Simultaneous Identification of the Diffusion Coefficient and the Potential for the Schrödinger Operator with only one Observation

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Abstract

This article is devoted to prove a stability result for two independent coefficients for a Schrödinger operator in an unbounded strip. The result is obtained with only one observation on an unbounded subset of the boundary and the data of the solution at a fixed time on the whole domain.

Let $\Omega = \mathbb{R} \times (d, 2d)$ be an unbounded strip of $\mathbb{R}^2$ with a fixed width $d > 0$. Let ν be the outward unit normal to Ω on $\Gamma = \partial\Omega$. We denote $x = (x_1, x_2)$ and $\Gamma = \Gamma^+ \cup \Gamma^-$, where $\Gamma^+ = \{x \in \Gamma;\ x_2 = 2d\}$ and $\Gamma^- = \{x \in \Gamma;\ x_2 = d\}$. We consider the following Schrödinger equation

$$\begin{cases} Hq := i\partial_t q + a\Delta q + bq = 0 \ \text{ in } \ \Omega \times (0, T), \\ q(x, t) = F(x, t) \ \text{ on } \ \partial\Omega \times (0, T), \\ q(x, 0) = q_0(x) \ \text{ in } \ \Omega, \end{cases} \tag{0.1}$$

where a and b are real-valued functions such that $a \in \mathcal{C}^3(\overline{\Omega})$, $b \in \mathcal{C}^2(\overline{\Omega})$ and $a(x) \geq a_{min} > 0$. Moreover, we assume that a is bounded and b and all its derivatives up to order two are bounded.
Our problem can be stated as follows:
Is it possible to determine the coefficients a and b from the measurement of $\partial_\nu(\partial_t^2 q)$ on Γ^+?

Let q (resp. $\widetilde{q}$) be a solution of (0.1) associated with $(a,\ b,\ F,\ q_0)$ (resp. $(\widetilde{a},\ \widetilde{b},\ F,\ q_0)$). We assume that q_0 is a real valued function.
Our main result is

$$\begin{aligned} \|a - \widetilde{a}\|^2_{L^2(\Omega)} + \|b - \widetilde{b}\|^2_{L^2(\Omega)} \quad &\leq \quad C\|\partial_\nu(\partial_t^2 q) - \partial_\nu(\partial_t^2 \widetilde{q})\|^2_{L^2((-T,T)\times\Gamma^+)} \\ &+ \quad C\sum_{i=0}^{2} \|\partial_t^i (q - \widetilde{q})(\cdot, 0)\|^2_{H^2(\Omega)}, \end{aligned}$$

where C is a positive constant which depends on (Ω, Γ, T) and where the above norms are weighted Sobolev norms.