Finite-frequency noise cross correlations of a mesoscopic circuit: A measurement method using a resonant circuit

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The measurement of finite frequency cross correlations noise represents an experimental challenge in mesoscopic physics. Here we propose a generalization of the resonant LC circuit setup of Lesovik and Loosen which allow one to probe directly such cross correlations by measuring the charge fluctuations on the plates of a capacitor. The measuring circuit collects noise contributions at the resonant frequency of the LC circuit. Autocorrelation noise can be canceled out by switching the wires and making two distinct measurements. The measured cross correlations then depend on four nonsymmetrized correlators. This detection method is applied to a normal metal three terminal device. We subsequently discuss to what extent the measurement circuit can detect electron antibunching and what singularities appear in the spectral density of cross-correlations noise.

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I. INTRODUCTION

The measurement of finite frequency noise in mesoscopic systems provides a useful diagnosis of quantum transport: it allows one to characterize the carriers which are involved. In normal metal conductors,\cite{1,2} at zero temperature (and in the absence of spatial averaging effects\cite{3,4}), finite frequency correlations exhibit a singularity at \(\omega = eV\). In normal metal-superconductor junctions,\cite{5} Andreev reflection gives rise to a singularity at 2 eV signaling that Cooper pairs enter the superconductor, in the fractional quantum Hall effect, the tunneling of quasiparticles in the vicinity of a point contact leads to a singularity at \(eV\), with \(\nu\) the filling factor.\cite{6} At the same time, “zero” frequency noise cross correlations can be used to directly probe the statistics of the charge carriers: are the carriers bunched or antibunched?\cite{7} This implies a “Y” geometry where carriers are injected in one arm, and correlations are measured in the two receiving arms. The purpose of the present work is to discuss an inductive coupling setup which can be used toward the measurement of high frequency noise correlations in such geometries.

Any setup for measuring noise involves the selection (filtering) of a range of specific frequencies by the electronic detection circuit. Provided that the electrical apparatus in an experiment can sample the current at discrete times, finite frequency noise can in principle be computed directly from this time series. Yet, a noise correlator is a quantum mechanical average of products of operators. Symmetrized or nonsymmetrized correlators can thus be constructed from the time series, provided that the frequency is much smaller than the inverse time step of the series. This recipe for computing the noise is not practical at high frequencies. Here, we are interested in systems where noise is detected via a measuring device—coupled to the mesoscopic device which should pick up the noise contribution at a specific frequency, via repeated measurements. Such proposals have been put forth and some have been implemented experimentally within the last decade. Reference\cite{8} effectively uses a LC circuit in order to measure the noise of a two terminal diffusive conductor. A theoretical suggestion due to Lesovik and Loosen\cite{9} consists of a LC circuit coupled inductively to the fluctuating current emanating from a mesoscopic conductor: the measurement of the charge fluctuations on the capacitor plates provides information on the finite frequency noise correlators at the resonant frequency of the LC circuit. The fact that this method samples contributions from the nonsymmetrized noise correlators, which are related to emission and absorption from the device, has been emphasized in Ref.\cite{10}. Aguado and Kouvenhoven\cite{11} have proposed to measure the noise of an arbitrary circuit by coupling capacitively this circuit to a detector circuit: a DC current—generated by inelastic electron tunneling events—flows in the detector circuit when a “photon” \(\hbar\omega\), is provided/absorbed by the mesoscopic circuit. This theoretical suggestion has been successfully implemented to measure the finite frequency noise of a Josephson junction using a large superconductor-insulator-superconductor (SIS) junction:\cite{12} quasiparticles tunneling in the SIS junction can occur only if it is assisted by the frequency provided by the antenna. A subsequent experiment allowed one to isolate the emission and absorption contributions to noise.\cite{13} High frequency noise measurements have been also performed using the detection of photons emitted by the mesoscopic conductor.\cite{14} Such photons propagate in coaxial lines and are analyzed/detected in a Hanbury-Brown and Twiss geometry for microwave photons which appeared in order to measure two terminal noise with an auxiliary mesoscopic circuit. Turning to noise cross correlations, low frequency noise measurement in such branched circuits have been performed,\cite{15} which showed that a fully degenerate electron gas has negative noise correlations. Noise cross correlations have obvious applications in the study of electron transport in Hanbury-Brown and Twiss type geometries. Below, we will mention two situations where they are useful: the study of electronic entanglement in mesoscopic devices,\cite{16,17} and the identification of anomalous charges in Luttinger liquid wires.\cite{18} Here we consider the case of a setup where the noise cross correlations are analyzed via inductive coupling to the mesoscopic circuit to be measured.

The paper is organized as follows. In Sec. II, the model with a measuring circuit is introduced, and it is shown how
cross correlations can be directly obtained. In Sec. III, we compute the measured noise cross correlations. In Sec. IV, we show a model with two separated circuits and we compute the measured noise cross correlations. Our results, in the case of one circuit, are then applied in Sec. V to a simple device: a three terminals normal metallic sample, where we stress the difference between the symmetrized noise, the nonsymmetrized noise, and the measured noise. Section VI gives two examples where cross correlations are needed. We conclude in Sec. VII.

