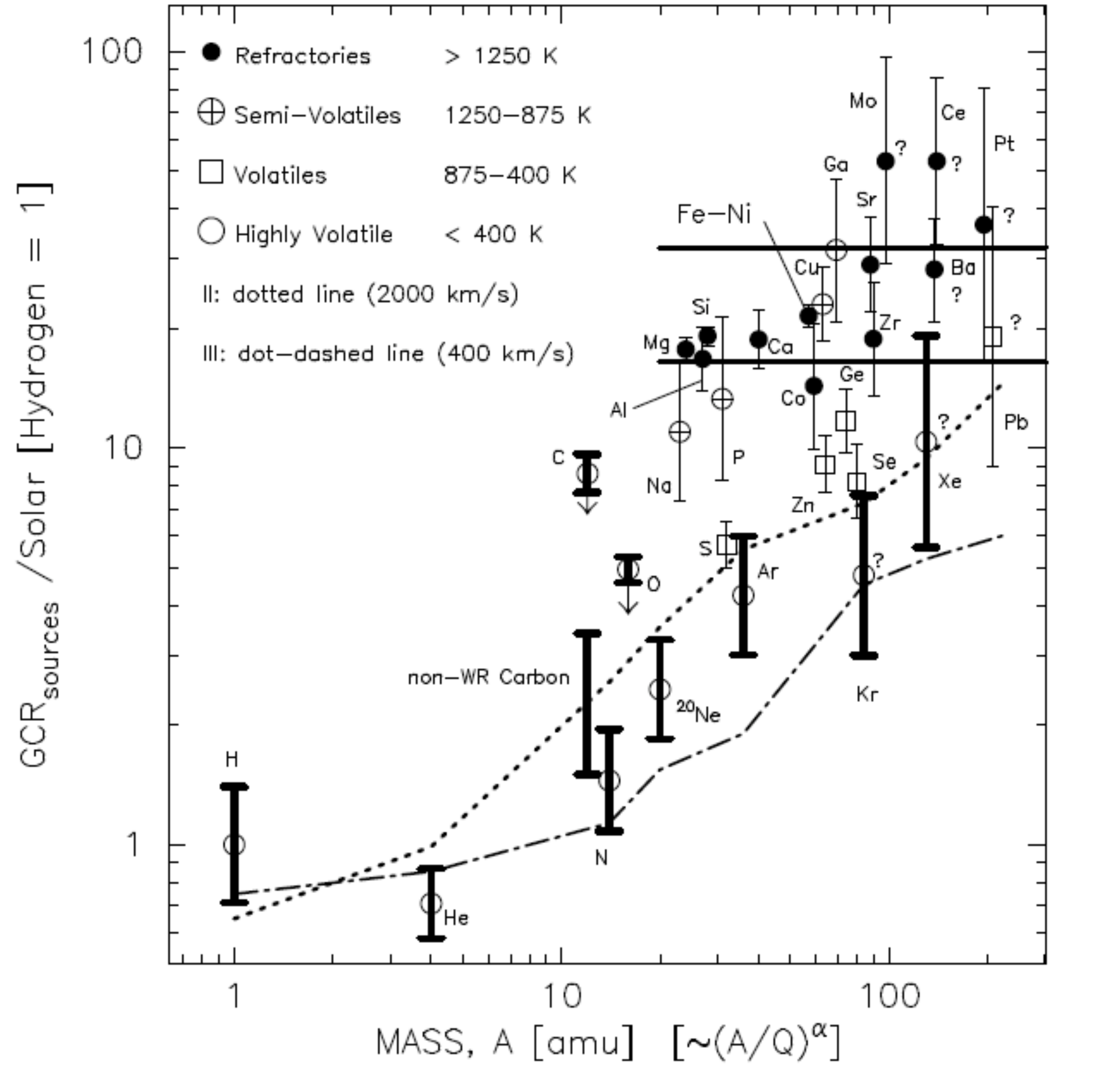
Received 2014 July 5; accepted 2014 August 11; published 2014 September 23

- Expect injection to be easiest for high rigidity species - compositional bias towards heavy ions.
- Fits qualitatively with the observed CR composition, but hard to make it work quantitatively unless dust is included.
- With limited acceleration and sputtering of dust grains can get very good fit to observed composition.



From Ellison, Drury and Meyer (1997) ApJ 487 197

- Real problem is to throttle back the injection of ions - easy to see that for typical SNR shocks if more than about 0.0001 of incoming protons become relativistic cosmic rays there is a severe energy problem!

For shock at 3000 km/s, 1% speed of light, mean kinetic energy per incoming particle is

$$10^{-4} m_p c^2$$

mean energy per CR several times $m_p c^2$

Conclusions

- Fermi's key insight, that differential motions of magnetised plasma can drive particle acceleration, remains fundamental.
- Additional key to DSA is that compression in physical space must drive expansion in momentum space (Liouville's theorem).
- Even if diffusive shock acceleration is the main game in town, second order Fermi has not gone away and must occur - just normally very slow.
- Many complications - see next lecture!