II. MODEL AND METHOD

For measuring cross correlations, two inductances \((L_1\) and \(L_2\)) and a single capacitor (C) are needed. The two inductors having coupling constants \(\alpha_1\) and \(\alpha_2\), are placed next to the two outgoing arms of the three terminal mesoscopic device (Fig. 1). We consider two cases: the two inductances are placed in series [Figs. 1(a) and 1(b)] or in parallel [Figs. 1(c) and 1(d)]. Depending on the wiring of these inductances [Figs. 1(a) and 1(c) or Figs. 1(b) and 1(d)], the two inductances “see” the outgoing currents with the opposite sign or with the same sign. Throughout this paper, we neglect the mutual inductance of the detector circuit: it is justified by the fact that such inductances are placed near opposite ends of the circuit whose noise correlations are measured. Classically, the charge \(x\) on the capacitor plates of the measuring circuit obeys the equation of motion:

\[
M \ddot{x}(t) = -D \dot{x}(t) - \alpha_1 \dot{I}_1(t) = \alpha_2 \dot{I}_2(t),
\]

where the “mass” \(M = L_1 + L_2\) if the circuit is in series [Figs. 1(a) and 1(b)] or \(M = L_1 L_2 / (L_1 + L_2)\) if the circuit is in parallel [Figs. 1(c) and 1(d)], and \(D = 1/C\). The characteristic frequency of the LC circuit is \(\Omega = \sqrt{D/M}\).

The currents which appear in the two coupling terms are \(I_{1(2)}(t) = I_{1(2)} e^{i\omega t} / \sqrt{\pi/2} \int_{-\infty}^{\infty} e^{i\omega t} dr\), where \(I\) is the length of the inductive coupling region. We assume in a standard way that the chemical potential of the leads is the largest energy scale, compared to the bias voltage and the frequency we want to probe. Spatial averaging effects for the currents have been discussed in Refs. 19 and 20: the current operator contains fast and slow oscillations, at least in the ballistic case. Here we claim to be measuring a current which is spatially independent of the integral over the inductive region \(I\). We first assume that \(I \gg \lambda_F\), so that short wave length oscillations are averaged out. Furthermore, the slowly oscillating terms in the current will be reduced to a constant contribution if we require that \(I \ll v_F / \omega\). Reference 3 has studied the spatial oscillations of the current in the presence of Coulomb interactions: interactions modify the wavelength \(\pi v_F / \omega\) of such oscillations, but the amplitude of its signal is reduced with increasing interaction strength.

The derivation of Ref. 9 is generalized to the two-inductance situation. To quantize the measuring circuit, we note that the equation of motion can be derived from the Hamiltonian:

\[
H = H_0 + H_{int} = \frac{p^2}{2M} + \frac{Dx^2}{2} + H_{int},
\]

with

\[
H_{int} = -\frac{p[\alpha_1 I_1(t) \pm \alpha_2 I_2(t)]}{M} + \frac{[\alpha_1 I_1(t) \pm \alpha_2 I_2(t)]^2}{2M}.
\]

The mesoscopic circuit plus measuring circuit are assumed to be decoupled at \(t = -\infty\). The coupling between the two is switched on adiabatically, and one can monitor the charge of the capacitor, and its fluctuations, in the presence of the fluctuating currents at time \(t = 0\): it is a stationary measurement.

The \(n\)th power of the capacitor charge reads, in the interaction representation:

\[
\langle x^n(0) \rangle = \text{Tr} [e^{-iH_0 U^{-1}(0)} x^n(0) U(0)],
\]

with the evolution operator

\[
U(0) = T \exp \left( -\frac{i}{\hbar} \int_{-\infty}^{0} dt' H_{int}(t') \right).
\]

We calculate perturbatively the \(n\)th power of the charge, expanding the evolution term \([U(0)]\) in powers of \(H_{int}\). Considering the average charge and the average charge square, one obtains:

\[
\langle x(0) \rangle = \langle x(0) \rangle_1 + \langle x(0) \rangle_3 + \cdots,
\]

\[
\langle x^2(0) \rangle = \langle x^2(0) \rangle_0 + \langle x^2(0) \rangle_2 + \cdots
\]

where the different orders in \(H_{int}\) are linked to the higher cumulants of the current: \(\langle x(0) \rangle_1\) contains information about the average current, \(\langle x^2(0) \rangle_2\) about the current fluctuations, and \(\langle x(0) \rangle_3\) about the third moment, and so on.

The zero order contribution of the charge fluctuations gives:
\[ \langle x^2(0) \rangle_0 = \frac{\hbar}{2M\Omega} [N(\Omega) + 1/2], \quad (2.8) \]

with \( N(\Omega) = 1/(e^{\beta\Omega} - 1) \) is the Bose Einstein distribution at the detector circuit temperature, which is not necessarily the mesoscopic device temperature. The next nonvanishing term, which depends on current of operators is:

\[ \langle x^2(0) \rangle_{2\Delta} = \left\langle \frac{1}{2M} \left( -\frac{i}{\hbar} \right)^2 \int_{-\infty}^{t_1} dt_1 \int_{-\infty}^{t_2} dt_2 \times \left[ \langle x(0), p(t_1) \rangle (\alpha_1 I_1(t_1) + \alpha_2 I_2(t_1)) \right] 
\times \left[ \langle x(0), p(t_2) \rangle (\alpha_1 I_1(t_2) + \alpha_2 I_2(t_2)) \right] \right\rangle, \]

\[ p(t) = i \sqrt{\frac{\hbar M \Omega}{2}} \left( a e^{i\Omega t} - a^\dagger e^{-i\Omega t} \right), \quad (3.2) \]

where \( a \) is the destruction operator which satisfies \( \langle a^\dagger a \rangle = N(\Omega) \). The first commutator in Eq. (2.9) becomes:

\[ \langle [x^2(0), p(t_1) I_1(t_1), p(t_2) I_2(t_2)] \rangle \]

\[ = -\hbar^2 \cos(\Omega t_1) (I_1 I_1(t_1)) \]

\[ \times \left[ \langle x(0), p(t_2) \rangle (\alpha_1 I_1(t_2) + \alpha_2 I_2(t_2)) \right] \]

\[ \times \langle N(\Omega) + 1 \rangle e^{i\Omega t_2} - N(\Omega) e^{-i\Omega t_2} \]

\[ \times (I_2 I_2(t_2)) \], \quad (3.4) \]

and the average of the two interlocked correlators of Eq. (2.9) reads:

\[ \langle [x^2(0), p(t_1) I_1(t_1), p(t_2) I_2(t_2)] \rangle \]

\[ = -\hbar^2 \cos(\Omega t_1) (I_1 I_1(t_1)) \]

\[ \times \left[ \langle x(0), p(t_2) \rangle (\alpha_1 I_1(t_2) + \alpha_2 I_2(t_2)) \right] \]

\[ \times \langle N(\Omega) + 1 \rangle e^{i\Omega t_2} - N(\Omega) e^{-i\Omega t_2} \], \quad (3.3) \]

with \( i,j=1,2 \) (i \( \neq \) j). Substituting this commutator in the expression of charge fluctuations, four correlators of current derivatives appear in the result. Translation invariance motivates the change of variables: \( \{t_1, t_2\} \rightarrow \{t_1-1/2, t_2+1/2\} \). The charge fluctuations become:

\[ \langle x^2(0) \rangle = \frac{\alpha_1 \alpha_2}{(4M)^2} \int_{-\infty}^{+\infty} dt \int_{-\infty}^{0} dT e^{i\Omega t} \left( e^{i\Omega t} + e^{-i\Omega t} \right) \]

\[ \times \left[ \langle x(0), p(t_1) \rangle (\alpha_1 I_1(t_1) + \alpha_2 I_2(t_1)) \right] \]

\[ \times \langle N(\Omega) + 1 \rangle (I_1 I_2(t_2)) + (I_2(t_1) I_2(t_2)) \]. \quad (3.5) \]

In a manner similar to Refs. 9 and 10, the following nonsymmetric current correlators are introduced in Fourier space:

\[ S_{ij}^c(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} (I_i(0) I_j(t)), \quad (3.6) \]

\[ S_{ij}^s(\omega) = \int \frac{dt}{2\pi} e^{i\omega t} (I_i(t) I_j(t)), \quad (3.7) \]

Translation invariance relates these two correlators: \( S_{ij}^c(\omega) = S_{ij}^s(-\omega) \), and furthermore \( I_i(\omega) I_j(\omega) = \delta(\omega_1 - \omega_2) S_{ij}^c(\omega) \), where \( I_i(\omega) \) is the Fourier transform of \( I_i(t) \). Note that only in the case of autocorrelation: \( S_{ii}^c(\omega) \) \( S_{ii}^s(\omega) \) can be identified as an emission (absorption) rate from the mesoscopic device at positive frequencies. At this point, both integrals in Eq. (3.5) can be performed. The integration over \( t \) gives two contributions: one is a delta function, and the other gives products of principal parts, which cancel out. The charge fluctuations take the final form:

\[ \langle x^2(0) \rangle = \frac{\pi \alpha_1 \alpha_2}{2\eta (2M)^2} \left( \langle N(\Omega) + 1 \rangle (S_{12}^c(\Omega) + S_{21}^c(\Omega)) \right. \]

\[ - N(\Omega) (S_{12}^c(\Omega) + S_{21}^c(\Omega)) \]. \quad (3.8) \]

To show that \( \langle x^2(0) \rangle \) is real, we use the properties of the noise correlators: \( [S_{ij}^c(\Omega)]^* = S_{ij}^s(\Omega) \), and the measured charge fluctuation reads:
The measurement of the product of the two charges. We calculate perturbatively the $n$th power of the charge, expanding the evolution term $[U(0)]$ in powers of $H_{\text{int,1(2)}}$. Considering the average charge and the average charge square, one obtains:

$$
\langle x_0(0) x_0(0) \rangle = \langle x_{1(2)}(0) \rangle + \langle x_{1(2)}(0) \rangle_3 + \cdots,
$$

(4.2)

$$
\langle x_1(0) x_2(0) \rangle = \langle x_1(0) x_2(0) \rangle_2 + \cdots,
$$

(4.3)

where the different orders in $H_{\text{int,1(2)}}$ are linked to the higher cumulants of the current: $\langle x_{1(2)}(0) \rangle_1$ contains information about the average current, the product $\langle x_1(0) x_2(0) \rangle_2$ about the current fluctuations, and $\langle x_{1(2)}(0) \rangle_3$ about the third moment, and so on.

The zero order contribution of the charge fluctuation $\langle x_1(0) x_2(0) \rangle_0$ becomes zero. The first nonvanishing term, which depends on products of current operators is:

$$
\langle x_1(0) x_2(0) \rangle_2 = \frac{1}{2} \int_{-\infty}^{0} dt_1 \int_{-\infty}^{t_1} dt_2
$$

$$
\times \left[ \begin{array}{cc}
-x_1(0) x_2(0) & -i \frac{p_1(t_1) I_1(t_1)}{h M_1} \\
-i \frac{p_2(t_1) I_2(t_1)}{h M_1} & -i \frac{p_2(t_2) I_2(t_2)}{h M_2} - i \frac{1}{h}
\end{array} \right]^2
$$

$$
\times \left[ \begin{array}{cc}
\left( \frac{\alpha I_1(t)}{M_1} \right)^2 & \left( \frac{\alpha I_2(t)}{M_2} \right)^2 \\
\left( \frac{\alpha I_2(t)}{M_1} \right)^2 & \left( \frac{\alpha I_2(t)}{M_2} \right)^2
\end{array} \right].
$$

(4.4)

The calculation of the charge fluctuations gives four terms: two autocorrelation terms which correspond to the fluctuations due to a single inductor (in terms of $\alpha_1^2$ or $\alpha_2^2$), and two terms associated with the correlation between the two inductors, which are proportional to $\alpha_1 \alpha_2$. In the two autocorrelation terms, we calculate the average of an odd number of creation and destruction operators for each circuit. The autocorrelation is zero. The combination of current cross correlators will be referred to from now on as the measured cross correlations.

Choosing a particular case where the two $LC$ circuits have the same characteristic frequency, the charge fluctuations correspond to the result obtained in the presence of a single capacitor with two inductances [see Eq. (3.8)]. In practice, it is challenging to build two $LC$ circuits with exactly the same characteristic frequency. The result of Eq. (3.8) for $\Omega_1 = \Omega_2$ has also appeared in Ref. 20. For the case of different frequencies, the charge fluctuations have a general form:
which result in delta function contributions, as well as (unwanted) principal parts: strictly speaking the charge fluctuations do not simplify. However, a practical measurement always involves some bandwidth averaging, which could possibly lead to a result close to that of the case $\Omega_1=\Omega_2$.

Furthermore, the use of two separate capacitors should also consider the multiplication process of the two charge operators. Multiplication of two signals involves adding and subtracting the two signals, then squaring and subtracting. In these steps of the measurement, the noise from the amplifiers will be detrimental in the same manner as in the case of a single LC circuit.

V. APPLICATION TO A THREE TERMINAL NORMAL CONDUCTOR

Next, we consider noise measurement with the single capacitor and two inductances circuit initially proposed. It is tested on a system of three terminals (Fig. 3), a so-called “Y junction,” electrons are injected from terminal 1, which has a higher chemical potential than terminals 2 and 3. The noise cross correlations are measured. This corresponds to the setup where a fermionic analog of Hanbury-Brown and Twiss type experiments was proposed and measured. Without loss of generality, one considers that the three bias voltages, $\mu_{ij}=\mu_i-\mu_j$ ($i,j=1,2,3$), are chosen such that $\mu_{13} > \mu_{12}, \mu_{23}$.

Using scattering theory, one can readily obtain the general expression for the nonsymmetrized finite frequency noise (Ref. 21):

$$S_{ab}^+(\omega) = \frac{e^2}{2\pi\hbar} \sum_{\gamma,\delta} \int dE [\delta_{\alpha\gamma} \delta_{\alpha\delta} - s^\dagger_{\gamma\delta}(E) s_{\alpha\delta}(E - \hbar \omega)]$$

$$\times [\delta_{\beta\gamma} \delta_{\beta\delta} - s^\dagger_{\beta\delta}(E) s_{\beta\gamma}(E - \hbar \omega)]$$

$$\times f_\gamma(E) [1 - f_\delta(E - \hbar \omega)],$$

(5.1)

where Greek letters represent the terminals and $s_{\alpha,\beta}$ is the scattering amplitude for electrons incoming from $\beta$ and ending in $\alpha$. $f_\gamma(E)$ is the Fermi-Dirac distribution function associated with terminal $\gamma$ whose chemical potential is $\mu_\gamma$. In what follows we assume that the temperature is much smaller than the applied biases. One also neglects the energy dependence of the scattering matrix over the energy ranges specified by the voltages biases $\mu_{ij}$. Furthermore, Eq. (5.1) neglects $\pm 2\hbar f_\gamma$ oscillating terms in the noise fluctuations spectrum: this assumes that the region over which current is measured is much larger than the Fermi wave length, $l \gg \lambda_F$.

For $\mu_{23} < \mu_{12}$ and at negative frequencies, cross correlations between terminals 2 and 3 yield:

$$S_{23}^+(\omega) = S_{32}^+(\omega)$$

$$= -\frac{e^2}{2\pi} \omega (T_{21}T_{13} - (2 - R_2 - R_3)T_{23})$$

$$- \frac{e^2}{2\pi\hbar} \left\{ \begin{array}{ll}
T_{13}R_{3}(\mu_{13} - \hbar \omega) - \hbar \omega (2T_{12}R_2 + 2T_{23}R_2) & \text{if } \hbar \omega < -\mu_{13}, \\
T_{13}R_{3}(\mu_{13} - \hbar \omega) - \hbar \omega (2T_{12}R_2 + 2T_{23}R_2) & \text{if } -\mu_{13} < \hbar \omega < -\mu_{12}, \\
T_{13}R_{3}(\mu_{13} - \hbar \omega) + T_{13}R_{3}(\mu_{13} - \hbar \omega) + T_{12}R_{2}(\mu_{12} - \hbar \omega) - \hbar \omega (2T_{23}R_2) & \text{if } -\mu_{12} < \hbar \omega < -\mu_{23}, \\
T_{13}R_{3}(\mu_{13} - \hbar \omega) + T_{13}R_{3}(\mu_{13} - \hbar \omega) + T_{12}R_{2}(\mu_{12} - \hbar \omega) & \text{if } -\mu_{23} < \hbar \omega < 0,
\end{array} \right.$$
\[ S^c_{23}(\omega) = S^c_{32}(\omega) = -\frac{e^2}{2\pi\hbar} \begin{cases} T_{23}R_2(\mu_{23} - \hbar\omega) + T_{13}R_3(\mu_{13} - \hbar\omega) + T_{12}R_2(\mu_{12} - \hbar\omega) & \text{if } 0 < \hbar\omega < \mu_{23}, \\ T_{13}R_3(\mu_{13} - \hbar\omega) + T_{12}R_2(\mu_{12} - \hbar\omega) & \text{if } \mu_{23} < \hbar\omega < \mu_{12}, \\ T_{13}R_3(\mu_{13} - \hbar\omega) & \text{if } \mu_{12} < \hbar\omega < \mu_{13}, \\ T_{12}R_2(\mu_{12} - \hbar\omega) & \text{if } \mu_{13} < \hbar\omega, \end{cases} \]

where \( R_\alpha = s^\dagger_\alpha s_\alpha \) is the reflection probability from lead \( \alpha \) and \( T_{\alpha\beta} = s^\dagger_\alpha s_\beta = T_{\beta\alpha} \) is the transmission probability from \( \alpha \) to \( \beta \).

As expected, the frequency dependence is given by a set of continuous straight lines with singular derivatives. At zero frequency, the nonsymmetrized cross correlations are:

\[ S^c_{23}(\omega = 0) = -\frac{e^2}{2\pi\hbar} (T_{23}R_2\mu_{23} + T_{13}R_3\mu_{13} + T_{12}R_2\mu_{12}). \]

(5.4)

The cross correlations are negative regardless of bias voltage and transmission of the sample: one recovers the result of Ref. 7.

On the other hand, the symmetrized finite frequency cross correlations are defined by:

\[ S^s_{23}(\omega) = \int d\omega e^{i\omega t} (\Delta I_2(t)\Delta I_3(0) + \Delta I_3(0)\Delta I_2(t)), \]

(5.5)

with \( \Delta I(t) = I(t) - \langle I(t) \rangle \). From Ref. 2, when \( \mu_{23} < \mu_{12} \) and at zero temperature, one obtains:

\[ S^s_{23}(\omega) = -\frac{e^2}{2\pi} \omega [T_{21}T_{13} - (2 - R_2 - R_3)T_{23}] - \frac{e^2}{2\pi\hbar} \begin{cases} + T_{23}R_2\mu_{23} + T_{13}R_3\mu_{13} + T_{12}R_2\mu_{12} & \text{if } |\hbar\omega| < \mu_{23}, \\ \hbar\omega(T_{23}R_2) + T_{13}R_3\mu_{13} + T_{12}R_2\mu_{12} & \text{if } \mu_{23} < |\hbar\omega| < \mu_{12}, \\ \hbar\omega(T_{12}R_2 + T_{23}R_2) + T_{13}R_3\mu_{13} & \text{if } \mu_{12} < |\hbar\omega| < \mu_{13}, \\ \hbar\omega(T_{13}R_3 + T_{12}R_2 + T_{23}R_2) & \text{if } \mu_{13} < |\hbar\omega|. \end{cases} \]

(5.6)

The frequency dependence of \( S^s_{23}(\omega) \) is symmetric in \( \omega \). The nonsymmetrized cross correlations, given by Eqs. (5.2) and (5.3), coincides with the symmetrized cross correlations at \( \omega = 0 \), as illustrated in Fig. 4. The nonsymmetrized cross correlations behavior is quite different from the symmetrized cross correlations, although the locations of their singularities are the same. The symmetrized and nonsymmetrized cross correlations are both negative. The nonsymmetrized cross correlations are monotonously increasing and equal to zero for \( |\hbar\omega| > \mu_{13} \).

We now consider the measured cross correlations, given by Eq. (3.8), as a function of temperature of the measurement circuit. Because of the symmetry of the transmission probabilities, we have in this case \( S^s_{23}(\omega) = S^c_{23}(\omega) \) (Refs. 9 and 10). With the above symmetry consideration, the charge fluctuations resemble the formula for autocorrelation noise:

\[ \langle \chi^2(0) \rangle = \frac{\alpha_1\alpha_2\sigma^2}{\eta(2M)^2} [S^c_{23}(\omega) + \Delta S_{23}(\omega)N(\omega)]. \]

(5.7)

Note that it is the difference of the two nonsymmetrized correlators \( \Delta S_{23}(\omega) = S^c_{23}(\omega) - S^c_{23}(-\omega) \) which is multiplied by the Bose distribution \( N(\omega) \):

\[ \Delta S_{23}(\omega) = -\frac{e^2}{2\pi} \omega [T_{21}T_{13} - (2 - R_2 - R_3)T_{23} - 2T_1R_3 - 2T_1R_2 - 2T_{23}R_2], \]

(5.8)

i.e., it is linear with frequency with a positive slope, and does not have any singularities.

Taking \( \mu_2 = \mu_3 \) for simplicity, the measured cross correlations are plotted in Fig. 5.

For temperatures such that \( k_BT > \mu_{13} \), the Bose distribution is large, \( N(\omega) \sim \frac{k_BT}{\hbar\omega} \gg 1 \). \( \Delta S_{23}(\omega)N(\omega) \) is thus larger than \( |S^c_{23}(\omega)| \). The effect of the increasing temperature is to change the sign of the charge fluctuations which becomes

FIG. 3. A system with three terminals (Y junction) with chemical potentials \( \mu_1, \mu_2, \mu_3 \) with \( \mu_1 \geq \mu_2 \geq \mu_3 \).
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positive, This may seem a change given the fact that one is computing the fluctuations of charge at the plates of a capacitor: recall that our measurement implicitly assumes two experiments with different wiring, whose results are subtracted one from another. The main message of Fig. 5 is that the sign of the measured correlation can be misleading if the temperature of the measuring device is too large: one can observe positive cross correlations in a normal fermionic fork although this is a system where antibunching is expected. For positive cross correlations in a normal fermionic fork, the mesoscopic circuit itself.

At a low temperature, \( k_B T \ll \mu_{13} \), and low frequency, \( \omega \ll \mu_{13} \), \( \Delta S_{23}(\omega)N(\omega) \) is smaller than \( |S_{23}(\omega)| \) and the measured signal remains negative (see Fig. 6). When the temperature goes to zero, the measured charge fluctuations equal \( S_{23}^+ \).

For completeness, the case where the voltage biases satisfy \( \mu_{13} > \mu_{12} > \mu_{23} > 0 \) is discussed (see Fig. 7). The fact that terminals 2 and 3 have a different chemical potential will lower the contribution of \( S_{23}^+(\omega) \) to \( \Delta S_{23}(\omega)N(\omega) \), and increase the amplitude of charge fluctuations. At low temperature, the charge fluctuations stay negative and singularities are present at frequencies equal to \( \mu_{23}, \mu_{12}, \mu_{13} \). When the temperature becomes larger than the bias voltages, the charge fluctuations become positive. However, when the temperature goes to zero, the charge fluctuations become equal to \( S_{23}^+(\omega) \). When the difference between the chemical potentials of terminals 2 and 3 increases the effect of temperature is more important at small temperature and the amplitude is bigger regardless of the effect of \( S_{23}^+(\omega) \) and it is less important for large temperatures.

VI. WHEN ARE FINITE FREQUENCY NOISE CROSS CORRELATIONS USEFUL?

When probing quantum nonlocality with a source of electrons (for instance a \( S \)-wave superconductor or any other source of electrons) with the help of Bell inequalities, one is confronted with the fact that particle number correlators must be converted into noise correlators. The particle number operator reads:

\[ \hat{N}_{12} = \hat{n}_{12} - \frac{1}{2}, \quad \hat{N}_{23} = \hat{n}_{23} - \frac{1}{2} \]

where \( \hat{n}_{12} \), \( \hat{n}_{23} \) are the number operators for particles 1-2 and 2-3, respectively. The noise correlator reads:

\[ \langle \hat{N}_{12}\hat{N}_{23} \rangle = \langle \hat{n}_{12} \hat{n}_{23} \rangle - \frac{1}{2} \]

The noise correlator is related to the correlation function by:

\[ \langle \hat{N}_{12} \hat{N}_{23} \rangle = S_{23}(\omega)N(\omega) \]

where \( S_{23}(\omega) \) is the coherence function and \( N(\omega) \) is the noise spectrum. The coherence function is defined as:

\[ S_{23}(\omega) = \langle \hat{n}_{12} \hat{n}_{23} \rangle \]

and the noise spectrum is defined as:

\[ N(\omega) = \frac{1}{2} \left( \langle \hat{n}_{12} \hat{n}_{23} \rangle + \langle \hat{n}_{12} \rangle \langle \hat{n}_{23} \rangle \right) \]

\[ = \frac{1}{2} \left( S_{23}(\omega) + 1 \right) \]

The coherence function is related to the correlation function by:

\[ \langle \hat{n}_{12} \hat{n}_{23} \rangle = \langle \hat{n}_{12} \rangle \langle \hat{n}_{23} \rangle + S_{23}(\omega)N(\omega) \]

When the noise correlator is small, the coherence function becomes equal to the noise spectrum. This is the case for large temperatures and low frequencies. However, when the noise correlator is large, the coherence function becomes equal to the correlation function. This is the case for small temperatures and high frequencies. The noise correlator is used to study quantum nonlocality and quantum correlations in mesoscopic circuits. The noise correlator is also used to study the effect of temperature and frequency on quantum correlations.

FIG. 6. (Color online) Same as Fig. 5, for a smaller frequency range and at lower temperatures (in units of \( \mu_{13} \)).
\[ N_a(\tau) = \int_0^\tau I_\alpha(t')dt' = \langle N_a(\tau) \rangle + \delta N_a(\tau), \]  

(6.1)

where \( I_\alpha(t) \) is the current operator in lead \( \alpha \). The irreducible particle number correlator is expressed in terms of the finite frequency shot noise power:

\[ \langle \delta N_a(\tau) \delta N_{\beta}(\tau) \rangle = (1/2\pi) \int_{-\infty}^{\infty} d\omega S_{a,\beta}(\omega)4 \sin^2(\omega\pi/2)/\omega^2. \]

(6.2)

It is therefore only in the limit of relatively “large” acquisition times that the Bell inequality can be cast in terms of zero frequency correlations only,\(^24,25\) thus the need in general to measure in general finite frequency noise correlations. This is implicit in the work of Ref. 26, where entanglement occurs with a normal source of electrons, provided that short time dynamics can be analyzed. Reference 24 missed this subtlety concerning normal electron sources.

Another situation of interest for finite frequency noise cross correlations deals with the detection of anomalous (noninteger charges) in a carbon nanotube. A Hanbury-Brown and Twiss experiment has been proposed where a scanning tunnel microscope (STM) tip injects an electron in the bulk of the nanotube and noise cross correlations are measured at the extremities of the nanotube.\(^18,27\) In Ref. 27, the case of an infinite nanotube has been considered. Schottky-type relations for the zero-frequency Fourier transforms of the autocorrelation noise and the noise cross correlations were derived within the Tomonaga-Luttinger model:

\[ S_{auto}(\omega = 0) = \frac{1 + (K_+)^2}{2} e^{i\langle I(x) \rangle}, \]

(6.3)

\[ S_{cross}(\omega = 0) = \frac{1 - (K_+)^2}{2} e^{i\langle I(x) \rangle}, \]

(6.4)

where \( \langle I(x) \rangle \) is the charge current through the nanotube and \( K_+ \) is the Luttinger liquid interaction parameter. \( S_{auto} \) is the term of autocorrelations and \( S_{cross} \) is the cross-correlations term. However, in the presence of electrical contacts at the extremities of the nanotube, the zero-frequency Fourier transforms for noise and cross correlations lose their \( K_+ \)-dependence: at order 2 in the perturbative expansion with the tunneling amplitude from the tip to the nanotube, they reduce to \( S_{auto}(\omega=0) = e^{i\langle I(x) \rangle} \) and \( S_{cross}(\omega=0) = 0 \). It has been shown in Ref. 18, that one has to consider finite-frequency Fourier transform in order to recover nonzero cross correlations and Coulomb interaction effects in such a system.

VII. CONCLUSION

In summary, we have shown that the noise cross correlations of a mesoscopic circuit can be measured by coupling the latter to a resonant circuit which is composed of two inductances and a capacitor. Each inductance is attached to the arms where correlations are measured. As in Ref. 9, the proposed noise measurement is made by monitoring the charge fluctuations on the capacitor. Two distinct measurements, with different wiring of the circuit, are necessary to isolate the noise cross correlations.

Granted, we have not described the experimental apparatus which is needed to convert the signal into classical information, i.e., the amplification scheme: this goes beyond the scope of the present paper, whose goal it is to convey the information that noise cross correlations can be converted into a charge signal on a capacitor. A general approach for analyzing the quantum behavior of electronic circuitry has been proposed,\(^29\) allowing one to treat both nonlinear and dissipative elements, hence useful for signal conversion. The transition from mesoscopic to macroscopic quantum transport has been addressed in the context of scattering theory.\(^29\)

Note that in our analysis of the setup, the variable \( \hat{x} \) denotes the current flowing in the capacitor’s circuit. It is assumed to be a constant current in space. This description may be inappropriate at high frequencies, setting some upper limit to the frequencies we want to probe.

Note that the quality of the diagnosis of cross correlations presented in this work depends, to a large extent, on how the mesoscopic circuit is perturbed when one switches from one wiring configuration to the other. Indeed, it is necessary to minimize the changes in configuration (the values of \( \alpha_1 \) and \( \alpha_2 \) ) of the circuit which occurs between the two measurements. Here one could assume ideally that the inductances are built “on chip” with the mesoscopic circuit, or with the wires connected to it, which is a challenge in practice: small inductors working at high frequencies (\( \omega = 100 \text{ GHz} \)) may be difficult to achieve. While it may be more realistic to couple the two circuits away from the mesoscopic device, on-chip inductances of small scale are nevertheless used in qbit circuitry.\(^30\) Moreover, the wires of the measurement circuit are assumed to have a low impedance compared to that of the mesoscopic sample: within these working conditions the change of wiring does not significantly affect the inductive couplings, and the measurement will be reliable.

As a first necessary application of this measurement scheme, we considered a three terminal normal metal conductor, which is known to exhibit negative noise correlations at zero frequency. The symmetrized noise differs strongly from the nonsymmetrized noise and the measured noise. For a mesoscopic circuit at zero temperature, the nonsymmetrized noise contains singularities at frequencies corresponding to the chemical potential differences. However, when considering the measured noise, care must be taken to work with a detector circuit whose temperature is below these relevant biases. In this case the singularities in the derivative can still be detected, and upon slowly increasing the temperature, the measured noise deviates from the nonsymmetrized noise. This provides a condition for the observation of negative noise correlations—electron antibunching—in such three terminal devices. Because of the above mentioned temperature effects of the device, the amplifiers needed for signal conversion would need to be cooled down in order to avoid such problems.

The present analysis can be extended to study the measured noise correlations in other mesoscopic devices, in particular three terminal structures where electrons are injected in one lead from a superconductor.\(^31\) Finite frequency noise...
correlations in this case contain information of the time dynamics of the two electrons subject to crossed-Andreev transport.

APPENDIX

We discuss the case where the resonant LC circuit has a finite line shape. For a distribution of oscillators, the charge operator now takes the form:

\[ x(t) = \sum_\omega x_\omega(t) = \sum_\omega \sqrt{\hbar/2M} \omega (a_\omega e^{-i\omega t} + a_\omega^\dagger e^{i\omega t}), \]

where the interaction coupling, \(-\alpha ip/M\), becomes \(-i\sum_\omega \alpha_\omega p_\omega/M\).

These expressions are substituted in the interlocked commutators Eq. (2.9), which now contains a sum over 4 frequencies \(\omega_1, \omega_2, \omega_3, \) and \(\omega_4\). Because of time integrations, delta functions appear and give equalities between the frequencies. We are left with two summations:

\[
\langle [x^2(0), p(t_1) I(t_1)], p(t_2) I(t_2)] \rangle
= \sum_{\omega_1, \omega_2} \hbar^2 \cos(\omega t_1)
\times \langle [I(t_1) I(t_2)] [N_{\omega_1} e^{-i\omega_1 t_2} - (N_{\omega_1} + 1) e^{i\omega_1 t_2}] - [I(t_2) I(t_1)] [N_{\omega_2} + 1] e^{-i\omega_2 t_2} - N_{\omega_2} e^{i\omega_2 t_2}] \rangle.
\]

We substitute this expression in the charge fluctuations and proceed with the change of variable, we make use of the time translational invariance: \(t = t_1 - t_2\) and \(T = t_1 + t_2\) and \(\langle I(t) I(t)\rangle = \langle I(0) I(t)\rangle\). Making use of the definitions of Eqs. (3.6) and (3.7), the integral over \(T\) leads to two contributions:

\[
K_1 = \int_{-\infty}^{\infty} dT e^{i(\omega_1 + \omega_2) T/2} e^{\eta T + i(\omega_1 - \omega_2) \operatorname{sign}(t) T/2}
= \frac{e^{i(\omega_1 + \omega_2) t/2}}{\eta + i(\omega_1 - \omega_2) \operatorname{sign}(t)/2},
\]

\[
K_2 = -\int_{-\infty}^{0} dT e^{i(\omega_1 + \omega_2) T/2} e^{\eta T - i(\omega_1 + \omega_2) \operatorname{sign}(t) T/2}
= -\frac{e^{i(\omega_1 + \omega_2) t/2}}{\eta - i(\omega_1 + \omega_2) \operatorname{sign}(t)/2}.
\]

The line shape \(L(\omega - \Omega)\), which is sharply peaked around the resonant circuit frequency \(\Omega\), is introduced when converting discrete sums over frequencies into integrals. The integral over \(t\) is performed subsequently.

The denominators of \(K_1\) and \(K_2\) yield a real part which are a principal part and an imaginary part which are a Dirac delta function. We obtain four contributions for the charge fluctuations:

\[
\langle x^2(0) \rangle = A_1 + A_2 + A_3 + A_4,
\]

where

\[
A_1 = 4 \pi^2 \int d\omega L_2(\omega - \Omega) \frac{\alpha_1 \alpha_2}{(2M)^2} \times \text{Re}[(N_{\omega_1} + 1) S_{12}^+(\omega) - N_{\omega_2} S_{12}^-(\omega)],
\]

\[
A_2 = -2 \int d\omega_1 d\omega_2 d\omega_3 L(\omega_1 - \Omega) L(\omega_2 - \Omega) \frac{\alpha_1 \alpha_2}{(2M)^2} \times \text{Re}[(N_{\omega_1} + 1) S_{12}^+(\omega_3) - N_{\omega_2} S_{12}^-(\omega_3)],
\]

\[
A_3 = 4 \pi^2 \int d\omega L(\omega - \Omega) L(-\omega - \Omega) \frac{\alpha_1 \alpha_2}{(2M)^2} \times \text{Re}[(N_{\omega_1} + 1) S_{12}^+(-\omega) - N_{\omega_2} S_{12}^-(\omega)],
\]

\[
A_4 = -2 \int d\omega_1 d\omega_2 d\omega_3 L(\omega_1 - \Omega) L(\omega_2 - \Omega) \frac{\alpha_1 \alpha_2}{(2M)^2} \times \text{Re}[(N_{\omega_1} + 1) S_{12}^+(\omega_3) - N_{\omega_2} S_{12}^-(\omega_3)],
\]

where the function \(P\) gives the principal part. The contribution which dominates is the contribution where the two line shape functions \(L(\omega - \Omega)\) are peaked at the same frequency, i.e., Eq. (A6). The quantity \(\eta\) which appears in the single oscillator model thus corresponds physically to the width of the line shape, as expected.
21 T. Martin, in Nanophysics: Coherence and Transport, edited by H. Bouchiat, Y. Gefen, S. Gueron, G. Montambaux, and J. Dali-